**Chaotic Dynamical Systems**
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**Lecture – 08**
**A Population Dynamics Model**

Welcome to students, today we will be looking into the population dynamics model.

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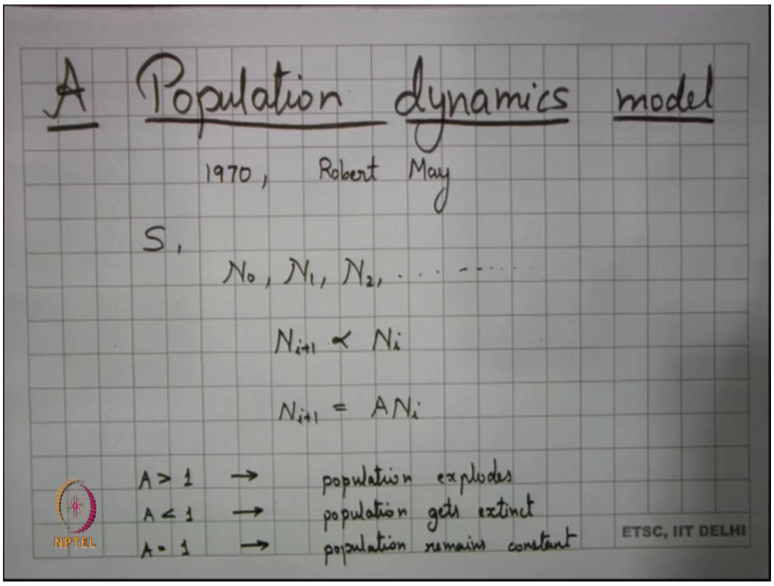


So, it was started in 1970 when Robert may, he proposed to mathematical model to study the complex behavior exhibited by the growth of population in certain biological species. So, let us assume that there is a particular specie S and for this particular species S, we can have observe that, the individuals here at the born and they die right within the same year. So, this can be actually observed in many insects and all.

So, we start with N 0 say for example, and the very first year the number of species is an 0, then it has been observed that whatever the number of species comes up next year depends on the number of species this year. So, supposing in the next year we have the number of species to be N 1 after that here we have the number of species to be N 2 and so on. So, he observed that this N i plus 1 at the i plus oneth year is basically proportional to Ni. So, the number of spaces in the corresponding the next year will be basically depending on the number of species in the present year.

Now. So, this gives us some kind of a proportion consensus. So, this gives us an equation and we can say that N i plus 1 is basically A times N I, where N i is the constant of proportion now what happens if this a is greater than 1. So, if a is greater than 1, then it is observed that the population in the subsequent years right we start with N 0 right a one N 2 and so on. So, in the corresponding years right the population becomes tends to infinity right and basically the population explodes.

So, in this case the population explodes, if A is less than 1 then it is observed that, what happens to the spaces is with the corresponding year you will have less number of individuals than the present year and that continues and since A is less than 1 it would happen that as time tends on the population gets extinct. And if my A is equal to 1 then the population remains constant does not matter; however, the season changes whatever happens to the year the population continuously remains the constant. So, population remains constant.

Now, it was observed that this is not a very practical model for population. So, what is a practical model for population? So, it was observed that population also depends on how the individuals interact among each other. Say if 2 people are interacting right there will be a competition for food, there will be a competition for shelter, there will be competition for space. So, it also depends on the number it also depends on the mutual interaction.

So, it was observed that we need another term in this model which is proportional to N square basically representing the interaction.

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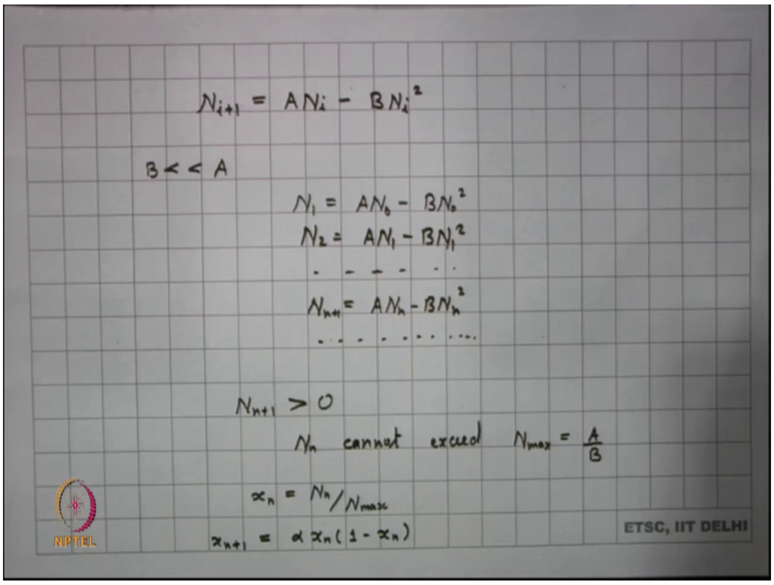


So, it was then proposed that N i plus 1 should be equal to something like a times N i minus B times N i square. Now what happens here is; why do we have minus sign here, because as the interaction increases you can observe a decrease in the population.

Now, this if you look into this model, if my B is very much less than A, then as long as my N i is very small the second term is not going to make any difference, but what happens here is that if N i become sufficiently large. So, once N i becomes sufficiently large N i square dominates right and then that plays into the population in the corresponding year.

So, we observe here that I can write my N 1 to be equal to A times N naught minus B times and naught square, N 2 can be written as A times N 1 minus B times N 1 square right so on. At the nth of the stage I can write Nn to be equal to A times Nn sorry Nn plus 1 to be equal to A times Nn minus B times Nn square and so on.

