Introduction to Probability Theory and Stochastic Processes
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Lecture - 22

In this module we have discuss the various Standards Distributions, both the discrete type as well as the continuous type in last 2 lectures. That is the in the first lecture we have discussed various discrete type common distributions, starting from constant random variable, then Bernoulli distribution, then binomial distribution, geometric distribution, then discrete uniform distribution, then we discuss the Poisson distribution. So, these are all the standard discrete type distribution which we have discussed in the lecture 1.

In the lecture 2 we have discussed standard or a common distributions of continuous type random variables. In that lecturer we have discussed continuous uniform distribution between the intervals, then we discuss the exponential distribution, then we discuss the gamma distribution, beta distribution, cos a distribution, and we have given some list of distributions.

In particular we have discussed the normal distribution that is very important distributions of a continuous type random variable which is a common distribution. So, we have discussed normal distribution also. From the normal distribution how one can get the standard normal distribution then we have solved 1 or 2 problems in the normal distributions also.

Now, in this lecture we are going to give a few problems then one can identify what is the correct distribution attached with those problems, then using those common distributions one can get the solution. That means, we are going to use this common distributions to get the solution of the given problem.

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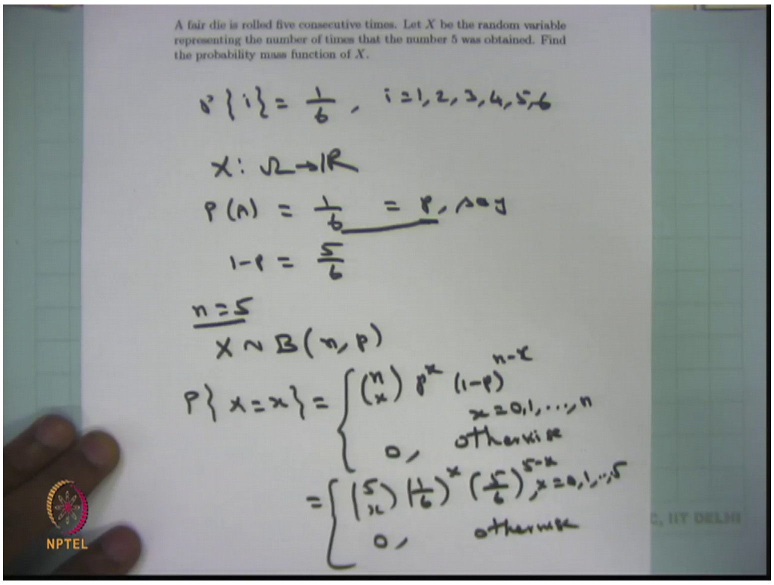

The first problem is a fair die is rolled 5 consecutive times. Let X be the random variable representing the number of times that number 5 was obtained. The question is find the probability mass function of the random variable X. Here the fair die means it is unbiased; that means, the probability of occurrence of each possible outcomes or a seen and since there are 6 possibilities therefore, the probability of each possible outcome is going to be 1 by 6.

So, a fair die is rolled 5 consecutive times therefore, the probability of each possible outcome that is equal to 1 by 6, when i takes the value 1, 2, 3, 4, 5 and 6. So, this is going to be the collection of all possible outcomes and the probability is going to be 1 by 6 because it is a fair die is rolled consecutively 5 times. So, now, the random variable X is defined from omega 2 or where omega is a collection of all possible outcomes based on the this random experiment that random experiment is a fair die is rolled 5 consecutive times. Therefore, let X be the random variable representing the number of times the number 5 was obtained.

So, you can make a event A that is nothing but the number 5 was obtained, the event A is the number 5 is obtained. The probability of event A is going to be 1 by 6. So, that can be treated as the probability of success for each roll. Like that we are making a 5 independent rules; that means, the probability of success is 1 by 6 and the probability of failure is 5 by 6 in each Bernoulli trial here when I say Bernoulli trial the getting the number 5 with the probability 1 by 6 that is going to be the probability of success and the probability of a failure is not getting the number 5 that is with the probability 5 basics.

So, therefore, each Bernoulli trial with the probability P that is 1 by 6 and the failure probabilities 5 by 6. Like that we have n independent Bernoulli trials here n is going to be 5. So, the random variable say is representing the number of times the number 5 was obtained that means, it is same as a 5 independent Bernoulli trials and the X represents the total number of n independent Bernoulli trials gives the values.

Therefore, we can conclude X follows binomial distribution with the parameters n comma P here n is 5 and P is on basics. In this problem the X follows a binomial distribution with a parameters n and P where n is a 5 because 5 consecutive times we are rolling the dice all are independent and the probability of success in each role getting the number 5 that is 1 by 6.

