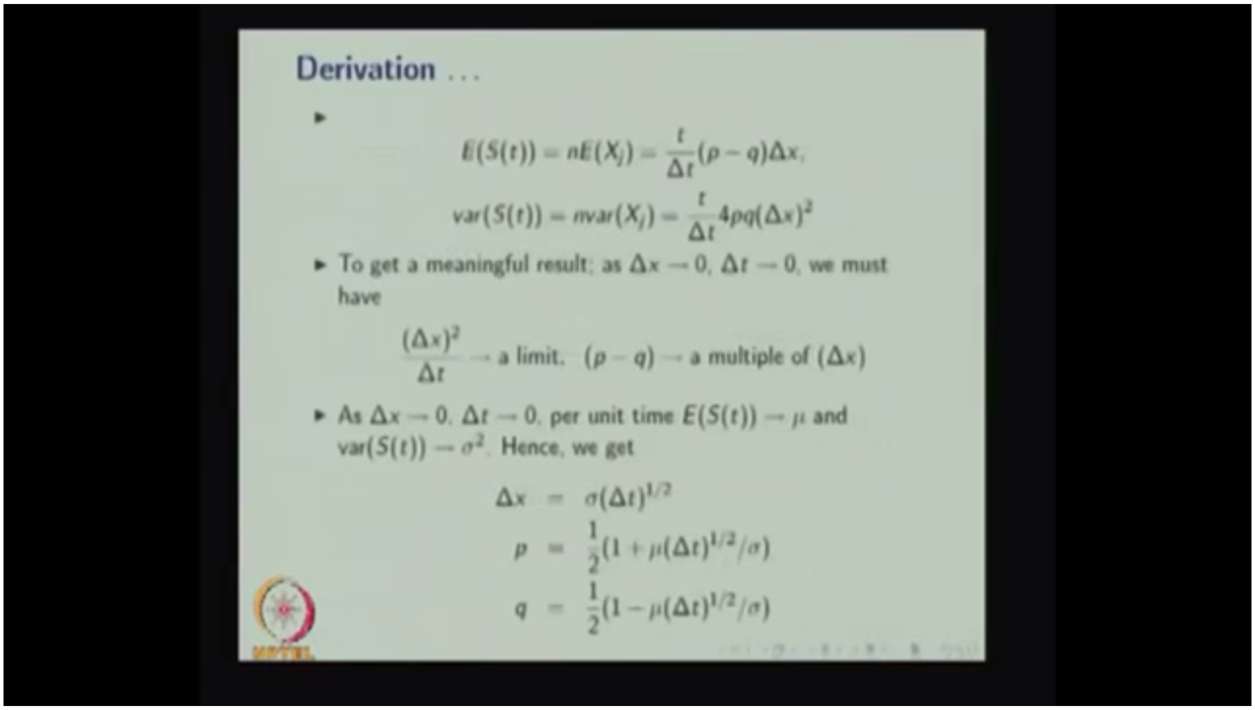
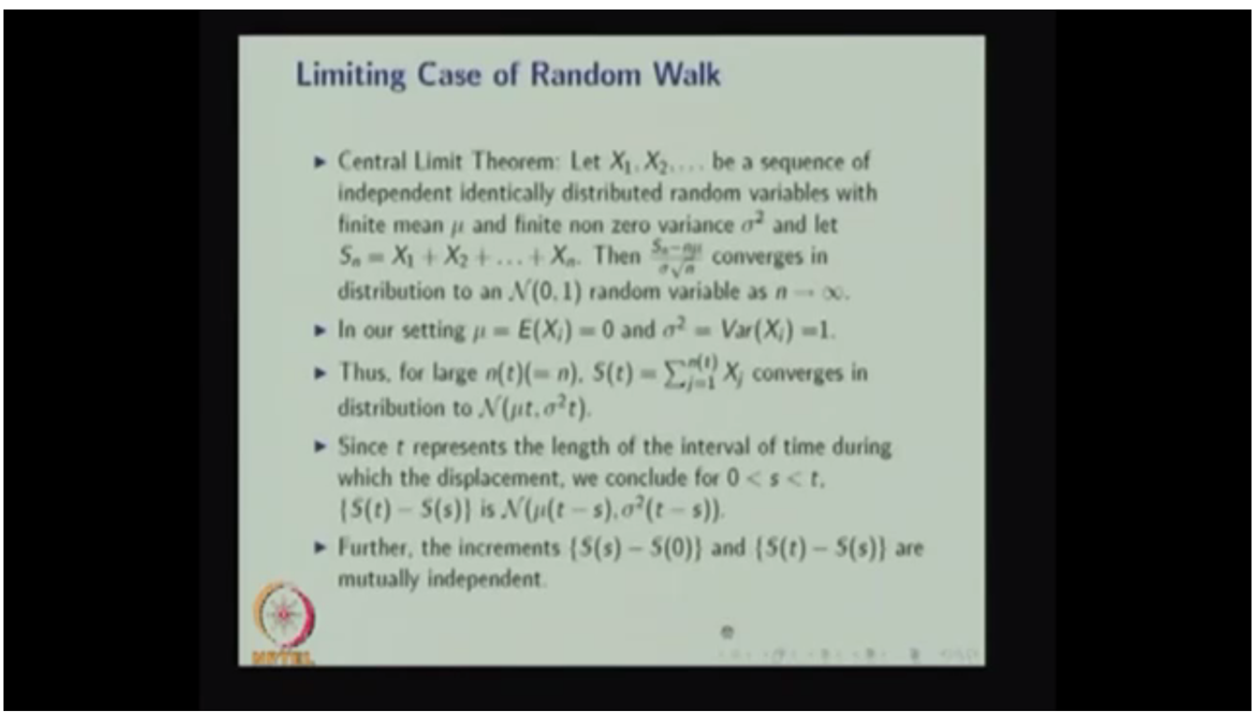
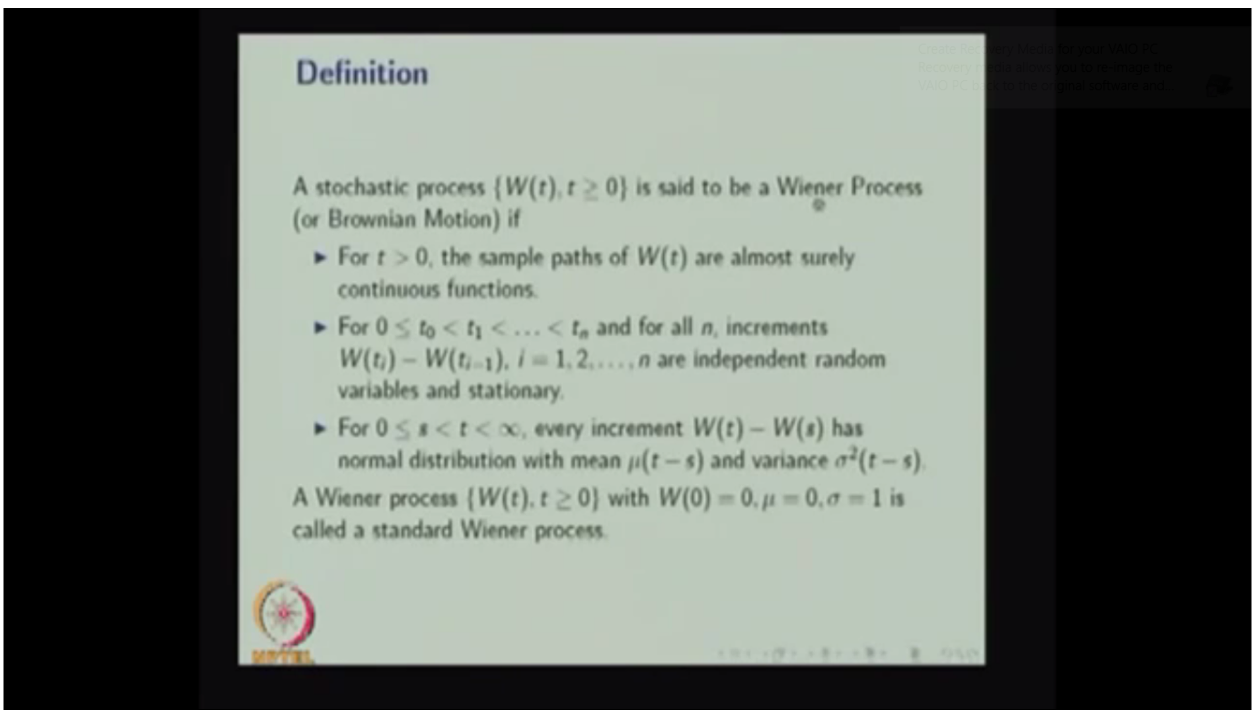