So, it was observe that you can write you can look into the corresponding population in this particular manner. We want the population to still remain positive. So, supposing I want my Nn plus 1 to remain positive, what happens in that particular case? Then one can observe that simply calculating the stuff one can observe that in that case my Nn cannot exceed what I called is are as a N maximum, which happens to be equal to A by B right I think that is what it turns out to be years turns out to be A by B.

So, if my population turns out to be equal to A by B right we can find that this becomes extinct, and hence we can say that here right our this remains positive only if your population at any given time does not exit A by B. Now what we trying to do here is, we try to put here x and now I am putting another variable here xn to be equal to Nn upon N maximum.

Now if we try to put this in this particular model right try to put this a variable in this particular model, then that gives me an equation xn plus 1 is equal to I am writing the constant I am changing the constant of proportionality although it would turn out to be a, but this happens to be alpha times xn into 1 minus xn. We find that the population dynamics right it observes sort of an equation or sort of a function right given in terms of xnn given in terms where xn plus 1 can be given in terms of xn.

So, this follows a certain function and that particular function is something which depends on x and we call this function as the logistic map. So, we have the logistic map here.

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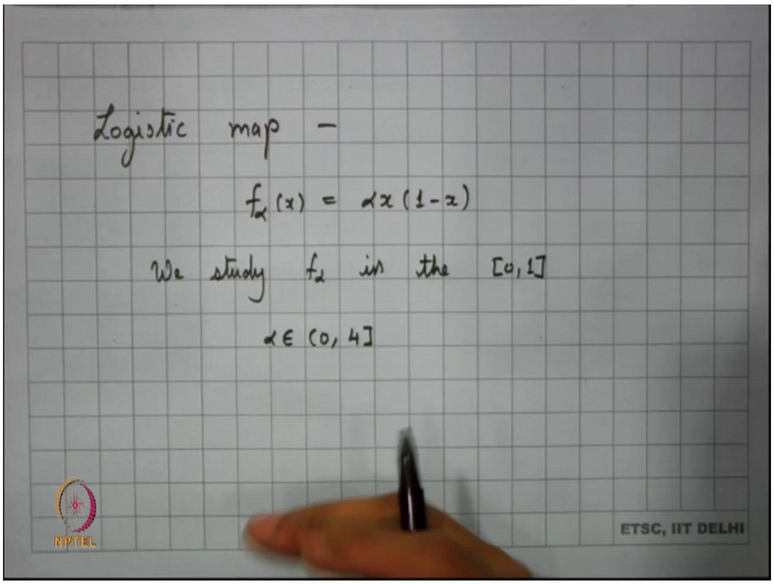


So, what I have here is that once I fix the constant of proportion alpha, I find this map to be f alpha x is alpha x times 1 minus x. Now we are trying to study population and when we want to look into population of course, we cannot study population if there is no spaces, then nothing can exist and we can say that fine sort of normalizing it. So, we say that the maximal possible population is 1 and the minimum possible population is 0.

So, we try to study this logistic map in the interval 0 1. So, we study this we study f alpha right in the interval 0 1, also we try to see that we will study this logistic map right maybe for alpha in the range of 0 to 4. Of course, what happens beyond 4 is something which you can easily analyze, but we will start with we will study this for the constant of proportion to be from 0 to 4.

So, what are we essentially studying here? So, we are essentially looking into maybe this is the graph that we have of the logistic map.

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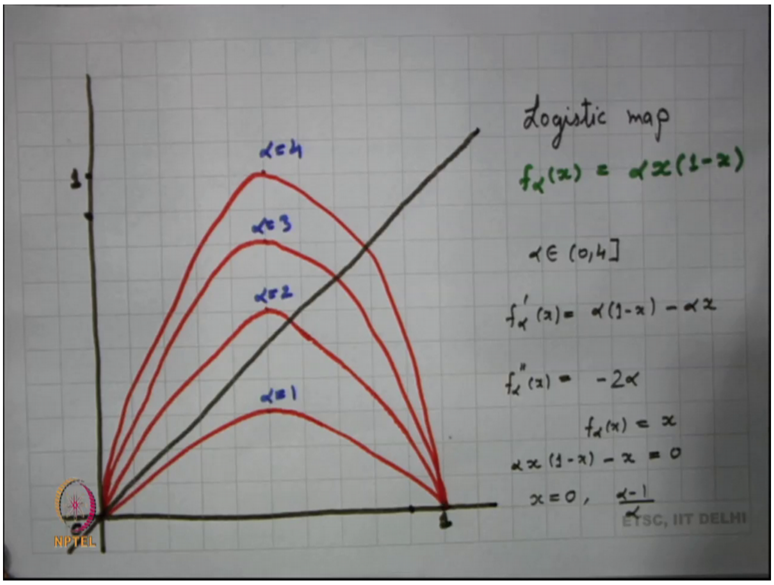


So, we have this logistic map here and we want to study this for various alpha. So, we are looking for alpha belonging to 0 4. So, when alpha equal to 1 we have this curve, in alpha equal to 2 we have this curve, alpha equal to 3 we have this curve alpha equal to 4 we have this curve.

Also we observe that; what is the derivative of f in this particular interval. So, we find that the derivative of f that is f prime f alpha prime of x happens to be equal to, alpha times 1 minus x right minus alpha x this is your derivative and if I look into the second derivative yes what should be the second derivative here.

Student: Minus.

Minus 2 alpha. So, the second derivative here turns out to be minus 2 alpha. Also we will want to look into is there possible is there any possibility of a fixed point here. So, we are trying to look into what happens when f alpha x is equal to x. So, what kind of equation we are solving? We are solving this equation that x right alpha x 1 minus x, minus x is equal to 0 and what values does this give for the for x? So, one of the values one can easily take x common out of here. So, one of the values turns out to be x equal to 0 right which you can easily see from the graph, that this is a fixed point here right and the other value which you can think of is or I would put it up as alpha minus 1 by alpha.

