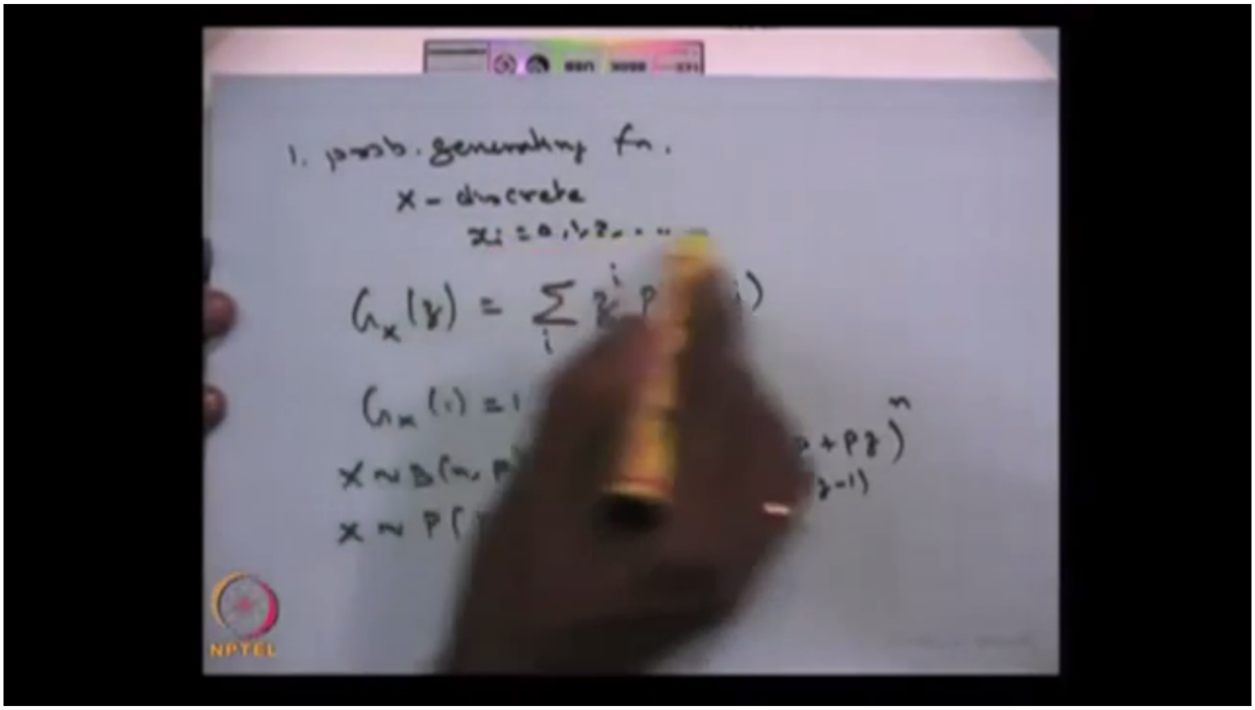
Now we are going to discuss few generating functions. So the first one is called probability generating function. So this is possible only with the random variable is a discrete random variable and the possible values of Xi's has to take 0 or 1 or 2 like that that means if the possible values of the random variable X takes a value only 0, 1, 2, and so on then you can able to define what is the probability generating function for the random variable X is with the notation Gx(z) that is a probability generating function for the random variable X as a function of Z that is nothing but summation Z power I and what is the probability X takes the value I for all possible values of I. That means if the discrete random variable takes a only countably finite value then the probability generating function is a polynomial. If the discrete random variable takes countably infinite values then it is going to be the series.