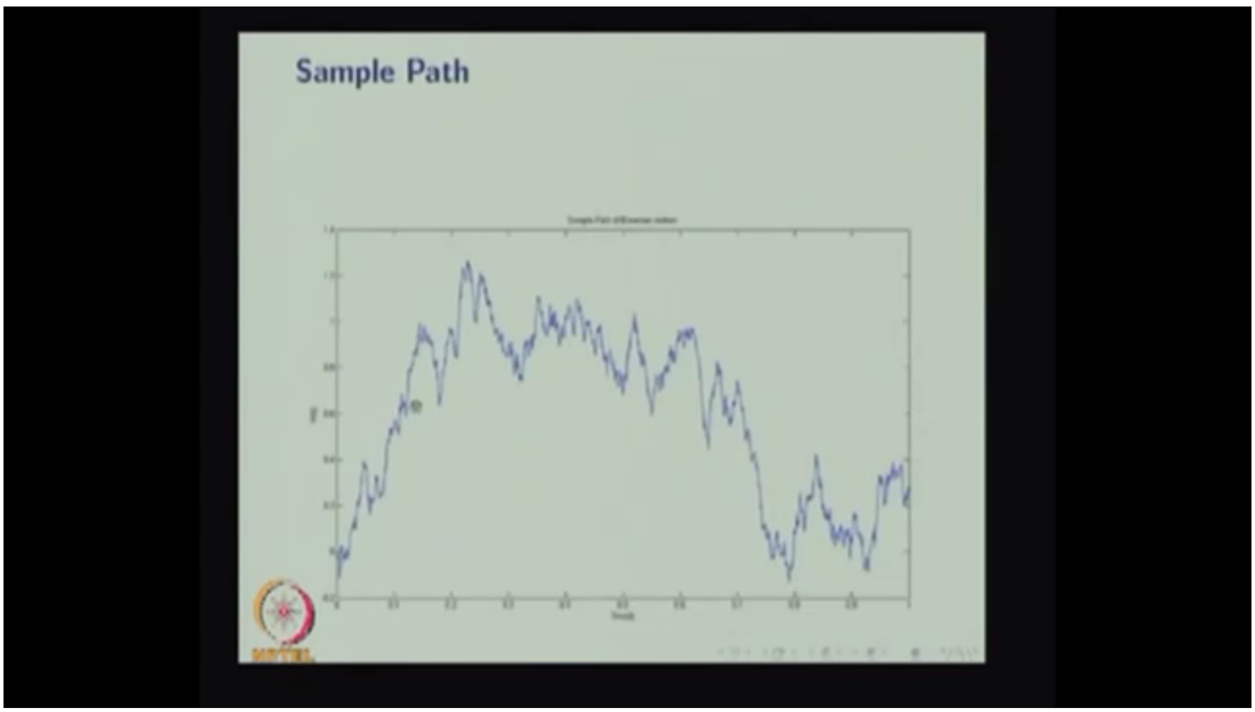

Now you can make a delta x tends to 0 as well as delta t tends to 0 therefore you will get a limit by using the simple calculation the Delta x is this much where p and q is equal to off times 1 plus mu times this and 1 minus mu times divided by delta.



