Introduction to Probability Theory and Stochastic Processes

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Lecture - 03

Now, we are going to discuss some important properties of the probability.

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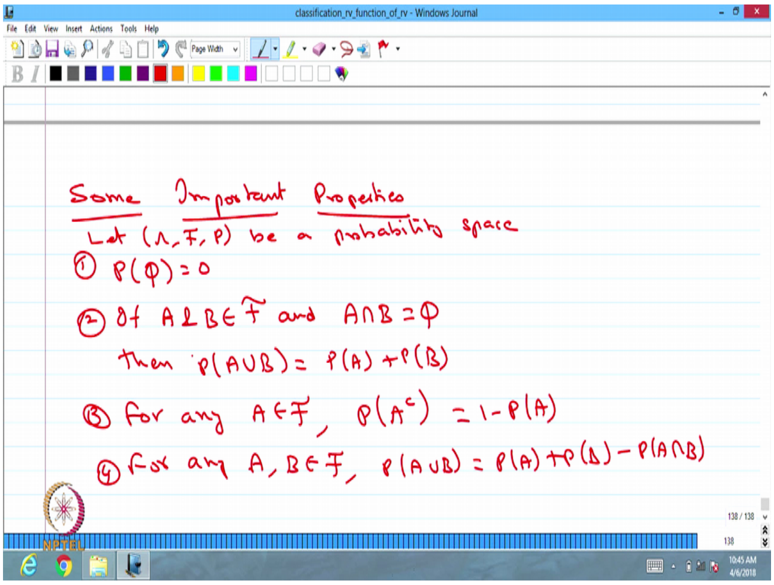

Some important properties: the first property. So, before that let me start with let omega be capital f P be a probability space, the first properties P of empty set is 0; in the probability is space definition we have a P of A omega is equal to 1. So, one can prove P of empty set is 0, because empty set is the magnesia of the whole set the whole set probability is 1. Therefore, this probability is going to be 0.

Second property if a two events A and B belonging to F and both are mutually disjoint events intersection of a events is empty. Then if you want to find out the P of A union B, that is same as a P of A plus P of B. If 2 events are mutually exclusive mutually disjoint events, then the P of A union b that is same as P of A plus P of B otherwise in general for any 2 events that is P of A union B is P of A plus P of B minus P of A intersection B. Since P A inter section B is empty set and the P of empty set is 0.

Therefore, P of A union B is same as P of A plus P of B. Third result for any event A belonging to F, P of A compliment that is same as 1 minus P of A if you know the probability of A, if you want find out the probability of A compliment that is same as 1 minus P of A the same concept is easy to find out the probability of empty set is 0 also.

The fourth result for any 2 events A coma B belonging to F, you can always have a P of A union b that is same as P of A plus P of B minus P of A intersection B. As in the result 2 and A intersection B is empty set, then it is going to be just P of A plus P of B there is no minus P of A intersection b then next result.

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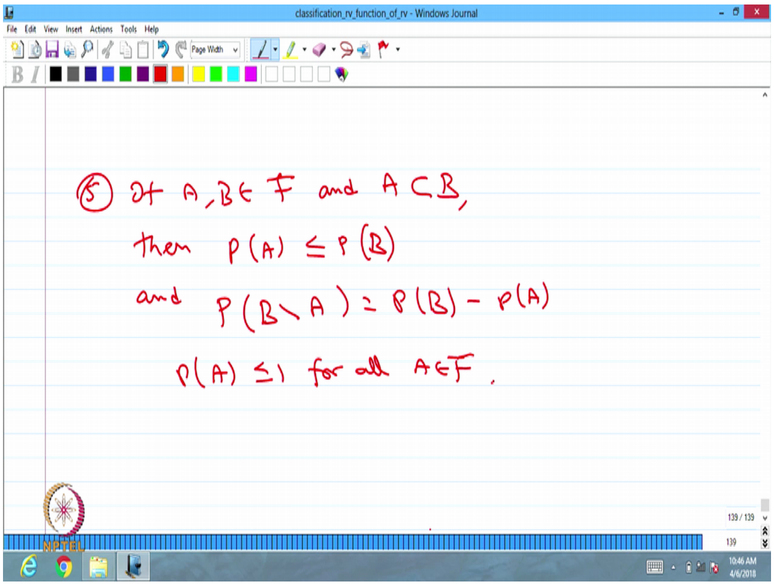

Site number 5 if A coma B belonging to the F and A contained B, then one can conclude P of A always less than or equal to P of B. Not only that P of B slash A that is same as P of B minus P of A, in particular P of A is always less than equal to 1 for all A belonging to F.