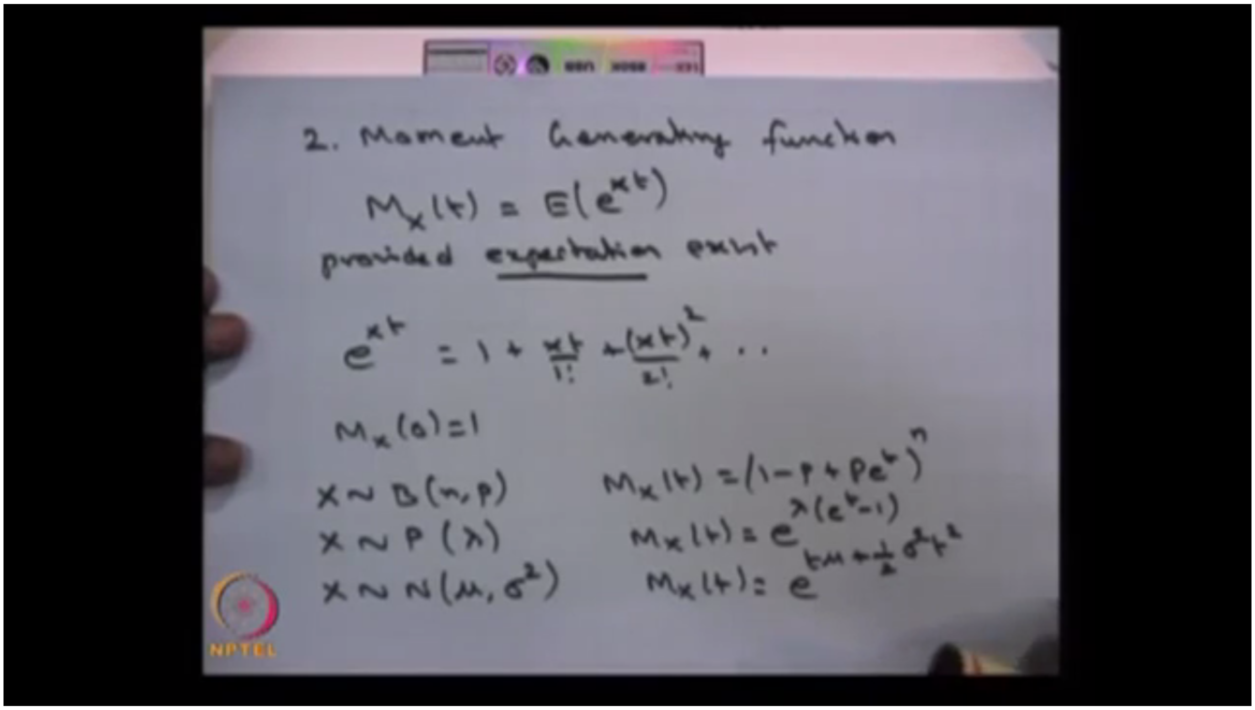
So this series is going to be always converges and you can able to find out what is the value at 1 that is going to be 1 and since it is going to be Z power I by differentiating you can get there is an easy formula or there is a relation between the moment of [Indiscernible] [00:01:36] function in the derivative of [Indiscernible] [00:01:39] derivative and substituting set is equal to 1 and suppose X is going to be a binomial distribution with the parameters n and p then you can find out what is the probability generating function for the random variable X that is going to be 1 minus P plus P times Z power n because the binomial distribution as the possible values are going to be 0 to n. Therefore, you will get the polynomial of degree n.

Suppose X is going to be a Poisson distribution with the parameter lambda because this is also a discrete random variable and the possible values are going to be countably infinite whereas here the possible values are going to be countably finite. So here also you can find out what is the probability mass function, sorry what is a probability generating function for random variable X and that is going to be e power lambda times z minus 1. So like that you can find out a probability generating function for only of a discrete type random variable with the possible values has to be countably finite or countably infinite with the 0, 1, 2, and so on.




The next generating function which I am going to explain that is a moment generating function. The way we use the word moment generating function it will use the moments of all [Indiscernible] [00:03:09] that means it uses the first order moment, second order moment and the third order moment and you can define the moment generating function for the random variable X as a function of T that is nothing but expectation of E power X times T provided the expectation exist. That is very important. That means since I am using the expectation of a function of a random variable and that to this function is e power Xt you can expand the e power Xt as 1 plus Xt by factorial 1 plus Xt power 2 by factorial 2 and so on. Therefore, that is nothing but the moment generating function for the random variable X is nothing but expectation of this expansion. That means expectation of 1 plus expectation of this plus expectation of this plus 1. that means if the moment of all order n exist then you can able to get what is the moment generating function for the random variable X. That is the provided condition is important as long as the right hand side expectation exists we can able to give the moment generating function for the random variable X. So here also many properties are there. I am just giving one property Mx of 0 is going to be 1 and there are some property which relate with the moment of order n with the derivative of moment generating function and I can give one simple example if X is going to be binomial distribution with the parameters n and p then the moment generating function for the random variable X that is going to be 1 minus P plus P times et e power t power n. Similarly if X is going to be a Poisson with the parameter lambda then you may get the moment generating function is going to be e power lambda times E power t minus 1.

