Introduction to Probability Theory and Stochastic Processes

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Lecture – 10

Now, we will move into the third type of a random variable.

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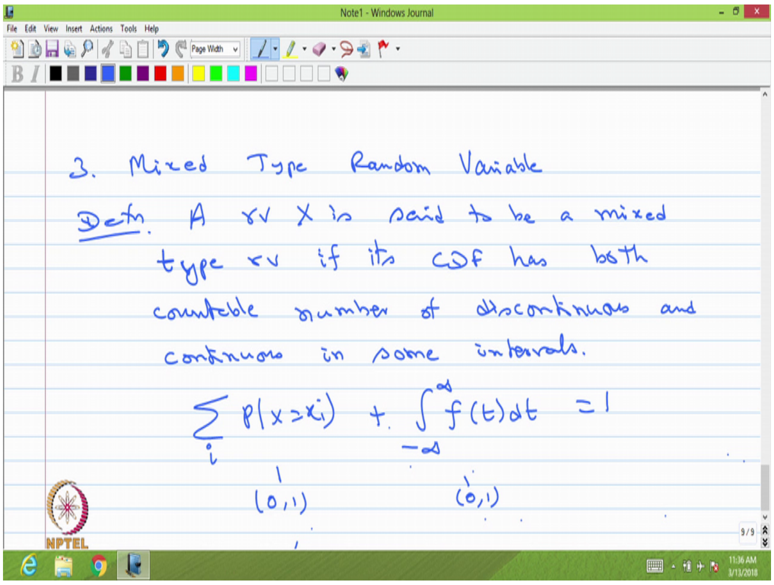

That is mixed type random variable; the definition a random variable X is said to be a mixed type random variable, if its CDF has both countable number of a discontinuities and continuous in some intervals.

You see the definition very carefully for a discrete type random variable it has countable number of discontinuities, for a mixed type random variable it is a continuous function in X that means, in the whole real line. Whereas, mixed type random variable is the combination of both; that means, the CDF has a discontinuities, as well as a continuous function in some interval. In that case that random variable is going to be call it as a mixed type random variable.

In this case, if you add the masses at countably countable number of discontinuities points. And if you integrate the probability density; both will give the value 1 that is very important I am writing integration from minus infinity to infinity f of t; here the f of t value will be greater than 0 in some interval. Definitely the integration alone will not give the value 1, as well as the summation of i P of x equal to x i alone may not give value 1.

If you add both it is going to be 1; that means, this value is lies between open interval 0 to 1 and this value is also lies between open interval 0 to 1 and put together that is going to be 1. That means, the unit mass is distributed over the countable number of a points in the real line as well; as in few intervals or some intervals; the whole unit mass is distributed both in finite sorry countable number of a points those are jump points and the values are jump values and the density between some interval.

Whereas for discrete type random variable the unit mass is distributed over countable number of points; for a continuous type random variable the unit mass is distributed over some intervals. Whereas, for mixed type random variable it has both jumps as well as density in the interval.

So, I am going to give one simple example through that example you will understand how the mixture type random variable look like. The way I started today's class I have given 5 different CDFs in the previous class. So, out of those 5 in different CDF; few CDFs are going to be a discrete type, few CDFs are going to be continuous type and few CDFs are going to be mixed type. So, I am going to give the example for the mixed type random variable.

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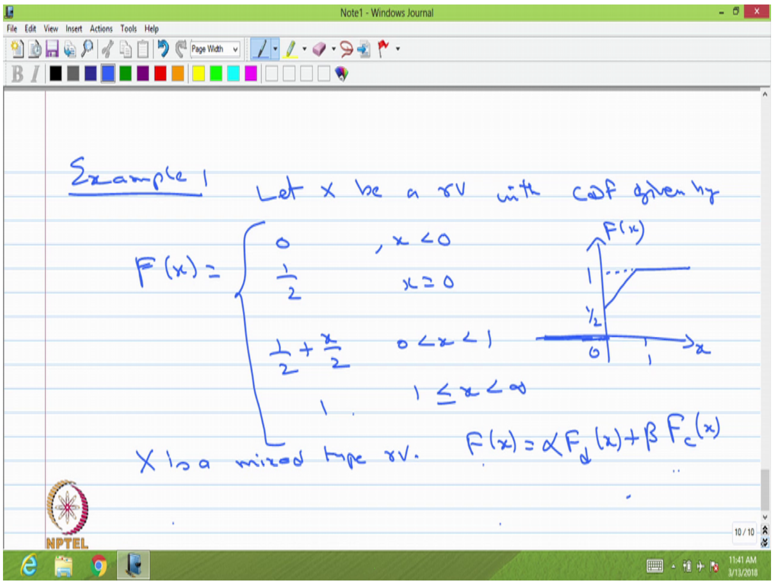

Example only one example; I am going to introduce for this mixed type random variable let X be a random variable with the CDF is given by CDF given by capital F of x; it takes a value 0, it is less than 0 it takes a value 1 by 2 when x takes the value 0. Between the interval 0 to 1 it is 1 by 2 plus x by 2 from 1 onwards the value will be 1.

