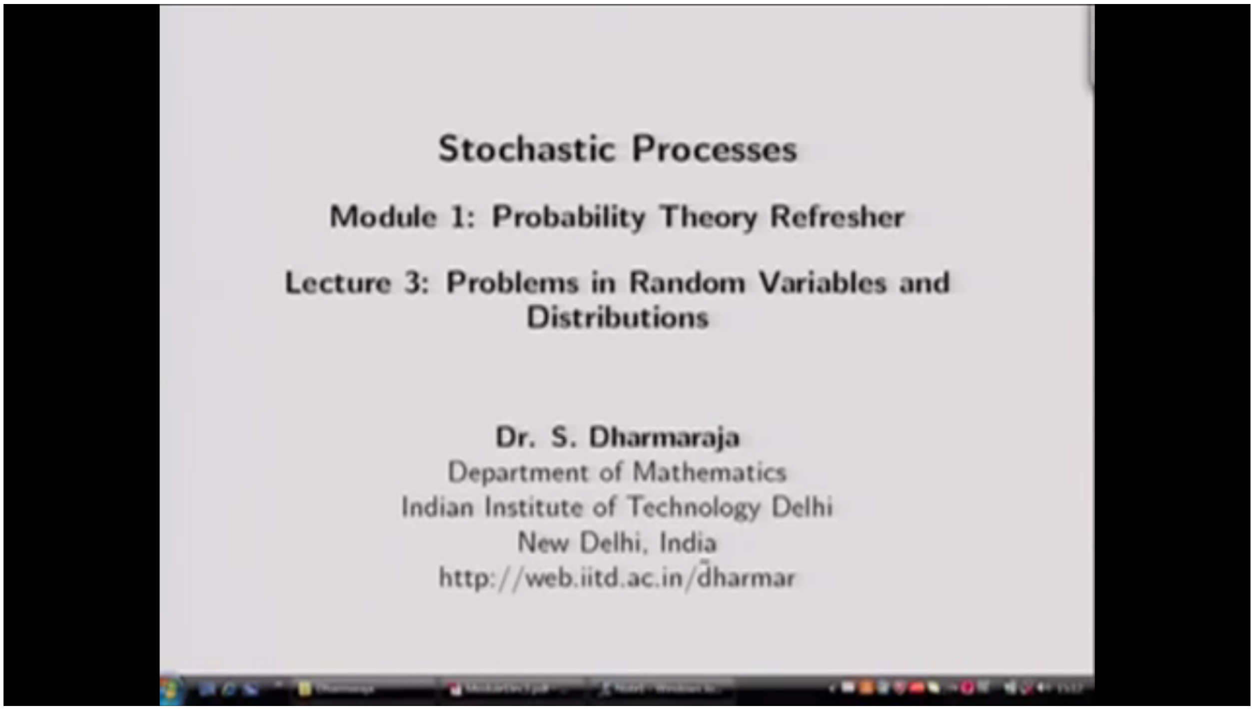

Video course on

Stochastic Processes -1

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Module 1 : Probability Theory Refresher

Lecture #3

Problems in Random Variables and Distributions

This is a stochastic processes model 1 probability theory refresher. Lecture 3 problems in random variables and distributions.

Let as a first problem, X be a random variable having geometric distribution with the parameter P. Our interest is to find – our interest is to prove that probability of X is equal to n plus K given X takes the value greater than n that is same as the probability that X takes the value K for every integers N and K.

We can prove this result by starting from the left hand side that is probability of the X takes a value n plus K given X greater than n by definition. This is same as probability of X is equal to n plus K intersection X greater than n divided by probability of X greater than n. That is same as the numerator X is equal to greater than N means all possible values. N is equal to n plus K that means that the intersection is going to be probability of X takes the value, n plus K that is the denominator is probability of X is greater than n. That is same as since X is a geometric distribution with the parameter P. The probability of X equal to n plus K that is nothing but 1 minus P time P power n plus K minus 1 into P; whereas the denominator probability of X is greater than n that means summation I is equal to n plus 1 to infinity 1 minus P power I minus 1 multiplied by P. That is same as emulator you can keep it as it is whereas the denominator since the summation I is equal to n plus K to infinity you can take P times 1 minus P power n common outside the remaining terms are 1 plus 1 minus P. The third term will be 1 minus P whole square and so on. Therefore, you can still simplify you will get one minus P power n plus K minus one divided by one minus P power n keep it as it is. This series will have the value 1 minus 1 minus P. Therefore if you further simplify you get 1 minus P power K minus 1 multiplied by P. That is nothing but probability of X is equal to K.

So this result says the probability of X equal to n plus K given X is greater than n that is same as probability of X is equal to K for all n at K. This is the important property of geometric distribution and this property is called a memoryless property.

We move into the next problem. Let X be a random variable having Gamma distribution with the parameter n, we assume that n is a positive integer and the other parameter is lambda. Then the cumulative distribution function CDF of X is given by capital F of X for the random variable X that is 1 minus summation I is equal to 0 to n minus 1 lambda X power I E power minus lambda times X divided by I factorial. So when every X is a gamma distribution with the parameters N and lambda then the CDF can be written in this form. We know that the probability density function of the Gamma distribution is lambda power n X power n minus 1 e power minus lambda X divided by Gamma of n. Since n is a positive integer Gamma of n is n minus 1 factorial.

Now we can find out the CDF of this random variable. That is nothing but minus infinity to X the probability density function. That is same as since the F of X is this is valid for X is greater than 0 and lambda is greater than 0 so this integration is valid from 0 to X lambda power n e power n minus 1 e power minus lambda times t divided by Gamma of n dt. So now we have to integrate this one and get the expression for the CDF of the random variable X.

So what we can do make a substitution. Lambda times t that is same as you make it as sum mu. Therefore this integration becomes the integration from zero to lambda times X mu power n minus 1 e power minus mu divided by comma of n into d of mu. That is same as 1 minus integration goes from lambda X to infinity mu power n minus 1 e power minus mu divided by Gamma of n d mu. That is same as 1 minus since the n is a positive integer Gamma of n is n minus 1 factorial. So you can take it outside. You can do this integration by parts. So you will get mu power n minus 1 e power minus mu divided by minus 1 between the limits lambda X to infinity minus integration from lambda X to infinity n minus 1 times mu of n minus 2 e power minus mu divided by minus 1 d mu. So the whole thing is multiplied by n minus 1 factorial.

Now you can integrate the second term again by integration by parts and when you substitute the limits for mu is infinity and as well as mu is equal to lambda X and then subsequently if you do the integration by parts you will get 1 minus n minus 1 factorial lambda X power n minus 1 e power minus lambda X. Then the next term will be minus lambda X power n minus 2 e power minus lambda X by n minus 2 factorial. Similarly the other terms.

The last term will be by doing integration by parts again and again, the last term you will get minus of lambda X power 0 e power minus lambda X by 0 factorial. This we can write it in the form 1 minus summation I is equal to 0 to n minus 1 lambda X power I e power minus lambda X by I factorial. So here we are finding the CDF of the Gamma distribution when one of the integer is a positive integer. One of the parameters is a positive integer. This result will be useful in finding the total time spend in the queuing system. That will be discussed in the later models.