And you can go for continuous random variable also. If X is going to be a normal distribution with the parameters mu and Sigma square then the moment generating function is going to be e power T times mu plus of sigma square t square. So this is very important moment generating function because we are going to use this moment generating function of normal distribution in the stochastic process part also.

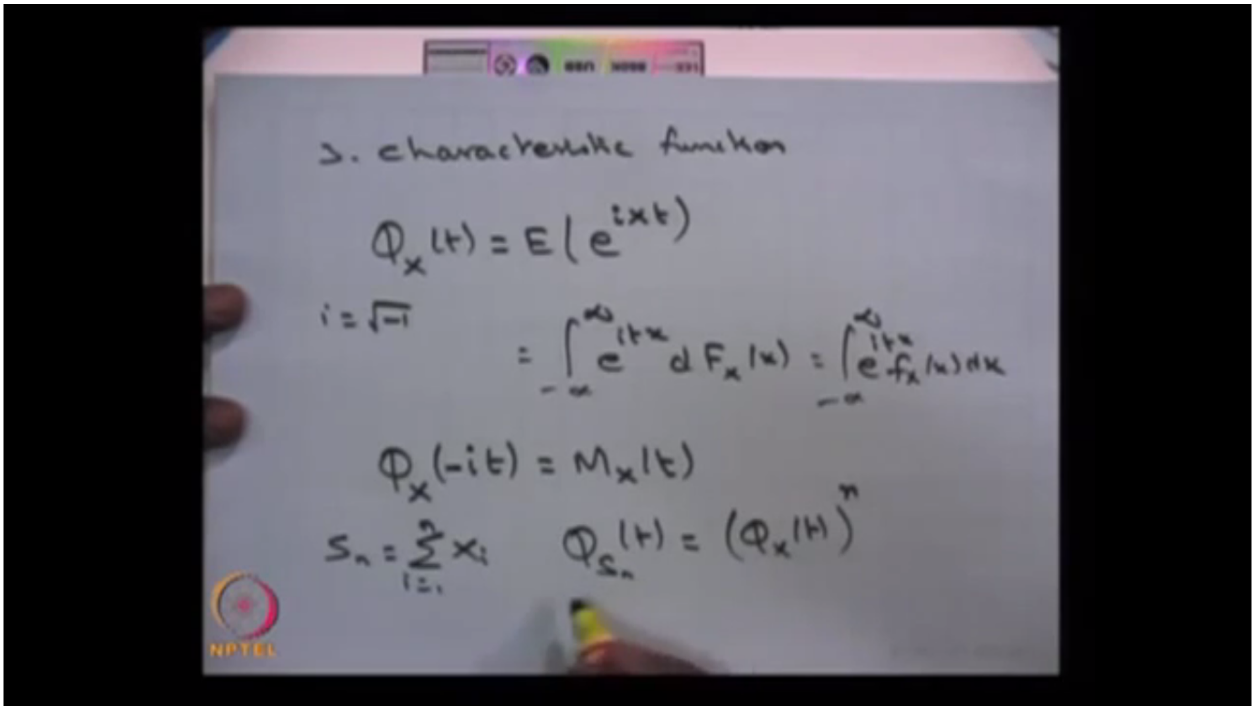



There is some important property over the moment generating function. Suppose you have a n random variables and all n Xi's random variables are iid. That means a independent identically distributed random variable. That means the distribution of when you say the random variable x and y are identically distributed that means that the CDF of X and the CDF of Y are same. For all x and y both the values are going to be same then we can conclude that both the random variables are going to be identically distributed. So here I am saying the n random variables are iid random variable that means they are not only identical they are mutually independent also. If this is a situation and my interest is to find out what is the MGF of sum of n random variables. That is Sn. So the moment generating function for the random variable Sn is going to be the product of the MGF of individual random variable.

Since they are identical the MGF is also going to be identical therefore this is same as you find out the MGF of any one random variable then make the power. So this independent random variables having the property when you are trying to find out the MGF of sum of random variable that is same as the product of MGF of individual random variables. Yes here there is a one more property over the MGF. Suppose you find out the MGF of some unknown random variable and that matches with the MGF of any standard random variables then you can conclude the particular unknown random variable also distributed in the same way. That means the way you are able to use the CDFs are same then the corresponding random variables are identical, same way if the MGF of two different random variables are same then you can conclude the random variables also identically distributed.