The CDF of this particular random variable is 0 till 0 at 0 there is a jump from 0 onwards till 1; it is half plus x by 2 from 1 onwards it the value is 1. So, I can draw the CDF of this random variable till 0; it is 0 then 0 at the point 0, there is a jump and from 0 to 1 1 by 2 plus x by 2. So, it is landing line till 1 then from 1 onwards it becomes 1.

You see that this particular CDF is 0 till 0 at 0 there is a jump then it is a continuous between 0 to 1; then from one onwards it is 1. So, basically it is a continuous from 0 to infinity whereas, 0 there is a jump and from minus infinity to 0 it is 0. You cannot conclude that this random variable is a discrete type random variable because it has jump as well as there is a continuous function between 0 to infinity.

You cannot consider this as a continuous type random variable because it has a jump also whereas; this is going to be a mixed type random variable because it has the one jump and the continuous between the interval 0 to infinity by seeing the CDF. The CDF has the jump as well as the continuous in the interval 0 to infinity therefore, it is a mixed type.

In general, you may have a countable number of jumps in the CDF and the continuous function in many intervals in this case it is only 1 interval. Therefore, this is the mixed type is a mixed type random variable in general; one can always right the CDF of any random variable of a discrete part and the continuous part. So, the suffix means the CDF with the discrete part discrete type; the CDF with the continuous type.

So, the mixed type random variable has a CDF in the form of sum of alpha times the discrete part as well as beta times the continuous. So, one can write for this example this is going to be capital F of x.

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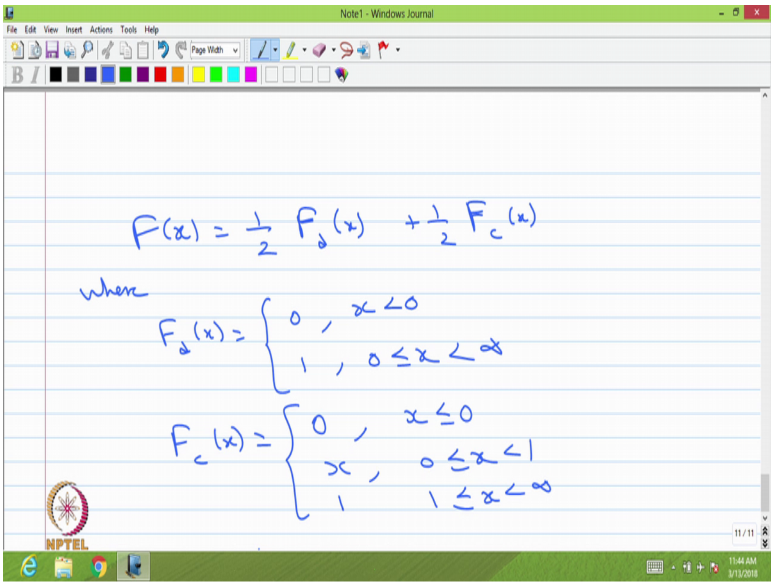

I can find out the alpha is going to be 1 by 2 of a discrete part times discrete part plus 1 by 2; the continuous part where the discrete part that itself CDF less than 0 till 0. And 1 from 0 onwards whereas, the continuous part of CDF that is again a CDF till 0 it is 0 and it take a value x between 0 to 1 and it becomes 1 from 1 onwards.

So, the discrete part F dx that alone it is a CDF of a discrete type random variable which has the mass at 0 and the jump value is 1. And F cx is it is a continuous part of CDF and that itself a CDF which is 0 till 0 and x between 0 to 1 and 1 onwards it is 1, but that is a continuous type random variable CDF for mixed type random variable it is both that is 1 by 2 times F d of x plus 1 by 2 times F c of x.

So, if you have any continuous type random variable you do not have a discrete part, if you have a discrete type random variable; there is no continuous part, if you have a mixed type random variable you will have a CDF in the some constant times discrete part plus constant times continuous part in this constant one cannot interfere. So, this call it as a canonical form of a CDF; the way we have written Fx is alpha times discrete type plus beta times the continuous type that we call it as a canonical form of a CDF. So, this is very important for immaterial of the random variable is a discrete type or continuous type or mixed type for every type of random variable; it will be simple fact.