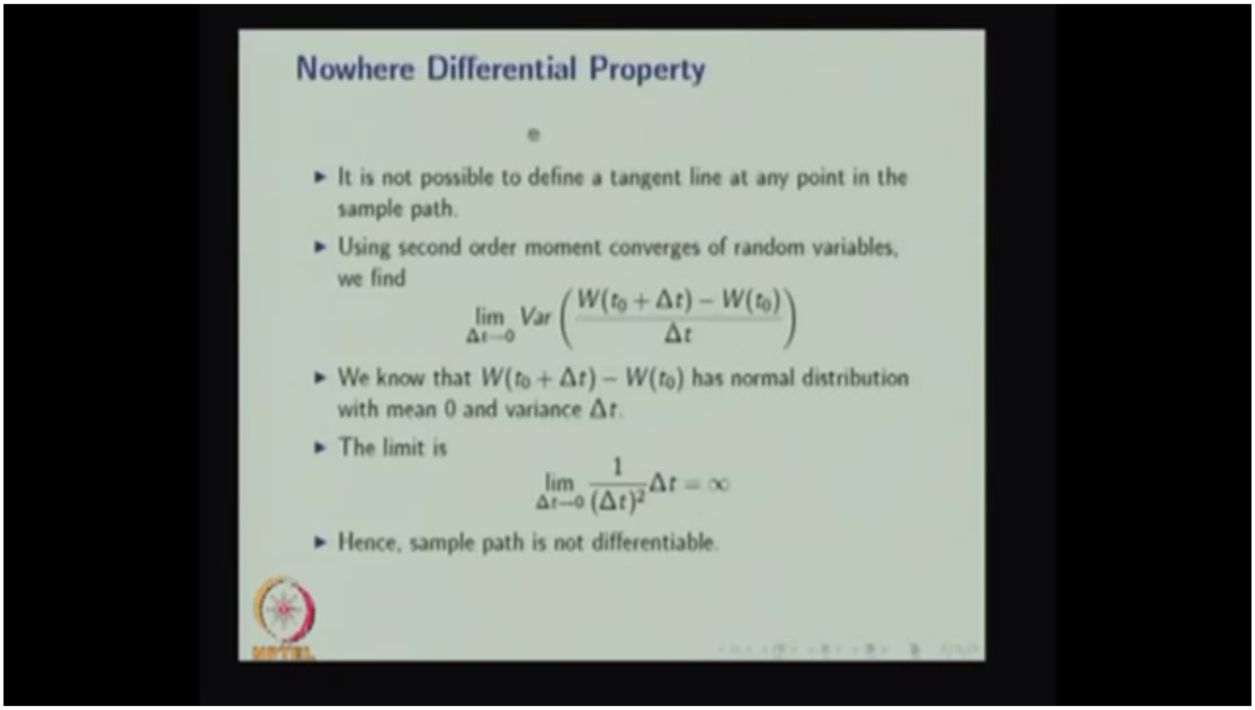
Now we are using the central limit theorem. Let X1, X2 be a sequence of independent identically distributed random variables with a finite mean mu and the finite non-zero variance Sigma square and let Sn be a sum of first n random variables then Sn minus the mean of this random variable divided by the standard deviation of this random variable convergence in distributed to the normal distributed random variable with the mean 0 variance. So we are going to use this central limit theorem for our random walk scenario and for the large N, N of t is equal to n where N is very large we can conclude the St converges in distribution to the mean of this random variable S of t that is mu times t and the variance of this random variable is Sigma square whereas here we have used the central limit theorem the random variable minus their mean divided by the standard deviation converges to the standard map but here we are saying the S of t converges to the normally distributed random variable with the mean mu times t and the variance is Sigma square t that is different from this mu and Sigma square where mu is discussed here and the Sigma is discussed here. Since t represents the length of the interval of a time during which the displacement therefore instead of the S of t you can go for S of t minus S of s since it is – t is the length of the interval therefore you can go for the S of t minus S of s that will converges to the normal distribution with the mean mu times t minus s and the variance sigma square times t minus s where s is less than the way we discussed the properties of random walk as the increments in the [Indiscernible] [00:03:05] it has the property of increments are stationary as well as independent the same logic can be used here so here the increments S of S minus s of zero are mutually independent increments are independent also.