Third we are going to consider another generating function that is called a characteristic function. This is important than the other two generating function because the probability generating function will exist only for the discrete random variable and the moment generating function will exist only if the moments of all order and exist whereas the characteristic function exists for any random variable whether the random variable is a discrete or their moments of all order they exist or not immaterial of that the characteristic function exists for all the random variable that I am using the notation psi suffix X as a function of T that is going to be expectation of E power I times Xt. Here the I is the complex number that is a square root of minus minus 1. So play a very important role such that this expectation is going to be always exist whether the moment exists or not. Therefore, the characteristic function always exists.

You can able to give the interpretation of e power this is same as minus infinity to infinity e power I times a TX T of CDF of the random variable. So that means whether the random variables are discrete or continuous or mixed you are integrating with respect to the CDF of the integrant function is e power I times the TX where I is the complex quantity and if you find out the absolute this absolute this is going to be using the usual complex functions you can make out this is going to be always less than or equal to 1 in the absolute sense. Therefore, this integration is exist and this integration is nothing but the [Indiscernible] [00:09:44] integration and if the function is going to be – if the random variables are continuous then you can able to write this is same as minus infinity to infinity e power I times a TX of the density function integration with respect to X. That means this is nothing but the Fourier transform of F and here we have this F is going to be the probability density function and you are integrating the probability density function along with E power I times tx and this quantity is going to be always converges whereas the moment generating function without the term I, the expectation may exist or may not exist. Therefore the MHF may exist or may not exist for some random variable. And I can relate with the characteristic function with the MGF with the form say X of minus I times T that is same as the MGF of the random variable T. That means I can able to say what is the MGF of the random variable X that is same as the characteristic function of minus I times T where I is the complex quantity. And here also the property of the summation of suppose I am trying to find out what is the characteristic function of sum of n random variables and each all the random variables are iid random variable then the characteristic function of Sn is same as when Xi's are iid random variable then the characteristic function of each random variable power n. And this also has the property of uniqueness that means if two random variables' characteristic functions are same then you can conclude both the random variables are identically distributed.