So, we have 2 equations, we have particularly 2 we can have possibly we can have just 2 fixed points here by looking into the equation of the logistic map. We try to look into this particular case when my alpha is varying from 0 to 1. So, what happens when alpha is varying from 0 to 1? We look into this particular case here, now when alpha varies from 0 to 1 there is only one fixed point.

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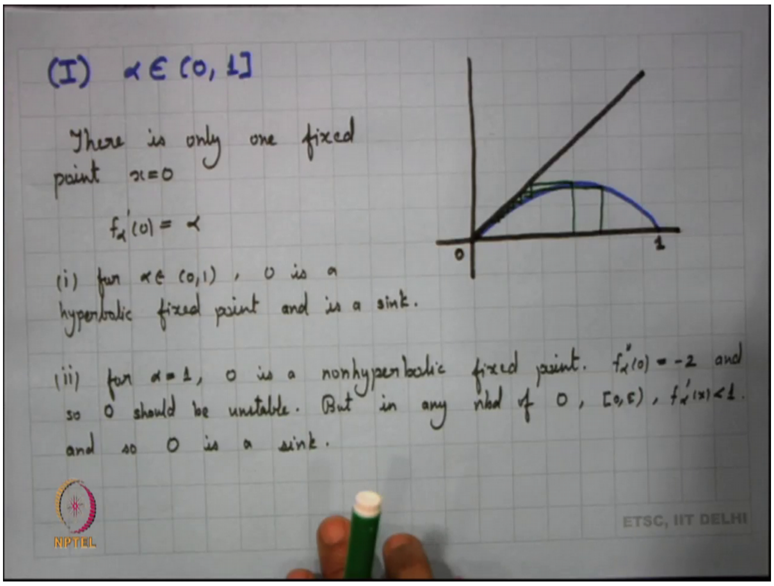


Because my alpha is 1. So, the other fixed point was basically alpha minus 1 which did not exist in the interval 0 1. So, there is only one fixed point. So, we have only one fixed point. So, there is only one fixed point and that fixed point is x equal to 0 and you can easily see this graph here right that this is the only fixed point x equal to 0 this is the only fixed here.

What happens to f alpha prime of 0 what is that equal to? So, let me again recollect right we have looked into this equation right f alpha prime of x happens to be equal to alpha times 1 minus x minus alpha x right. So, what is alpha prime of 0 here? Yes it is just alpha. So, this is just alpha, and hence we can say that our first observation here is that when alpha belongs to 0 1 right this module as will be less than 1. So, 0 happens to be. So, 0 is a hyperbolic fixed point and it is also a sink. What happens to alpha equal to 1? Now when equal to 1, 0 is a non-hyperbolic fixed point.

So, we have a look into the case of non hyperbolic fixed point, what is f alpha double prime at 0. So, we find here that f alpha double prime at 0, normally at x is equal to minus 2 alpha right. So, this happens to be equal to minus 2 which is not equal to 0 right. And so, we can say that this point right 0 should be unstable. According to the theory should be unstable right, but we observe here that what is a neighborhood of 0 that we are considering. We are considering values only between 0 and 1 right and in this neighborhood, but in the neighborhood of in any neighborhood of 0 that is my neighborhood of 0 happens to be something like 0 epsilon right. We find that f alpha prime at x is less than 1 f alpha prime decreases that is less than 1 and so 0 this fix point 0 is a sink here.

So, you can observe here that you start from any point here right you start from anywhere over here find, and you find that ultimately right the orbits are tending to 0 and start from anywhere over here ultimately the orbit is tending to 0. So, for alpha belonging to for alpha taking any value between 0 and 1 right there is only one fixed point 0, but dynamics is very simple right each and every orbit each and every other orbit is tending towards 0. So, the whole set 0 1 can be thought of as a stable set of 0.

Now, the case changes when my alpha becomes greater than 1. So, we look into this case alpha to be greater than 1.

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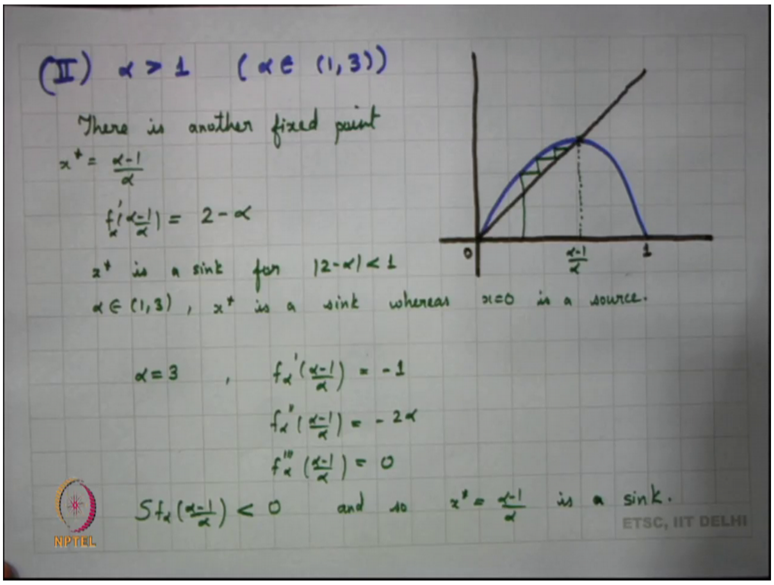


Now what happens when alpha is greater than 1? Yes we have another fixed point right. So, when alpha is greater than 1 right there is another fixed point, let me call it x star its basically alpha minus 1 by alpha. So, we have this another fixed point here right.