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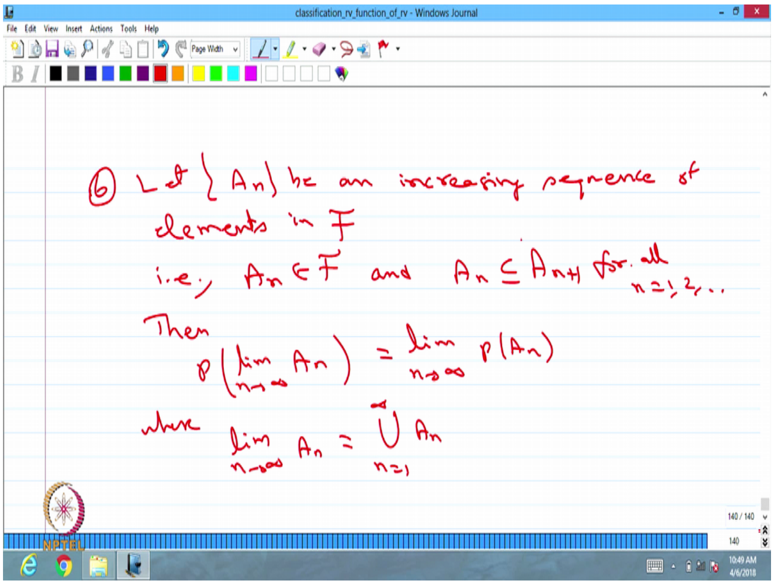

Then next result let A suffix n be an increasing seakales of events or elements in F, the element in the F is nothing but the events. So, let A and B increasing sequence of a elements in F.

That is if you take A n belonging to F and A n is contained in A n plus 1 for all n 1 2 3 and so on that is the meaning of A n B n increasing sequences of elements F. Then what the result says then P of limit n tends to infinity A n that is same as the limit n tends to infinity, P of A n where the limit n tends to infinity A n is nothing but; since A ns are the increasing sequence of elements limit n tends to infinity A n is nothing but union of n is equal to 1 to infinity A n similarly one can go for sequence of a elements which are a decreasing.

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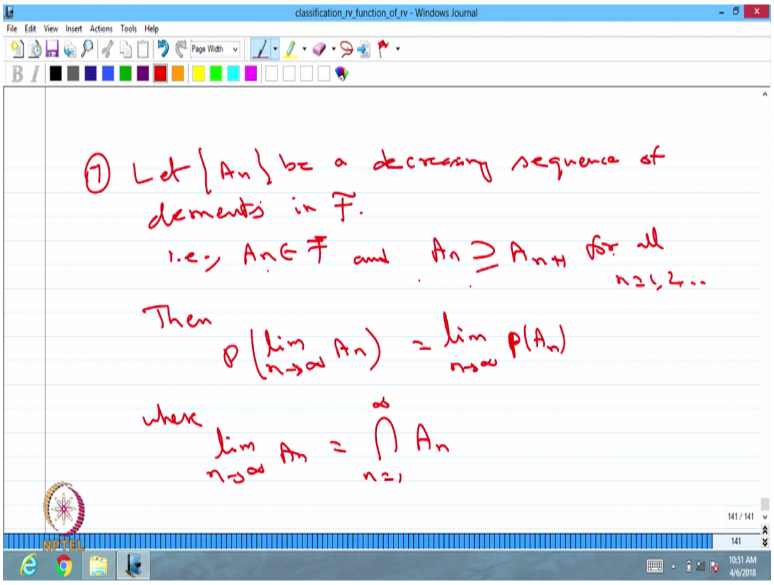

So, the next result or next property that is let A n be decrease a decreasing sequence of a elements in F that is A n belonging to F and it satisfies for all n is equal to 1 2 and so on. In this case P of limit n tends to infinity A n, that is same a again limit n tends to infinity of P of A n, where the limit n tends to infinity A n is nothing but intersection of n is equal to 1 to infinity of A ns, because a ns are the decreasing sequences of elements in f and not giving the proof of this results, but it can be proved easily.

