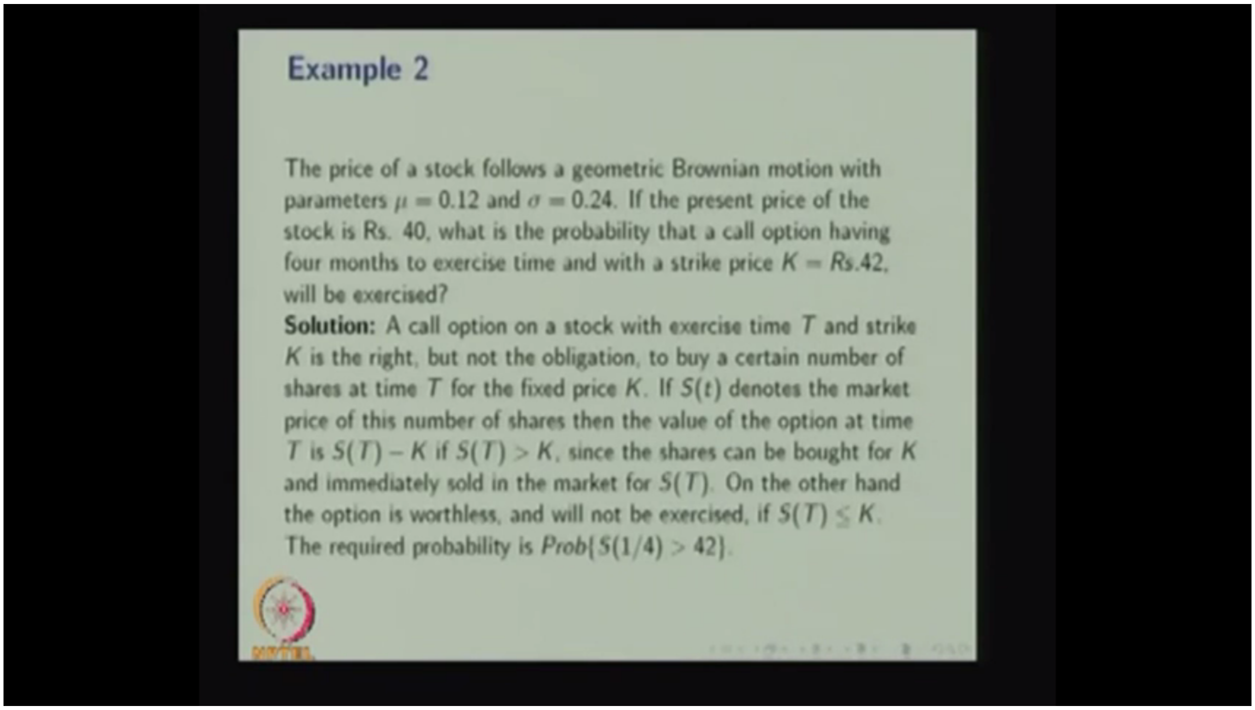
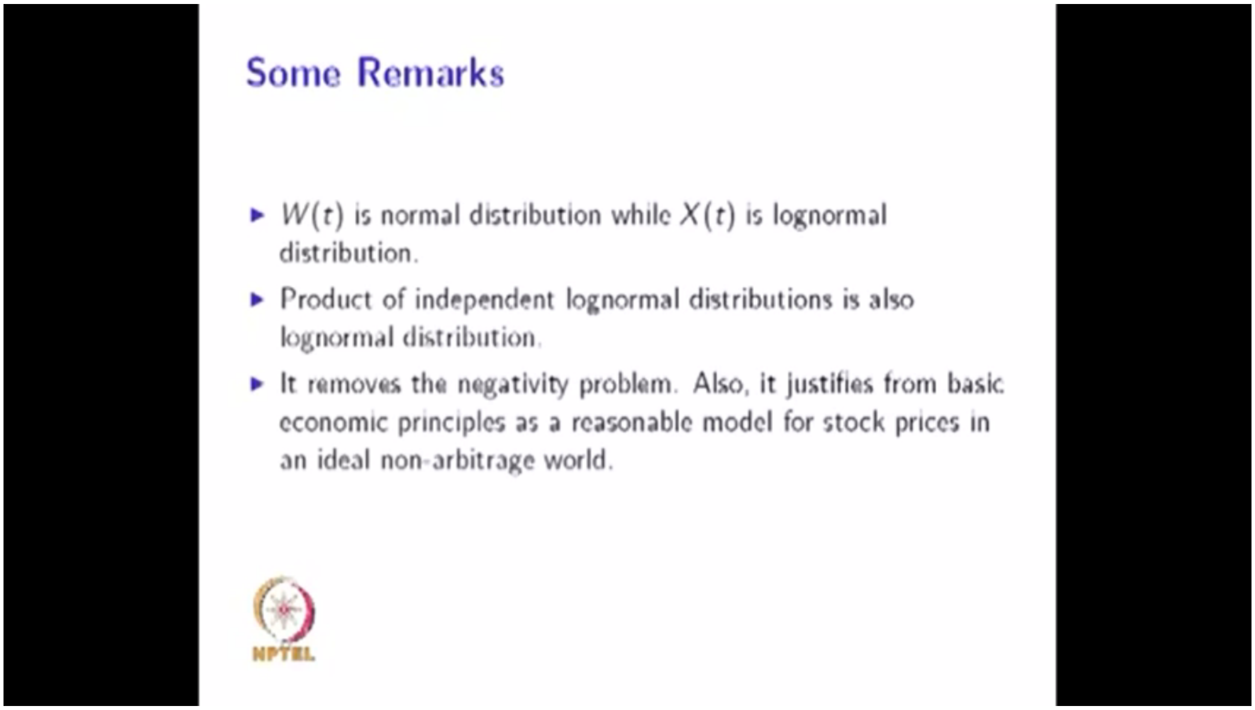
