Introduction to Probability Theory and Stochastic Processes

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Lecture – 28

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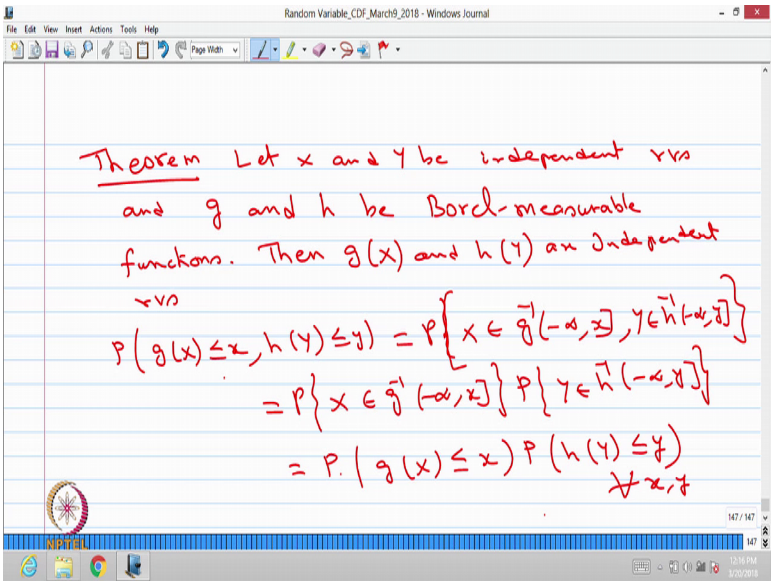

When 2 random variables are independent, I am going to give it as one important result as a theorem. Let X and Y be a independent random variables, and g and h be Boral measurable functions, then we will conclude after we get the results. So, let me keep the then word as it is let us go for finding out what is the probability of, what is the probability of g of X less than or equal to x with h of Y less than or equal to y.

Let us go for finding out function of a random variable with x in the form of g of X, function of a random variable with the random variable Y. In the form of h of Y we will try to find out the joint CDF of are CDF of the random variable g of X and h of Y. Since x is a random variable g is the Boral measurable function therefore, g of X is a random variable so, you can think of g of X is some other random variable. Similarly Y is a random variable h is a Boral measurable function therefore, h of Y is also a random variable.

So, you can think of some other random variable; that means, we can treat g of x is a another random variable, third random variable other than X and Y, h of y is a 4th random variable. We are trying to find out what is the joint CDF of this 2 random variable, whether that satisfies a the independent condition.

If they are going to be satisfies the independent condition, then you can conclude g of x and h of y are also independent random variable. That is probability of x belonging to g inverse of minus infinity to x closed interval, and since h of y is less than or equal to small y this also can be written y belonging to h inverse of minus infinity to small y closed interval. That is same as the probability of the way we write X belonging to g inverse Y belonging to h inverse.

We know that the random variable X and Y are independent, since X and Y are independent X belonging to some Boral set, Y belonging to some Boral set of the probability of that is same as probability of x belonging to the Boral set, that is g inverse of minus infinity to small x. Similarly, the product of probability of y belonging to h inverse of minus infinity small y because X and Y are independent, that is same as the probability of g of X is less than or equal to small x multiplied by probability of h of capital Y less than or equal to small y. And this is valid for all x and y.

This is valid for all x comma y ; that means, the probability of joint CDF is same as a probability of g of x and probability of h of y less than or equal to x and less than or equal to y respectively. Since this condition is satisfied we can conclude the random variables g of X and h of Y or independent random variables. This is a very important result whenever you have a independent random variable, if you create a Boral measurable functions on those dependent random variable, that is also going to be a independent random variables. As a example we can think of already we discussed a 3 examples.

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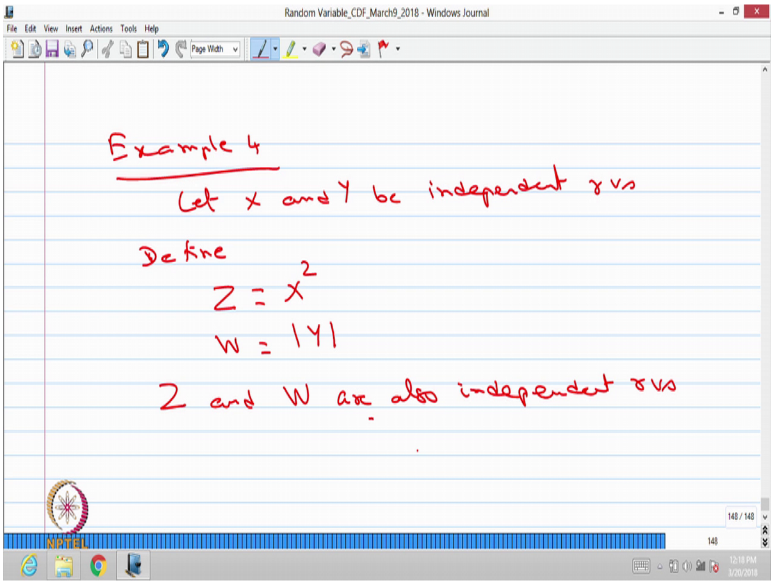