What happens to f prime of f prime alpha of alpha minus 1 by alpha what happens to this, what is the value of this one, what is the value here? We have f alpha prime x is alpha times 1 minus x minus alpha x. So, the value here turns out to be equal to 2 minus alpha. Now this is 2 minus alpha what is the value of value of f alpha prime x at 0 right, at 0 the value is certainly turning out to be greater than 1. So, the fixed point 0 now becomes a source what happens to this value. Now alpha minus 1 by alpha, if I look into this x star right. So, x star is a sink for mod of 2 minus alpha less than 1 right if this holds 2 than it is a sink.

So, what happens in that particular case? So, if my alpha belongs to now can think of this if my alpha belongs to 1 3 right in that particular case I will have this mod this equation being satisfied. So, if alpha belongs to 1 3 then x star is a sink whereas, x equal to 0 is a source. So, I have 2 fix points here right one a sink the other is source. So, if I start with any say you start with any point here right ultimately this converges to the fixed point of x star right.

What happens when alpha equal to 3. So, we are essentially looking now what happens to alpha equal to 3. When alpha is equal to 3 we find that f alpha prime of alpha minus 1 by alpha what does it turn out to be minus 1. So, this point x star is now a non hyperbolic fixed point and since this is a non hyperbolic fixed point, we now look into again what is f alpha double prime right at alpha minus 1 by alpha, which basically is my minus 2 alpha and my f triple prime f alpha triple prime at alpha minus 1 by alpha that happens to be equal to 0.

So, we find that when alpha is equal to 3, this fixed point x star is a non hyperbolic fixed point, but for this non hyperbolic fixed point we find the derivative to be equal to minus 1 right. So, we now need to look into the scuacian derivative of f alpha at this particular point. And we find that this scuacian derivative of f alpha right at this particular point alpha minus 1 by alpha is less than 0 because this term is missing right. So, this scuacian derivative is less than 0. So, this is scuacian derivative is less than 0. So, this fix point x star equal to alpha minus 1 by alpha right is a sink.

So, we find that this happens to this turns out to be sink. Now interesting cases what happens when alpha becomes greater than three. So, we go to the next case what happens when alpha is greater than 3. So, essentially we will be looking into this case that my alpha belongs to 3 and 1 plus root 6.

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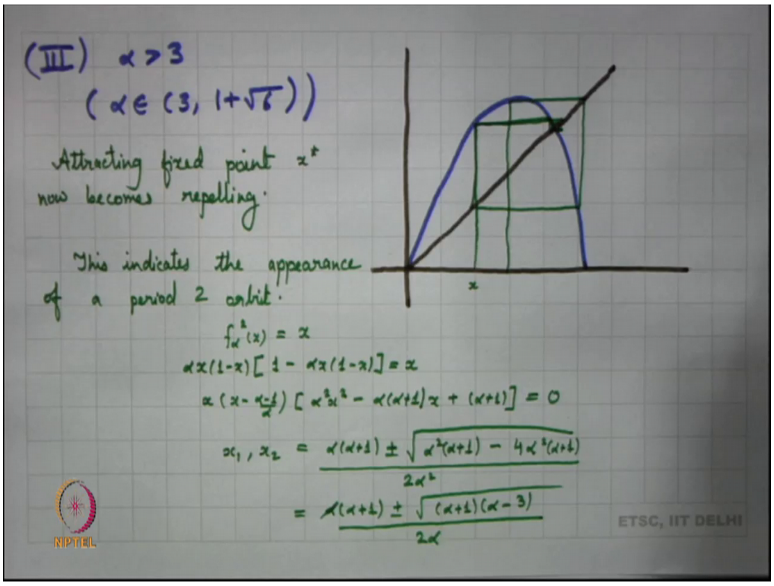


So, we will say this alpha being greater than 3, now what happens when alpha being greater than 3. Once alpha is greater than 3 we have already seen what happens to the fixed point. 0 is already a source right and now even x star becomes a source right because model as becomes better than 1 there right.

So, we find that x start happens to be. So, this attracting fixed point now becomes repelling, but we try to observe some particular orbit over here. So, let me take for any point say x here, let me try to look into what is its orbit here, how does it orbit behave then this goes to and what we observe here is that you start with any typical orbit, then it is basically starts oscillating around this fix point x star right giving us a kind of period 2.

So, what happens here is that this observation gives us. So, it settles down between 2 points its basically oscillating between 2 points and then settles down it these 2 particular 2 points. So, maybe we can think of another orbit here. So, let us try to look into this particular orbit. So, we find that here it comes back here right goes back here, goes back here again, goes back here and then you again its settles down to this 2 orbit.

So, what happens here is this indicates the appearance of a period 2 orbit. Now if really there is a period 2 orbit, we should be able to solve the equation right f alpha x square equal to x, and what essentially that means, that if I am looking into alpha x times 1 minus x into 1 minus alpha x times 1 minus x this should be equal to x. So, if I try to write down this equation right I know that this equation trying to simplify this equation and definitely have x common right. I can also say that x minus alpha minus 1 by alpha there is another common part and at the first part I see this this is alpha square x square minus alpha times alpha plus 1 into x plus alpha plus 1, this is equal to 0.

So, we can simplify this equation in this particular form, now we already know that we have this fixed points right alpha minus 1 by alpha and alpha and an x equal to 0 right. So, these are definitely going to be the fix points for f square f alpha square x equal to x they are going to be fixed points for that also. So, what we get is, we get another set of fixed points over here which come up from this particular equation.