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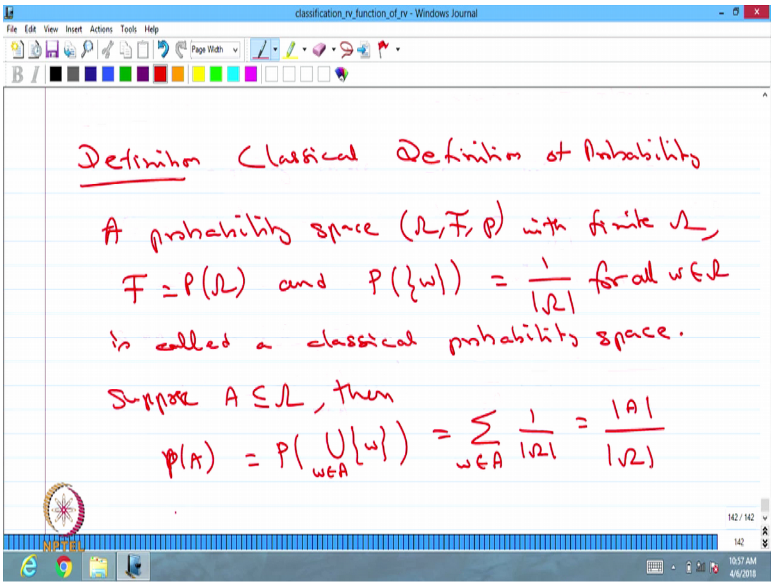

Now, we are moving into the next concept or next definition that is called a classic definition of probability. A probability space that is omega capital F capital P with finite omega coma the F is equal to power set of a omega the P of omega means power of omega and the probability measure of a singleton element w, that is equal to 1 divided by the number of elements in the omega for all w belonging to omega, that is called a classical probability a classical probability space.

Because we are defining a probability space here probability space omega F P, with the conditions on omega f as well as P, which basically as special case of a probability space by satisfying omega is finite and F is a largest sigma of field which is a power set of omega and the probability measured P is defined on each possible outcome as a 1 divided by total number of elements in the omega, which is finite for all w belonging to omega. If these 3 conditions are satisfied by a probability space, then that probability space is called a classical probability space, there is the another name for these that is called Laplace probability space.

In this situation the probability P is called a classical probability, the probability measure P which is defined in this probability space with omega is finite and f is a power set of omega and P on each possible outcome is equality that is 1 divided by number of elements the omega. Then this probability measure is called a classical probability measure and the probability space is called a classical probability space. So, the classical probability space is a special case of the probability space, which we have defined it earlier that is called asymmetric definition of probability the special case is the classical probability space.

Since, a P on each possible outcome is going to be a 1 divided by total number of elements in the omega. So, the probability of any event is nothing but number of favorable cases on event a divided by the total possible outcomes that is a whatever the finite number. Suppose, A is contained in omega where A is event then P of A is nothing but a P of A union of a singleton elements samples, where all the samples belonging to capital A the event that is nothing but summation of w belonging to A, 1 divided by that is same as cardinality of A divided by cardinality of omega.

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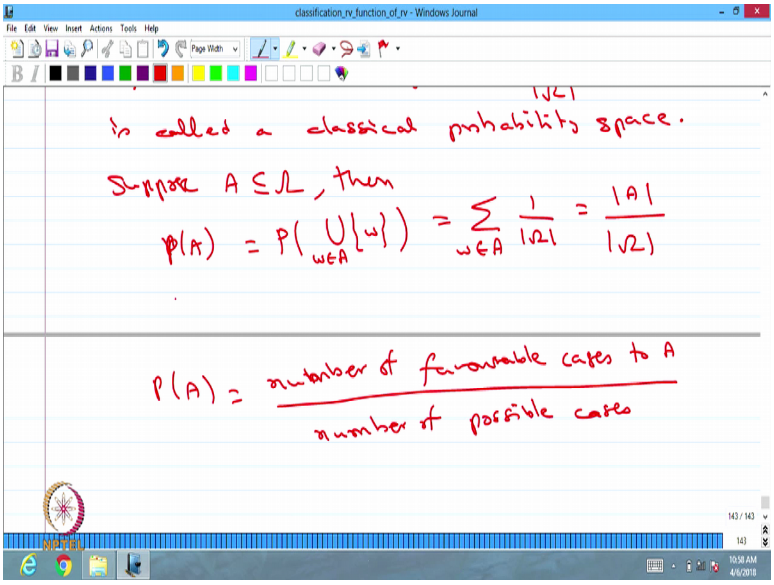

In other words P of A is nothing but number of favorable cases to the event A divided by number of a possible cases. So, this is what we have done it in the school days when we are computing the probability of event that is nothing but the number of favorable cases to the event A divided by the total number of all possible outcomes.

So, this is going to be the classical probability, with the assumption that omega is finite and sigma filled is the largest sigma filled which is nothing but the power set and equi probable outcomes. That means, a P of a each possible outcome that is same as the 1 divided by cardinality of omega. So, this is called the classical probability space.