Now we are defining the Brownian motion Wiener process. A stochastic process is said to be a Wiener process or Brownian motion if it satisfies these three conditions. For t greater than 0 the sample paths of Wt for almost surely continuous functions for the interval 0 to Tn in this form for all n the increments are independent as well as stationary. The increments are independent random variables as for the stationary and every increment as normal distribution with the mean mu times t minus s and variance Sigma square t minus s. This is what we have concluded in the limiting case of normal distribution. The increments is normally distributed with the mean this much and the variance this much. So this is what we are given as the conditions of a stochastic process will be a Wiener process. The Wiener process Wt with the mu of 0 is equal to 0, mu is equal to 0 and Sigma square is equal to 1 is call the standard Wiener process. Whenever it is normally distributed with the mean 0 and the variance t minus s that means the Sigma square will be treated as 1 and the mu will be treated as 0 it also W0 is equal to 0 then it is a standard Brownian motion or standard Wiener process. So any stochastic process satisfying these three conditions will be Wiener process or Brownian motion.



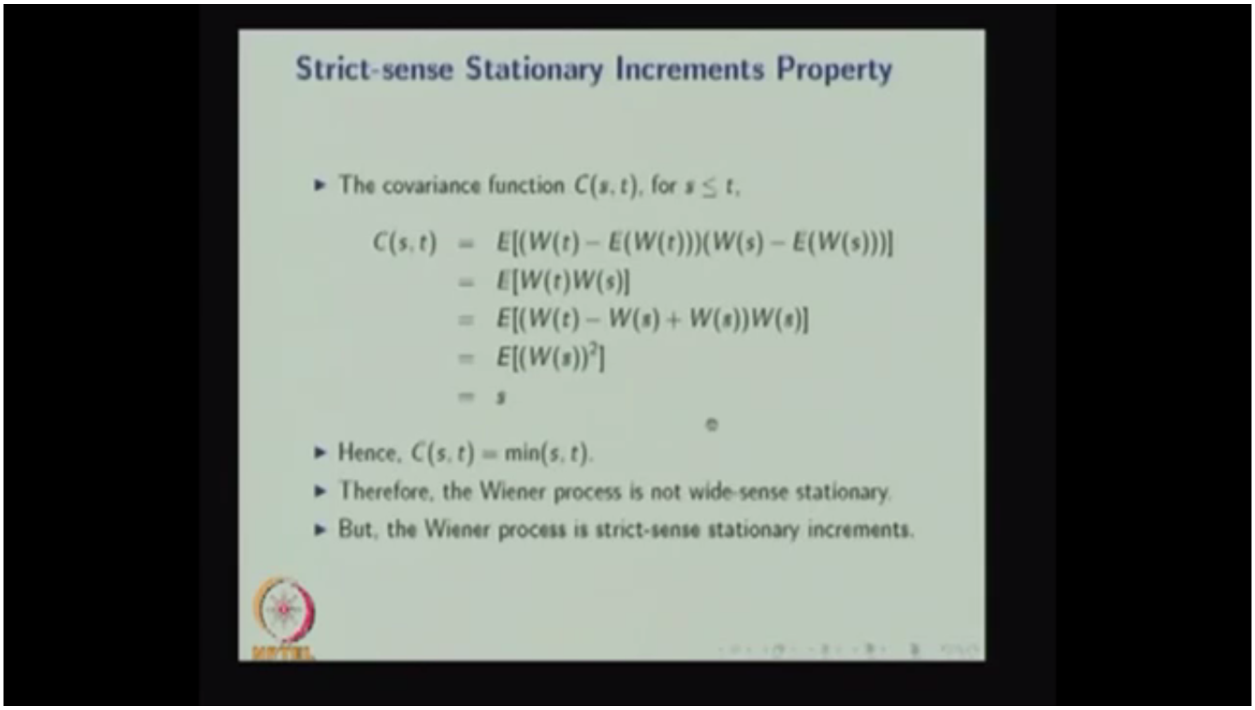

the sample path of Wiener process it looks like this. By definition Wt plus S minus Wt that is the increment follows normal distribution. It can take a positive and negative values. The sample path of Wt is continuous. There is no jumps and the limiting case of a random walk will be the Brownian motion. That also one can visualize in the sample path of Brownian motion.