Now, we try to solve this particular equation, then the solution of this particular equation gives us right 2 points x 1 x 2 which is we can write it as alpha times alpha plus 1, minus I have this root of maybe I should write it as alpha square into alpha plus 1 minus I have 4 times alpha square times alpha plus 1, it should be plus and minus here because I am looking into both x 1 and x 2 and then divided by twice alpha square.

So, now if I try to look into this part, if I try to solve this particulars say this equation then simplifying it we get that this happens to be alpha times alpha plus 1 plus or minus root of; now I have an alpha here maybe I can cancel this alpha out. So, this is just alpha plus 1 I have alpha square here which comes out and becomes alpha and there is an alpha in the denominator which can be cancelled out. So, what we get here is we just simply get here is alpha plus 1 plus or minus, alpha plus 1 into alpha minus 3 right divided by 2 alpha.

So, I get that x 1 and x 2 happen to be equal to alpha plus 1 plus or minus under root alpha plus 1 into alpha minus 3 upon 2 alpha. Very easy to see from this equation that this x 1 and x 2 will exist only when alpha is greater than 3 right. So, only when alpha is greater than 3, we have this x 1 x 2 existing and when this x 1 x 2 exist right we can now think of what happens to the dynamics or what happens to this particular period to orbit.

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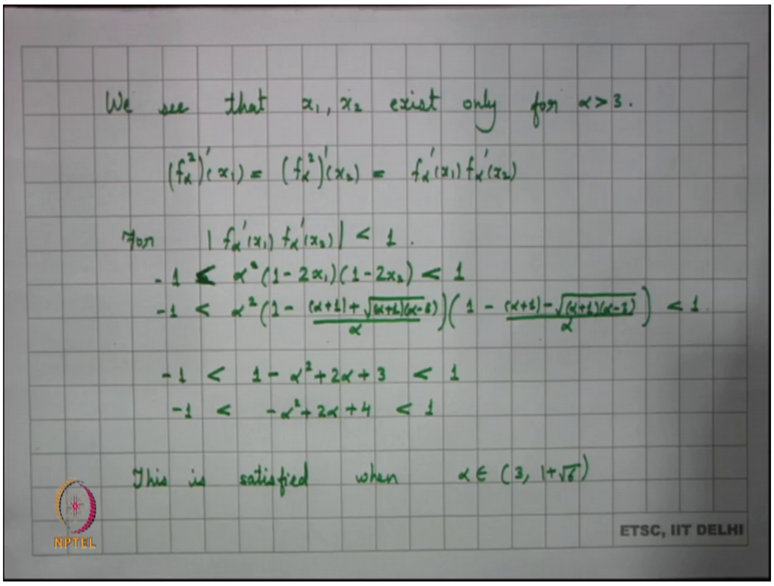


So, we see that right, only when alpha greater than 3 exists and now we can look into the dynamics. So, what happens at alpha squared right I am looking into this prime? At the point x 1 right and by isometry this will be same as f alpha square prime at x 2 which happens to be same as f alpha prime x 1 into f alpha prime x 2, and what is that equal to. Now we want to say that we will have this happens to be a hyperbolic periodic points right. So, this periodic orbit is hyperbolic when we have. So, for this I have f alpha prime x 1 times f alpha prime x 2 right I should have this to be less than 1, this basically gives me an sink there right.

So, if it gives less than 1 what do we want? We want it to be less than 1. So, basically if I look into this equation this basically means that minus 1 should be less than or less than basically I should say this is less than 1. So, minus 1 should be less than now I am multiplying the 2, and we know; what is the derivative here. So, the derivative here turns out to be alpha times 1 minus x minus alpha times x. So, the derivative if I look into that aspect I can say that this derivative, I can take alpha common here. So, I have alpha square times 1 minus twice x 1 into 1 minus twice x 2 and I want this to be less than 1.

I can pushing the value of alpha one and alpha 2 here. So, that gives me minus 1 is listed alpha square, I have 1 minus right. So, I can have I can write this as alpha plus 1 right plus root of alpha plus 1 into alpha minus 3 divided by alpha into again 1 minus alpha plus 1, minus root of alpha plus 1 into alpha minus 3 divided by alpha. So, this particular part should be less than. Now think of this part if I try to simplify this once again I have an alpha coming up over here. So, I have alpha minus say alpha plus 1. So, I have an alpha coming out here I have an alpha coming out here. So, this alpha square gets cancelled right then what I have is alpha minus alpha plus 1 which essentially turns out to be equal to minus.

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One right this turns out to be minus 1 right and then I am multiplying this 2 facts right. So, what I get here is just simplify this factor, I get that this is less than 1 minus alpha square plus 2 alpha plus 3 and I want this to be less than 1. Again simplify this once again what you get here is that minus 1 should be less than minus alpha square, plus 2 alpha, plus 4 and that is less than 1. Exactly this is satisfied when my alpha belongs to. So, this basically this is satisfied when my alpha belongs to of course, it should be greater than 3 and goes up to 1 plus root 6 this is satisfied for these values of alpha.

So, when whenever my alpha lies between 3 and 1 plus root 6 we find that this periodic orbit of period 2 happens to be a sink. So, everything all the orbits in 0 1 right excepting for this 2 the 2 fixed points are definitely there as it is right, but then these 2 fixed points are sort of source right. So, they are repelling, but all the other orbits are attracted into this particular periodic orbit. So, you have this periodic orbit and every point is attracted to this periodic orbit. What happens when alpha equal to 1 plus root 6? So, we shall look into that particular case.