So, the 4th example is a let X and Y be independent, random variables, define define Z that is equal to X square W is equal to mod y; that means, I am creating a g of X as a X square, and h of y as the mod y, and since this 2 are Boral measurable functions g of x and h of y form, we can conclude z and w are also independent random variables. Since X and Y are in independent random variable, Z is a Boral measurable function of x w is a Boral measurable function of y, therefore, Z and W are also independent random variables.

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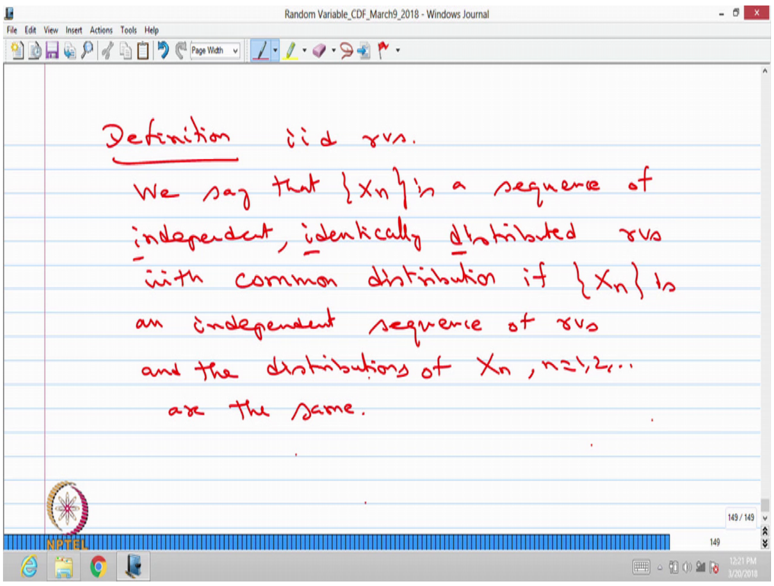

The next concept which we are going to discuss as their form of definition that is iid random variables; we say that we say that Xn is a sequence of independent identically distributed random variables with common distribution, if the sequence Xn is a independent sequence of random variables and the distribution of each random variable Xn or the same.

Whenever we say the sequence of random variables are iid; that means, the first I comes from here the second I comes here the d comes from, whenever we say the collection of random variable or sequence of random variables are iid; that means, they are mutually independent as well as distributions are same.

Whenever the distributions of few random variables are same, as well as all those random variables are mutual independent then we can conclude that collection of random variables are the iid random variables.

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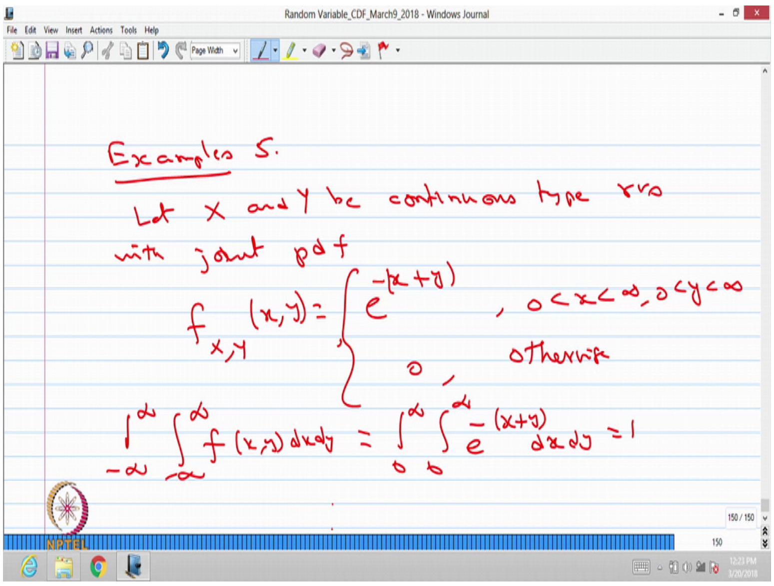

We can go for creating a simple example of iid random variables. So, this is a example number 5, let X and Y be a continuous type random variables with joint probability density function is given by e power minus x plus y.