Now we are going to discuss a few important properties of Brownian motion. The first important property is nowhere differential property. You can see the sample path of Brownian motion it is a continuous function but it has too many fluctuation at every point so this is a one sample path. So the first property says the sample path is not a differentiable anywhere or it is a nowhere differential. It is not possible to define a tangent line at any point in the sample. You can see it in the sample path the figure also. Using second order moment convergence of random variable we find the limit delta t tends to 0 the variance of the difference of this random variable divided by delta t you find out this limit. If this limit is a finite then we can conclude it is differentiable at the point t naught. Suppose you have a real valued function F of x and if you want to conclude the real valued function F of x as a derivative at the point X naught then you should find out limit delta t tends to 0 F of t naught plus Delta t minus F of t naught divided by delta t if this limit is a finite then you can conclude the real valued function F as the limit at t naught. Since the W's are the random variables and we know the mean and variance and also the distribution and the difference is going to be a random variable as a delta T tends to 0 it is going to be we should find out the convergence of these difference of random variable divide delta t. So one can use any mode of convergence to conclude to find out the limit delta t tends to 0 of this quantity but here we are using the second order moment convergence therefore we are finding limit delta t tends to 0 variance of this random variable. The difference is a random variable difference divided by Delta t is random variable. So we are finding what is the convergence of the function of random variable via second order moment of convergence.

So if you find out this quantity since this difference as a normal distribution with the mean 0 and the variance Delta t therefore the variance delta t has to be treated as a constant so the variance of a 1 divided by constant times this will be 1 divided by delta t whole square and the variance of the difference of this random variable is delta t therefore you will get infinity as a delta t tends to 0.

Since this limit is equal to infinity we conclude the sample path is not differentiable at t naught since t naught is a obituary time point therefore it is a nowhere differentiable or it is the sample path is not a differentiable at every point every time.




The second important property that is a strict sense stationary increment. We are not saying the given stochastic process Brownian motion is a strict sense stationary means here we are saying the increments are strict sense. The increments are strict sense stationary that means the increments are [Indiscernible] [00:10:47] the time invariant property. The strict sense stationary means it has the time invariant property in the distortion. So far that we the covariance function. You know the definition of covariance. So covariance of s,t it is a random. It is going to be s. The covariance of s, t is equal to minimum of s,t because here we have concluded for s is less than t it is s to make it t is less than or equal to s we will get t hence C of s,t is minimum of s,t therefore Wiener process is not wide sense stationery whereas we can conclude it is a strict sense stationary increments. That means first you find out the increments then one can prove for any finite-dimensional the joint distribution is same as the joint distribution by shifting the time scale H. For every H the increments that explain the condition the joint distribution are same the original joint distribution as well as the incremented by H therefore it is going to be a strict sense stationary and using the covariance function we are concluding it is not a wide sense stationary.