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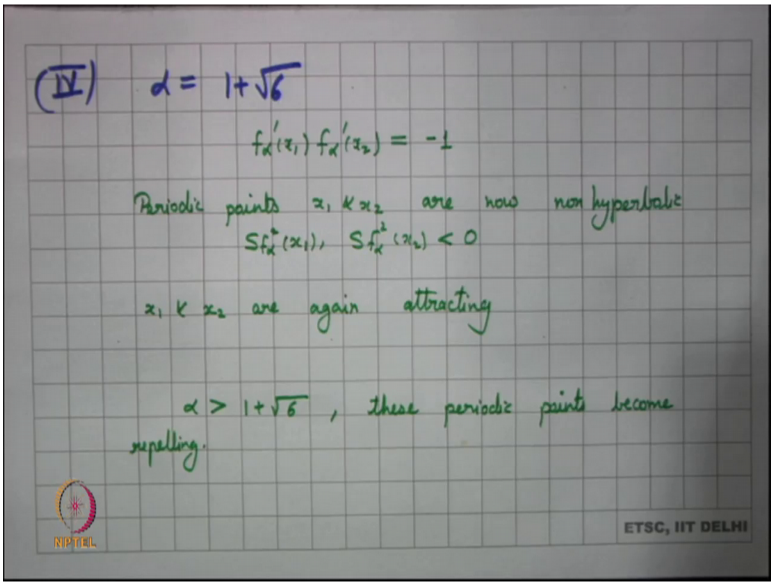


So, let me look into the next case what happens to. So, this is my fourth case what happens when alpha is equal to 1 plus root 6, what happens in this case now. So, if I look into what is f alpha prime of x 1 right into f alpha prime of x 2, we know what is that multiplication here right and in that particular case it turns out to be equal to minus 1. So, now, we have that this periodic orbit is non-hyperbolic right. It is a non-hyperbolic periodic point and for this point, this orbit. So, I should say that periodic points are now non hyperbolic, what is this scuacian derivative of f alpha at this point I mean x 1? Of course, we want the 6 f alpha square right.

So, this scuacian derivative at this particular point x 1 right and if we look into the scuacian derivative of f alpha square at this particular point x 2, we again find that this is less than 0. The simple reason being we know that the triple derivative happens to be equal to 0 right. So, this scuacian derivative is less than 0 and so, these periodic points are again attracting. So, they form a sink, this periodic orbit form. So, sink and so, they are attracting. So, x 1 and x 2 are again attracting.

But what happens now when my alpha is greater than 1 plus root 6, we find that this periodic with this periodic orbit becomes a source. There non hyperbolic now sorry there are hyperbolic now, where alpha greater than 1 plus root 6 their hyperbolic now and though they become a source right. So, when alpha is greater than 1 plus root 6 these periodic points, they become repelling. So, they are repelling now if they are repelling out; that means, the orbit is going somewhere else. So, what happens to this orbit? We are trying to look into this fact again. So, what happens when alpha becomes greater than 1 root 6?

So, we looked look into this next case. So, what happens when my alpha goes beyond 1 plus root 6 these 2 periodic points are basically these 2 are repelling.

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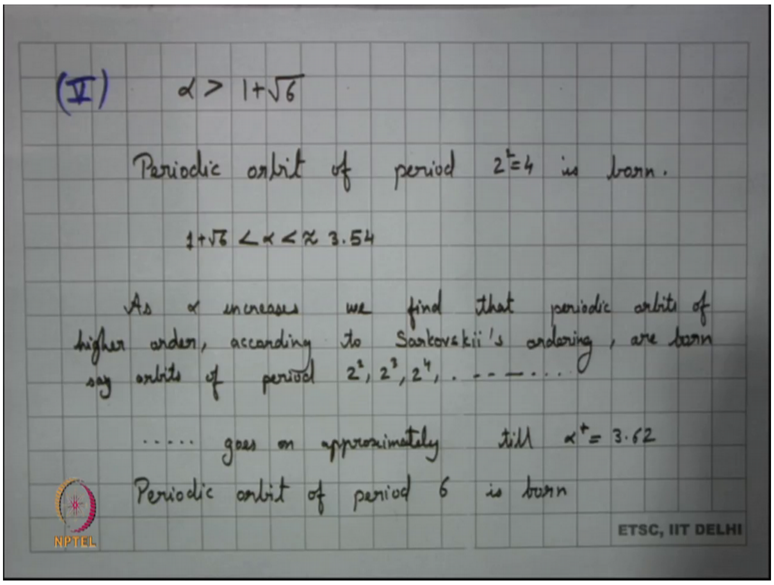


So, we find that there is another periodic orbit of period 2 square period 4 born right. So, a periodic orbit, now we have a new periodic orbit, again for this new periodic orbit we find that under certain conditions this is attracting, this is a sink. So, all the nearby orbits are basically being attracted to this, we do have our fixed points as it is we do have our period 2 points as it is, but these are source. So, they remain where they are all the points etcetera every other poi every other orbit is being attracted to this periodic point of period 4.

Now, if we proceed in this manner, we try to see what happens here in this particular case. So, we see what happens here is if you know that the graph of x verses alpha from s alpha varied from 0 to 1 we had only one fixed point 0 and that particular fix point 0 was we only attracting.