So, with this definition of a discrete and continuous and mixed type random variables and one or 2 examples I am concluding any random variable can be classified into the discrete or continuous or mixed type by seeing it is CDF. If it is a discontinuities only then it is a discrete if it is a continuous function then it is call it as a continuous type; if it has both then it is going to be call it as a mixed type random variable.

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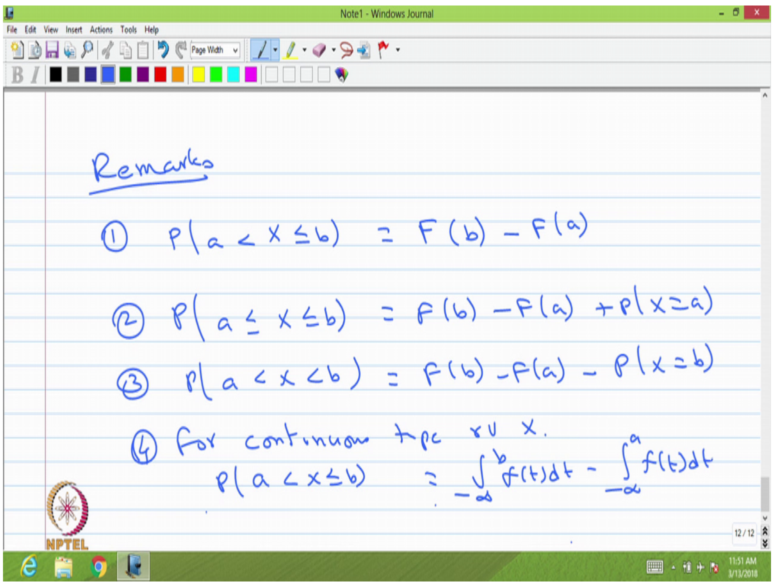

I have to give some more remarks over the random variable with respect to CDF. So, I am going to list out those remarks the first remark you can always find the probability of X lies between any interval. You can always find probability of X lies between a into b for a is less than b a can be a real as well as b can be real where a is less than b. One can find the probability of x lies between a to b that is nothing, but the CDF of the random variable at the point b minus the CDF at the point a; one can always find out the probability of x lies between a to b by using the CDF by substituting the value at x equal to b and x equal to a.

Whereas, if you want to find out the probability of x lies between a less than or equal to x less than or equal to b; then it is same as F of b minus F of a and you have to include the probability of x equal to a. Suppose you want to find out the probability of a less than x, less than b this also can be computed in the same way it is F of b minus F of a minus probability of x is equal to b.

Now, the question is when you need to add probability of x equal to a or when you have to subtract probability of x equal to b; based on which type of random variable you are in the discussion. If it is discrete type random variable and x is equal to a; where a is a jump point such that a probability of x is equal to a is greater than 0, then you can add. If X is discrete random type random variable X is a discrete type random variable where a is not a jump point; that means, probability of X equal to a is 0 then you do not need to add.

So, based on the discrete type random variable in which it is a jump point or not that is one discussion. The second discussion when X is a continuous type random variable when X is a continuous type random variable the probability of X equal to a that is nothing, but the integration and it is a Raymond integration. So, the probability of for continuous type random variable X for a continuous type random variable; the probability of a less than x less than or equal to b that is nothing, but the integration from minus infinity to b of f of t dt minus minus infinity to a f of t dt.

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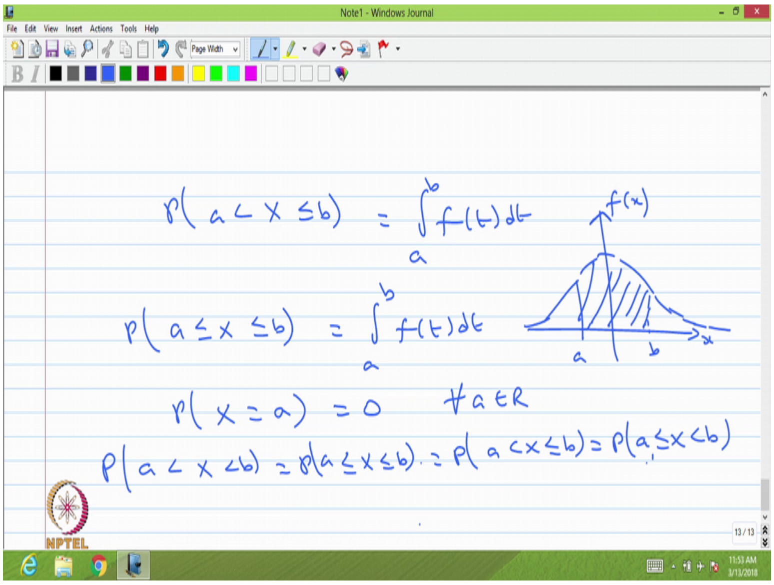

That is same as the probability of X lies between a to b that is nothing, but integration from a to b of f of t dt when x is a continuous type random variable.