Now, the question is a find the probability mass function of X. You know that since it is binomial distribution immediately you can write the probability mass function is of the form n c x P power x 1 minus p power n minus x, where x takes a value 0 1 and so on till n 0 otherwise. This is a probability mass function of binomial distribution. So, in this problem this is going to be 5 c x, 1 by 6 power x, 5 by 6 power 5 minus x, when x takes a value 0 1 and so on till 5, 0 otherwise.

In this problem I have stopped it a finding the probability mass function. Once you know the probability mass function suppose you want to find out the probability of X is less than or equal to 3 or if you want to find out the probability of X is greater than 4. So, all those things nothing but the probability of some events; so, once you know the probability mass function you can get the probabilities. Suppose the question is a find the mean variance because since it is a binomial distribution you can use the relation of mean and variance and so on and even you can find the further moments for this problem.

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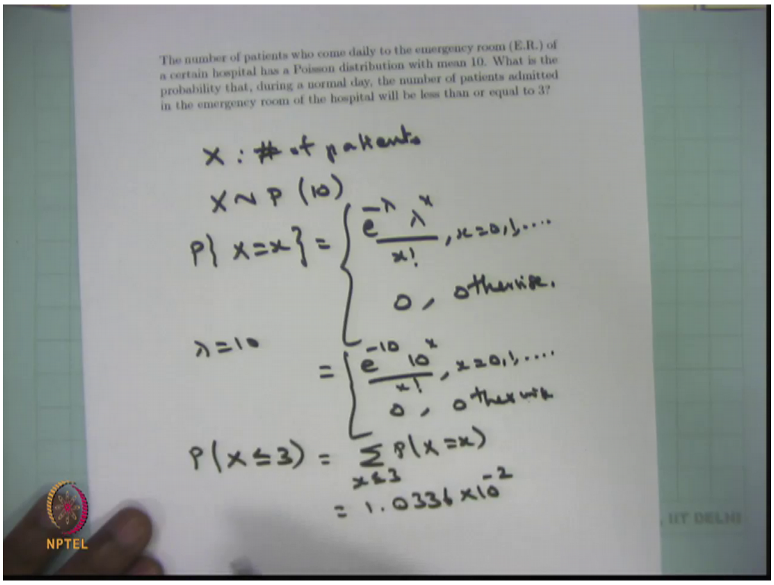

Now, we will move into the second problem. The second problem is a the number of patients who come daily to the emergency room of a certain hospital has a poison distribution with the mean time. What is the probability that during a normal day the number of patients admitted in the emergency room of the hospital will be less than or equal to 3?

This is a very difficult situation in which the number of people or number of customers or number of units entering into the system. Any system sometimes we can make the assumption of poison distribution because it comes in a very rare event and the possible values are 0 1 2 and so on countably infinite in that case it is good to make the assumption of at that follows poison distribution.

So, therefore, in this problem it is already made the assumption it follows a Poisson distribution with the mean time. So, the question is what is the probability that the number of patients entering into the hospital will be less than or equal to 3. Since already made the assumption of a it follows the poison distribution. So, the question is immediately you can make out you can create a random variable X is nothing but the number of patients, who come daily to the emergency room.

Since we made the already the assumption X follows poison distribution with the mean time usually we write the parameter. If you recall the Poisson distribution suppose the

Poisson distribution as the parameter lambda the mean is all lambda even the variance is also going to be lambda.

So, here the information is given the Poisson distribution with the meantime that means, it is a lambdaist. Therefore, you can immediately write down the probability mass function of the Poisson distribution that is e power minus lambda lambda power x by x factorial, when x takes the value 0 1 2 and so on, otherwise 0. So, this is a probability mass function. So, in this problem lambda is 10 therefore, this is e power minus 10, 10 power x by x factorial when x takes a value 0 1 so on, otherwise 0.

So, the question is what is the probability that the during a normal day in the number of patients admitted in the emergency room of the hospital will be less than or equal to 3. That means, you have to convert the given problem into the form of a that is X less than or equal to 3 this is the required property. This is same as summation of probability of X takes a value small x, when small x is going to be less than or equal to 3; that means, it is probability of X is equal to 0 plus probability of X equal to one probability of X equal to 2 plus probability of X equal to 3. You are substitute X is equal to 0, 1, 2, 3 and get the probability mass at those add all the values if you simplify you will get the answer it is a 1.0336, into 10 power minus 2.

Even this problem can be asked in find out the probability that no customer or no patient admitted in a normal day; that means, that is a probability of X equal to 0 or you can ask what is the probability that always some patients admitted in the emergency room and a normal day; That means, that is 1 minus of a probability of X equal to 0. Or there is a possibility of question what is the variance of X.