When x is lies between 0 to infinity and y is also lies between 0 to infinity, 0 otherwise, we started with the 2 dimensional continuous type random variable, random variables with the joint probability density function, that is e power minus x plus y within bracket, when x lies between 0 to infinity y between 0 to infinity, you can verify whether this is going to be the joint probability density function by integrating a joint probability density function with respect to x and y that is same as 0 to infinity 0 to infinity e power minus x plus y dx dy, that is same as 1.

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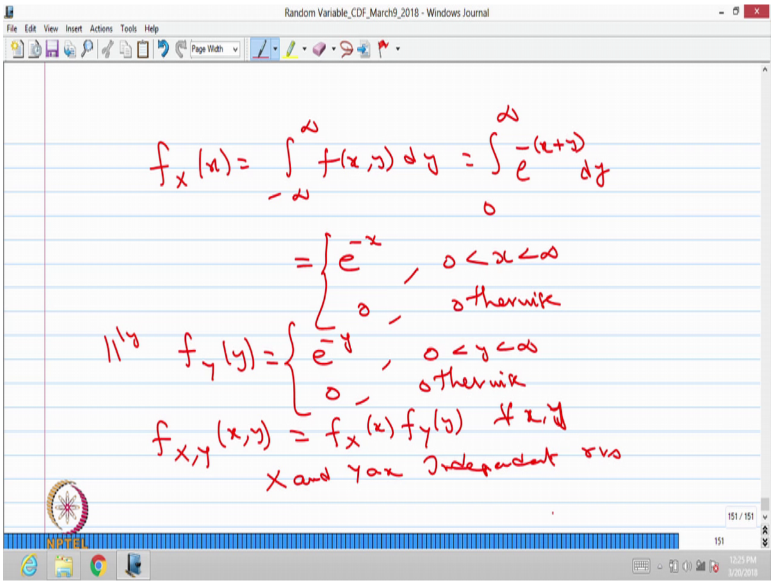

Therefore this is joint probability density function, let us go for finding out the marginal distribution by integrating the joint probability density function with respect to y, that is 0 to infinity e power minus x plus y integration with respect to y, if you do this integration you will get the answer that is e power minus x; when x is lies between 0 to infinity. So, this is the probability density function of x. Similarly if you do the exercise of finding the probability density function of y, you will get e power minus y, when y is lies between 0 to infinity, otherwise it is 0.

If you cross check the joint probability density function, whether that is same as product of probability density function of X and Y or all x and y, the joint probability density function is e power minus x plus y, and the probability density function of x is e power minus x probability density function of y is e power minus y so, this condition is true for all x comma y. Therefore, we can conclude x and y or independent random variables. Not only that probability density function of x is e power minus x, probability density function of y is e power minus y.

So, if you find the distribution; that means, the CDF, or the probability density function for a continuous type, or probability mass function for discrete type, the distributions are same the distributions of x and y are same, also they are independent random variable.

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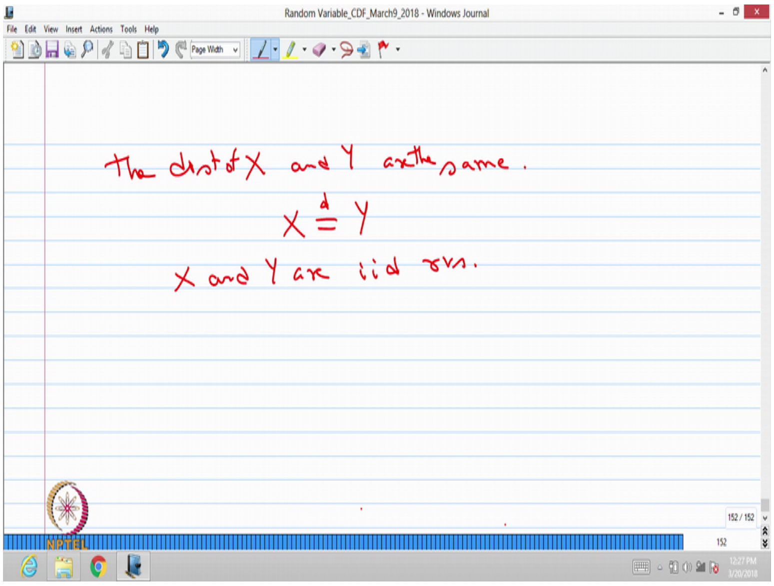

Therefore, we can conclude the distribution of X and Y are same, are the same we can conclude in notation, we can use X is capital Y in distribution, they are identical distribution the distribution of X and the distribution of y are same we can use the word d above the equal symbol; that means, both are having the identical distributions. And also they are independent therefore; we can conclude X and Y are iid random variables. So, whenever we write iid random variables that means, those random variables are mutually independent, also they are having the same distributions, then we call it as a iid random variables.