Since it is integration from a to b f of t dt that is nothing, but suppose you have a probability density function like this then a to b is nothing, but a suppose b somewhere here this is the. So, this is basically a Raymond integration of f of x between the interval a to b. Therefore, the probability of a less than or equal to X less than or equal to b that is also again it is a to b f of t dt when x is a continuous type random variable; whether you include the point or exclude the point the integration is going to be again a to be f of t dt.

Hence, the probability of X takes any point that is going to be 0 for all a belonging to R; hence for a continuous type random variable the probability of X lies between a to b open interval probability of X lies between closed interval or one side closed all the values are one and the same. Because the probability of X takes any point in the real line that value 0 for a continuous type random; that means, there is no mass at any point whereas, for a continuous type random variable there is a density between some interval therefore, that is going to be greater than or equal to 0; so, this is going to be the next remark.

Therefore, the previous result the result of 2 and 3 whether you have to add or subtract the probability of X equal to a or probability of X equal to b this is going to be the issue for a discrete type random variable, for continuous type random variable it is going to be 0. For a discrete type random variable if it is a jump point then there will be a addition, there will be a subtraction if that is not the jump point again it is going to be 0

The next remark the fifth one; the probability mass function is the probability at that point X takes the value x i for a discrete type random variable. Whereas, there is no probability of X takes a some value for a continuous type random variable the probabilities 0.

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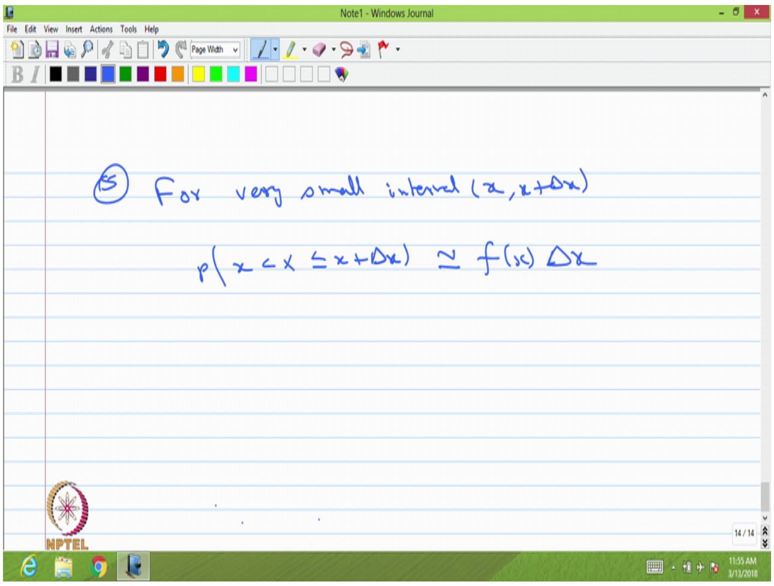

But still for a smaller for very small interval x to x plus delta x delta x is very small; then you can make out the probability of x lies between x to X plus delta x it whenever a delta x is very small that can be approximately say the f of x delta x; as it is f of x is not a probability, f of x is a probability density function. If you integrate between the interval you will get the probability in that interval, but if the interval is very small then you can make approximately it is going to be f of x delta x; that is for when delta x is very small.

So, if this is very important these 5 remarks about the random variable; whenever you know the CDF you can get the probability of a x lies between in interval in material of it is a discrete or continuous. If it is a continuous type random variable then whether it is open interval or say closed interval or semiclosed and so on everything is one and the same because the you are integrating the probability density between the interval.

Therefore, some books they write the probability density function in the open interval; otherwise it is 0, some books they use probability density function in the closed interval. So, whether you write the open interval or closed interval.

It is in material because you are going to do the Raymond integration to find out the probability of X lies between any interval; for a continuous type random variable. For a discrete type random variable; it is some of probability masses at the jump points. If it is a mixed type then it is combination of both; with these remarks along with the definition and the examples I am concluding the types of random variable.