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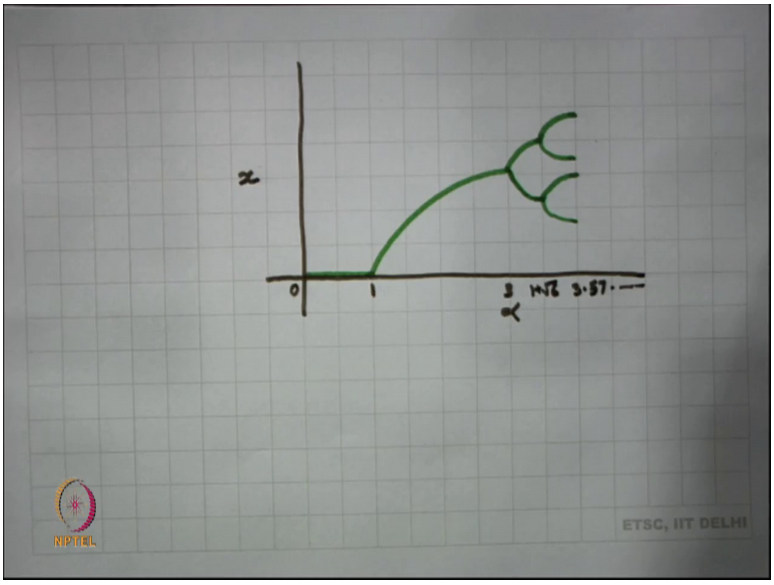


So, what happens at infinity, when you try to see what happens at infinity, you will see that the system is only turning out to be equal to 0. So, the entire thing is collapsing to 0. So, we can only see 0 here we cannot see anything else at infinity.

What happens when you alpha varies from 1 to 3. So, s alpha varies from 1 to 3 you find that, there is only one fixed point which is attracting and that fixed point is basically you can say that this is your alpha minus 1 by alpha. So, alpha minus 1 by alpha is only fix point right which is attracting.

So, at infinity you can see only this fixed points because all other orbits could have collapse to this particular fixed point and then what happens at 3. So, once you go beyond 3 there is a new periodic orbit of period 2 born, and once you have a periodic orbit of period 2 born this periodic orbit is sense of attracting right this loses the stability. So, this fixed point is definitely there, but now you cannot see this point because this point has lost it is this point has lost its attractive nature right.

So, what you find is you find only periodic orbit of period 2, and this goes on till 1 plus root 6. So, in 1 plus root 6 you find that this periodic orbit also loses its attractiveness right it becomes a source here and. So, what you find is that there is another period 4 born here right and this period 4 becomes attracting.

Now, we find that this continues till may be approximately value of alpha being 3 3.57. So, we try to into this facts the 3.52. So, we have this alpha. So, this continues. So, what we find here is that, whenever your 1 plus root 6 is less than alpha is less than I am looking now this to be approximately something like 3.54 right. So, we have this periodic orbit of period 4 born.

Now, what happens is this periodic orbit of period 4 again becomes repelling right when alpha increases beyond 3.54. So, once alpha is greater than this quantity it again becomes repelling now what happens in that case? Now again you find that there is another periodic orbit of period 8 coming up. So, the period 4 orbit becomes unstable, but there is another periodic orbit of period 8 coming up which again becomes stable. So, as alpha increases we find that periodic orbits of higher order.

Now, when I talk of higher order this is according to the Sharkovskii's order, these are born and then they eventually they become stable right. So, we have these are born. So, this happens say orbits of period 2 square, 2 cube, 2 4 etcetera these are born and then they eventually turn repelling the new orbits are born right which become attracting and so on and this goes on approximately. So, this goes on I am looking this to be approximately till your alpha star, and looking into this value of alpha star happens to be 3.62 I am just saying that this happens approximately till 3.62.

Now, what happens beyond that? So, what happens beyond that, we will try to look into this picture. So, let us look into the next case and maybe then we can go to the picture. So, what happens beyond that? So, if it look into now the sixth case.

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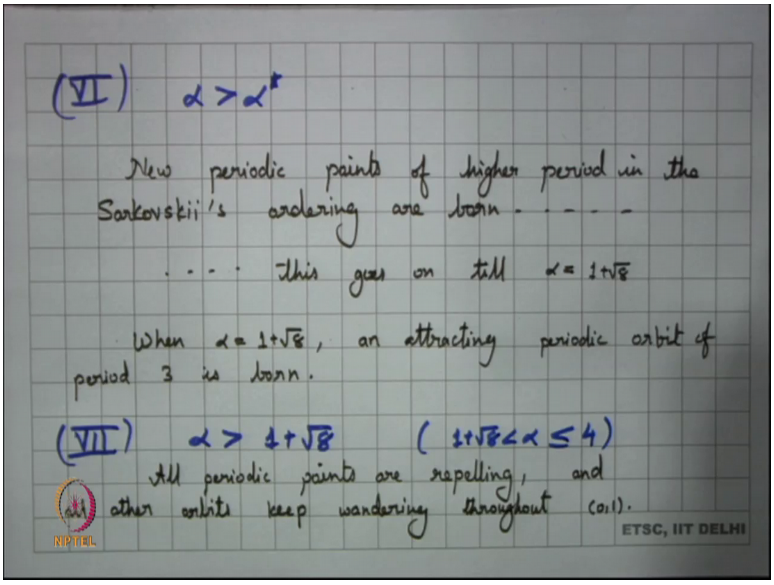


So, I am looking into alpha to be greater than alpha star, what happens in that case? Alpha at alpha star equal to 6.32 right we find that there is a periodic orbit. So, here we find a periodic orbit of period 6 is born. So, we find again periodic orbit of period 6.

Now, 6 is as we know it is an even number and this is the greatest even number in Sharkovskii's ordering. So, we find that this periodic orbit of period 6 being born, once it goes beyond this again there is lot of sort of kind of chaos you can think of that part right and the orbits become more and more complicated and we find that again we have sort of new periodic orbits being born, this is again going on right new periodic being born which are attracting right and the earlier ones they lose their attractiveness. So, they all become repelling. So, one is at. So, we get more and more stuff right till this goes. So, new periodic points of higher period in the Sharkovskii's ordering are born and this goes on till my alpha reaches the value 1 plus root 8.