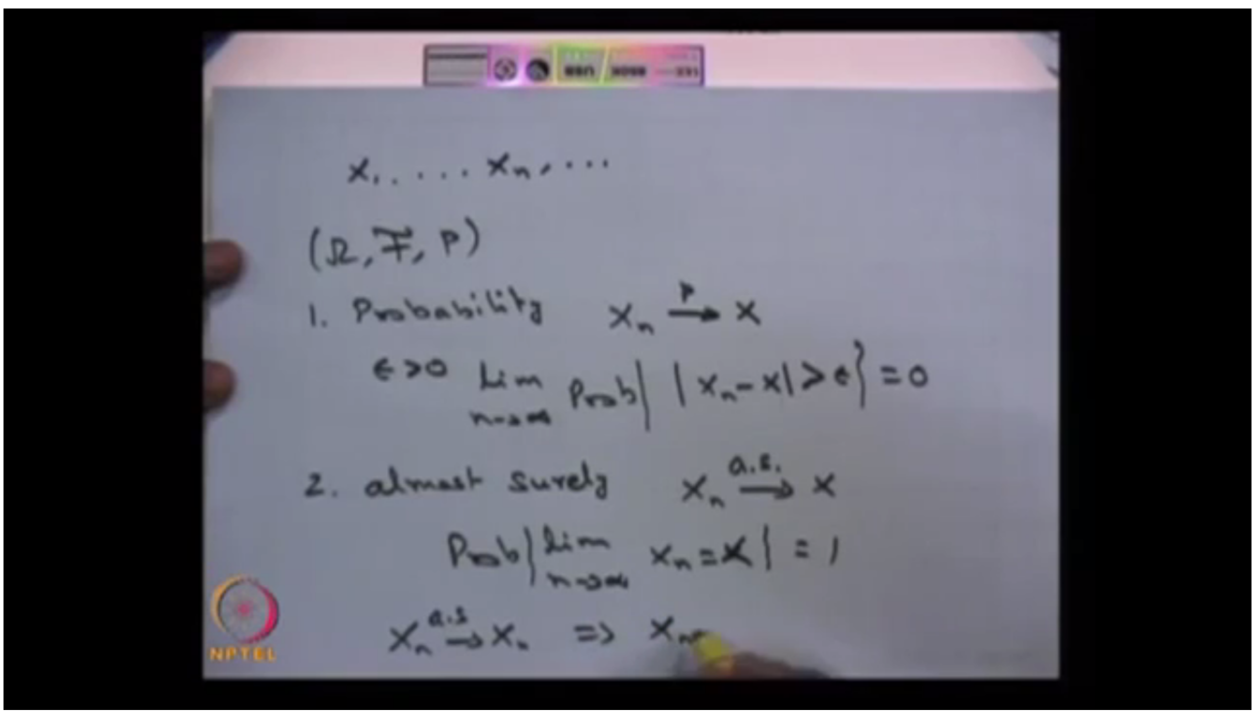



So as a conclusion we have discussed three different functions. First one is a probability generating function and the second one is the moment generating function and the third one is a characteristic function and we are going to use all those functions and all other properties of joint probability density function distribution everything we are going to use it in the – at the time of stochastic process discussion.

Next we are going to discuss what is the – how to define or how we can explain the sequence of random variable converges to one random variable. Till now we started with started with the one random variable. Then using the function of a random variable you can always land up another random variable or from the scratch you can create another random variable because random variable is nothing but a real valid function satisfying that one particular property inverse image is also belonging to F. Therefore, you can create a many more or countably infinite random variables or uncountably many random variables also over the same probability space. That means you have a one probability space and in the one single probability space you can always create either countably infinite or uncountably many random variables and once you are able to create many random variables now our issue is what could be the convergence of sequence of random variable. That means if you know the distribution of each random variable and what could be the distribution of the random variable the Xn as n tends to infinity. So in this we are going to discussed different modes of convergence that is the first one is called convergence in probability.

That means if I say sequence of random variable Xn converges to the random variable some X in probability that means if I take any epsilon greater than 0 then limit n tends to infinity of a probability of absolute of Xn minus X which is greater than epsilon is 0. If this property is satisfied for any epsilon greater than 0 then I can conclude the sequence of random variable converges to one particular random variable X in probability. That means this is the convergence in probability sense. That means you collected possible outcomes that find out the difference of X and minus X which is in the absolute greater than epsilon that means you find out what is the – what are all the possible event which is away from the length of two epsilon you collect all possible outcomes and that possible outcomes is that probability is going to be 0 then it is a convergence in probability that means you are not doing the convergence in the real analysis the way you do you are trying to find out the event then you are finding the probability. Therefore, this is called the convergence in probability.

The second one it is a convergence almost surely. So this is a second mode of convergence. This the notation is Xn converges to X. So that means the sequence of random variable has a n tends to infinity it converges to the random variable X as n tends to infinity that is almost surely provided the probability of limit n tends to infinity of Xn equal to X or Xn is equal to capital X that is going to be 1. That means a first you are trying to find out what is the event for n tends to infinity, the Xn takes a value Xn X that means you are collecting the few possible outcomes that has n tends to infinity what is the event which will give xn same as the X then that the event of probability is going to be 1 if this condition is satisfied then we say it is going to be almost surely. I can relate with almost surely with the if any sequence of random variable converges almost surely that implies Xn converges to X in probability also.




This is the third mode of convergence that means if the sequence of random variable CDF converges to the CDF of the random variable X then you can say that the sequence of random variable converges to the random variable in distribution and I can conclude the sequence of random variable converges in almost surely implies in probability that implies in distribution whereas the converse is not true and when I categorize this into the law of large numbers as a weak law of large numbers and a strong law of large numbers if the mean of X, xn that converges to mu in probability then we say it says it's a weak law of large numbers. Similarly if the convergence is almost surely then we conclude this is going to satisfy the strong law of large numbers. The final one that is the central limit theorem you have a sequence of random variable each are a iid random variables and you know the mean and variance and if you define there Sn is the form then Sn minus e n mu divided by Sigma times the square root of n that converges to standard normal distribution in convergence in distribution. That means whatever be there under variable you have as long as they are iid random variable and even these things can be relaxed the sequence of random variable the summation will converge this to the normal distribution or their mean divided by the standard deviation will converges to the standard normal distribution.

With this I complete the review of theory of probability in two lectures. Then thr next lectures onwards I will I'll start the stochastic process.

Thank you you.