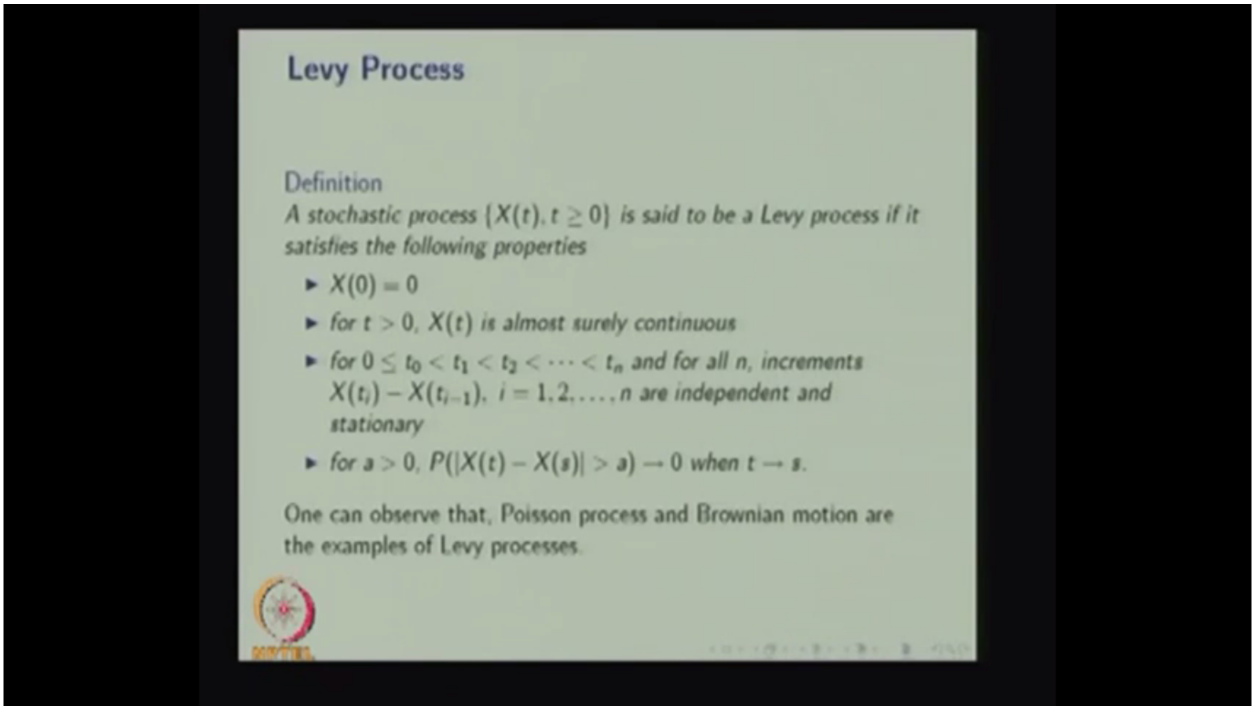