Now, what happens at 1 plus root 8? When alpha equal to 1 plus root 8 should say and attracting of period 3 is born an attracting periodic orbit of period 3 now is born. And we know that 3 is the highest largest number in Sharkovskii's ordering. So, once we have this attracting orbit we see that that everything is tending to up. So, the entire we have periodic point of all periods now, and all this periodic points are sort of repelling and the only point which is attracting the only periodic orbit which is attracting is this periodic point of period 3. So, periodic point of period 3 is attracting ad it is attracting all the orbits over there. So, all you can see is periodic point of period 3 and what happens beyond that?

So, we find that what happens beyond that is when your maybe I should call it the seventh case right when alpha becomes greater than 1 plus root 8. Now these when alpha become greater than 1 plus root 8 this periodic orbit of period 3 also becomes rippling. Now you have a very terrific case here that you have so many pu have periodic points of all periods and all these periodic points are repelling. So, what happens to the orbit there? So, you find the orbit of a typical point never settles down anywhere, it just keeps on wandering right throughout the interval 0 1.

So, our periodic points are repelling here and what happens here is all orbits I should talk of other orbits because periodic orbits definitely have a definite motions. So, all other orbits keep wandering and I should say where. So, it keeps wandering throughout 0 one right all the other orbits to keep wandering throughout 0 1 and they do not settle anywhere and this typically happens since we are looking into the case we are looking basically into this case when 1 plus root 8 its less than alpha and this is less than or equal to 4.

So, we stop at this case 4 here because ultimately what happens when the case is greater than 4, when alpha turns out to be greater than 4 is exactly what happens at alpha equal to 4. So, there is actually known changing the scenario there and we find that the orbits keep on repelling. So, if we try to sum up whatever we have done till now whatever we have observed till now, maybe we will look into this particular picture.

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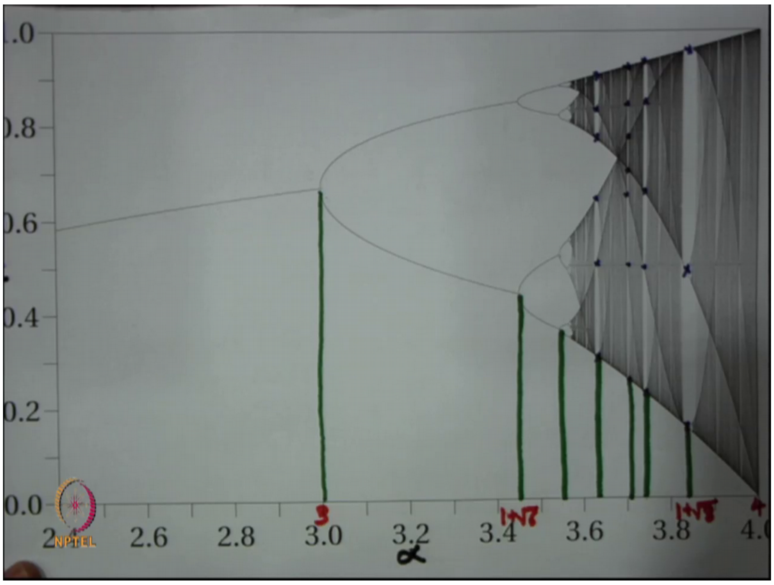


So, we try to analyze this particular picture here. So, here we find that you have a periodic orbit right we I should take up this blue.

So, up to 3 right up to from 0 to 1 you had this periodic orbit 0 was stable right, then you have another fixed point born and that is stable and that goes on till your alpha reaches 3 at 3 you find that there is a periodic orbit of period 2 born right. So, there is a periodic orbit of period 2 born here, and what you find is that up to from 3 to 1 plus root 6 this particular periodic orbit is stable. So, when you look into the dynamics what happens at infinity right only this stays this orbit stays nothing else stays right and what happens at 1 plus root 6? At 1 plus root 6 you again find that this is now repelling and there is another attacking periodic point of period 4 being born, and this continues till as we had looked into this case some 3.54 us right and there we find that again there is another periodic point of period 8 being born and so on.

So, this period doubling right we should we can say that there is a period doubling going on. So, this period doubling goes on right till maybe you are looking into the case of something like 3.6 here right 3.6 something here, where you have a periodic point of period 6 being born. So, you have a periodic point of period 6 being born here right and once you have a periodic point of period 6 being born the period no longer doubles right because we have now we basically have periodic points of all even period. So, period does not double, but still there are new and new periodic points being born and now these periodic points are basically periodic points of odd order right, or odd periods.

So, that they keep on being born till is here where you can find a periodic point of period say 7 right where whereas, here you find a periodic point of period 5 and ultimately at 1 plus root 8 right you find the periodic point of period 3. So, once you reach this point of period 3 you see that the dynamics becomes like you can you can almost see that almost every point is covered up over here.

So, with the full 0 1 is realized over here right. So, as and when you reach 4 of course, till you do not reach 4 the map is not on to. So, you find that everything is realized over here, and as you reach 4 you find that every single point is taken up by the orbit. So, if you will I try to look into the dynamics what happens at infinity you find that every point in 0 1 being taken up.

So, the population in in and now you can think of this in terms of the population. So, the population becomes sort of very stable here right. So, if I would look into alpha equal to 4 my population is very very stable in the sense that, it exactly remains the same though there is lot of dynamics going on we have this interaction also, we have the population decrease due to interaction also, but in all cases the population becomes as such it becomes stable it looks the same right because you have that it is taking up each and every value right I hope this is clear to all of you. So, today we stop here.