Since you know the poison distribution the mean and variance are same the variance is also going to be meantime. So, like that many more problems can be created once the situation is given and some information is provided you can find other (Refer Time: 12:11) in nice way. We will move into the third problem.

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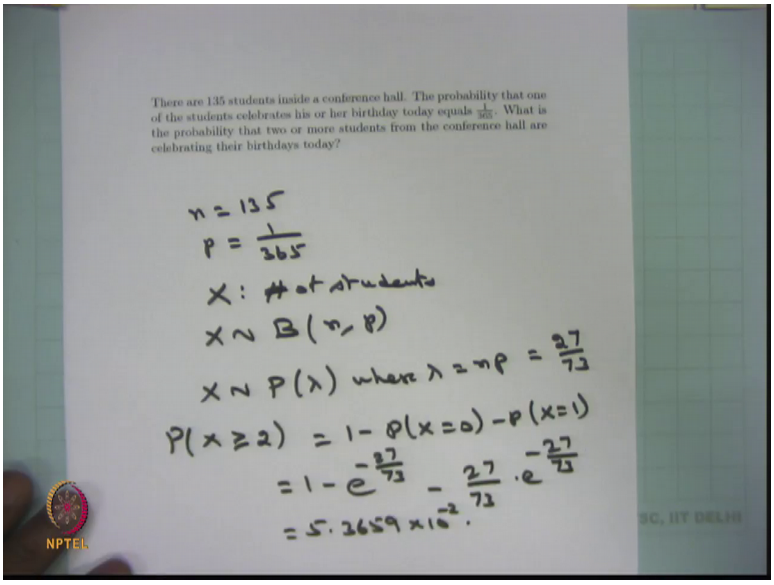

The third problem is a there are 135 students inside a conference hall. The probability that one of the students celebrates his or her birthday today equals 1 divided by 365 what is the probability that two or more students are from the conference hall are celebrating their birthdays today.

In this problem we made the some assumption the year is not a leap year therefore, we made the probability that the one of the student celebrate his or her birthday today equals to 1 divided by 365; that means, a we made the assumption the year has 365 days. The question is what is the probability that two or more students from the conference hall celebrating their birthday today.

Two or more that is equivalent of saying negation in the form of a 1 minus no and 1; that means, at the probability that two or more students from the conference hall celebrating their birthday today that is same as 1 minus probability that no one is celebrating minus only 1 person celebrating because a summation of all the probabilities is 1. So, this question either you can solve it what is the probability that 2 people having as a birthday today and 3, 4 and so on add all the values or we can go for 1 minus of a no one celebrating minus only 1 person celebrating.

So, let us find out the probability. The clue is there are 135 students and the probability of a success is 1 divided by 135. Suppose I make it n is equal to 135 and p is equal to 1 divided by 365 I can match this problem with the number of students celebrating the birthday that is number of students celebrating their birthdays that is going to be capital X. Then I can conclude X follows binomial distribution with the parameters n comma p, where n is a 135 and the p is a 1 divided by 365. By doing this I can get the probability of X is great or equal to 2 by applying the binomial distribution probability mass function and so.

Here we should observe the n 135 and small p is n is sort of very large and p is almost close to 0. Even though we say the p is open interval 0 to 1, here the p probability of success that is very small therefore, it is good to do by approximating this binomial distribution with the poison distribution, that is a X follows poison distribution with the parameter lambda. Now, the question is what is lambda? Where lambda is n into p that means, you multiply 135 with 1 divided by 365 that is going to be the lambda value.

So, one can simplify and get the value lambda. So, if you do the simplification that lambda is going to be 27 divided by 73. So, once you get the lambda let us a parameter for the poison distribution; that means, you know the probability mass function of the poison distribution therefore, the required probability is the probability that two or more students from this conference hall are celebrating their birthday today, that is nothing but X is a greater than or equal to 2. That is same as either one way find out the probability of X equal to 2, probability of X is equal to 3 and so on sum it up or the other way is 1 minus probability of X is equal to 0 and probability of X equal to 1.

So, this is easier than finding out the other side because the summation of probabilities 1 therefore, this is same as this. So, you can get a probability of X equal to 0 because you know the probability mass function. So, that is 1 minus probability of mass function for the Poisson is e power minus lambda lambda power X is 0 here divided by 0 factorial.

So, it is going to be e power minus 27 by 73 when you substitute probability of X equal to 1 you will get 27 divided by 73 times e power minus 27 by 73 if you simplify you will get the answer that is a 5.3659 times 10 power minus 2. So, this is a easy problem you can create a many more problems with the same first two statements. Since, the n is large and p is almost 0 we are going for binomial to the Poisson distribution.