As a second example the price of a stock follows a geometric Brownian motion with the parameters mu is equal to 0.12 and Sigma 0.24 if the present price of the stock is Rs. 40 what is the probability that a call option having four months to exercise time and with the strike price K is equal to Rs. 42 will be exercised? So a call option on a stock with the exercise time T and the strike price K is right but not the obligation to buy a certain number of shares at time T part of things to price T because the call option is there right but not the obligation to buy the meaning of call option. If S of t denotes the market price of this number of shares then the value of the option at time t is the value of the option S of t minus K if S of t is greater than K. That is if the value of the price is greater than K then the value of the call option is a S of t minus K.

Since the shares can be bought for a K and immediately sold in the market part S of t. On the other hand the option is worthless and will not be exercised if S of t is less than or equal to K. Call option is right but not the obligation to buy hence S of t is greater than K then the value will be S of t minus K the value will be 0 the option is worthless and will not be exercised if the S of t is less than or equal to T. Hence, the question is what is the probability that the call option having a four months to exercise time [Indiscernible] [00:02:30] K will be exercise that means the [Indiscernible] [00:02:34] probability is what is the probability that S of one fourth is greater than [Indiscernible] [00:02:40] because a is 42 and the time is four months and as all the other values are in years we can assume this parameter are in years therefore four months will one fourth – four months will be treated as a here it is given for so the probability of S of one fourth is greater than 42 is a vector [Indiscernible] [00:03:14]. we are not computing actual probability because you can use a earlier concept because S of t follows a normal distribution. So using the abnormal distribution one can find the probability of S of this greater than 42.



As a remarks we know that Wt is the Brownian motion and the increments are independent as well as increments are stationary and Wt is normally distributed with the mean 0 and the variance of t it is a standard one. If it is not a standard one then the Wt is a normally distributed with the mean mu times t and the variance is Sigma square. While the X of t [Indiscernible] [00:04:18] the X of t is a lognormal distribution because it is X of t is X of 0 e power Wt hence the X of t is a lognormal distribution. So that is a relationship between the Brownian motion and the geometric Brownian motion. The Brownian motion each random variable is normally distributed whereas in the geometric Brownian motion each random variable is lognormal distribution. Therefore you can use the central limit theorem as a sum of random variable will tends to again normal distribution if they are independent but here since X of t is a lognormal distribution you can use those properties with the [Indiscernible] [00:05:09] that's the second one. The product of independent lognormal distribution is also lognormal distribution. So the way we use the sum of independent normal distribution is a normal distribution and the normal distribution and the log normal distributions are [Indiscernible] [00:05:27] in the form of X of t is equal to X of 0 e power Wt hence the product of independent lognormal distributions is also a lognormal distribution. Therefore proved. Hence suppose you have a independent geometric Brownian motion if you make a product then that is also a geometric Brownian motion. The third remark it removes the negativity property. Means the Brownian motion has the range minus infinity to infinity since we made X of t is equal to X of 0 e power Wt the range of Xt is 0 to infinity it removes the negativity property. Also it justifies from the basic economic principles as the reasonable model or stock prices in real non arbitrate world. Hence it removes the negativity problem or it takes the non-negative values it justifies from the basic economic principles as the reasonable model for stock prices in an ideal non-arbitrary world.



Finally we are presenting here process derived from the Brownian motion. This is not the derived one but this is a very generalized stochastic process comparing with the Brownian motion and geometric Brownian motion. This has a lot of applications in finance. The definition; a stochastic process X of t is said to be a Levy process if it satisfies the following properties. The first property X of 0 is equal to 0. The same property we have used when we define the Poisson process. Then we define Brownian motion also. So here also the Levy process it satisfies the properties X of 0 is equal to 0. The second property for t greater than 0 Xt is almost surely continuous. So if you see the sample path of X of t it is almost surely continuous. The third property for 0 less than or equal to t naught less than t1, less than t2 and so on till less than tn for all n the increments that is X of ti minus X of ti minus 1 where i varies from 1 to n the increments are independent as well as stationary. This is proof for all arbitrary n time 1s for all. The fourth condition for a greater than 0 the probability of in absolute of X of t minus X of s greater than a, atends to 0 as t tends to s. So if these three properties are satisfied by any stochastic process then that stochastic process is said to be a Levy process. Any stochastic process satisfying these four properties will be called it as Levy process.

One can observe the Poisson process and the Brownian motion on the examples of Levy process. The Poisson process is their continuous time discrete state stochastic process whereas the Brownian motion is the continuous time, continuous state stochastic process. The Brownian motion the sample path are right continuous. Thus you see the sample path the inter-arrival of events are independent and they are exponentially distributed with the parameter lambda therefore whenever some event occurs the sample path that implement with the unit and it says the same value till the next event occurs. So in the Levy process the property is for t greater than 0 X of t is almost surely continuous. So that include the sample path is a continuous as well as the sample path is the right [Indiscernible] [00:10:48]. Hence the property we discuss in the Poisson process for the sample path as well as the property discuss in the Brownian motion for the sample path both are satisfied by saying X of t is almost surely continuous. The first property is used in both Poisson process and the Brownian motion because suppose N of t is a Poisson process then n of 0 is equal to 0. The Brownian motion is a Wt then to W0 is equal to 0. So the first property is the same for a Poisson process and the Brownian motion whereas the second property in the sample path after Poisson process are right continuous functions whereas the sample path of Brownian motion are continuous motion.

The third property is the same both Poisson process and Brownian because both the places we have the third property the increments are independent as well as stationary for all n. We won't discuss what is a distribution of the increments in the third problem. We discuss only the increments are independent and stationary. Whereas in the Poisson process definition we give the fourth property as the implements are Poisson distributed random variable with the parameter lambda times t minus s where Wt minus Ws is the increment for S is less than 1 whereas in the Brownian motion we say we supply the fourth property as the Wt minus Ws is the increment which is normally distributed which is normal distributed random variable with the parameters mu times t – s and the variance is Sigma square t minus s for the standard Brownian motion the mu is equal to 0, the Sigma square is equal to 1 whereas here for a greater than 0 the probability of in absolute a X of t minus X of s greater than a will tends to 0 when t tends to s. So this property is valid both for Poisson distribution random variable in the Poisson process as well as the normal distributed random variable in the Brownian motion. Hence, Poisson process and Brownian motions are the examples of Levy process. So this is a more generalized stochastic process then you are comparing with the Brownian motion and the [Indiscernible] [00:13:59] motion.

So in this lecture we have covered the definition of a geometric Brownian motion and the important property that is a Markov property of the geometric Brownian motion and then we have discussed applications of geometric Brownian motion, how one can use the geometric Brownian motion to model the stock price. And also we have given two examples. Finally we have discussed the Levy process. The definition as well as the standard examples of a Levy process. So with this we are completing the lecture two processes derived from Brownian motion.

Here is the list of references.

References

- S Karlin and H M Taylor, "A First Course in Stochastic Processes", 2nd edition, Academic Press, New York, 1975.
- J Medhi, "Stochastic Processes", 3rd edition, New Age International Publishers, 2009.