Stochastic Processes - 1

Arc Extensions in Petri Net, Stochastic Petri Nets and examples

by

Dr. S Dharmaraja
Department of Mathematics, IIT Delhi

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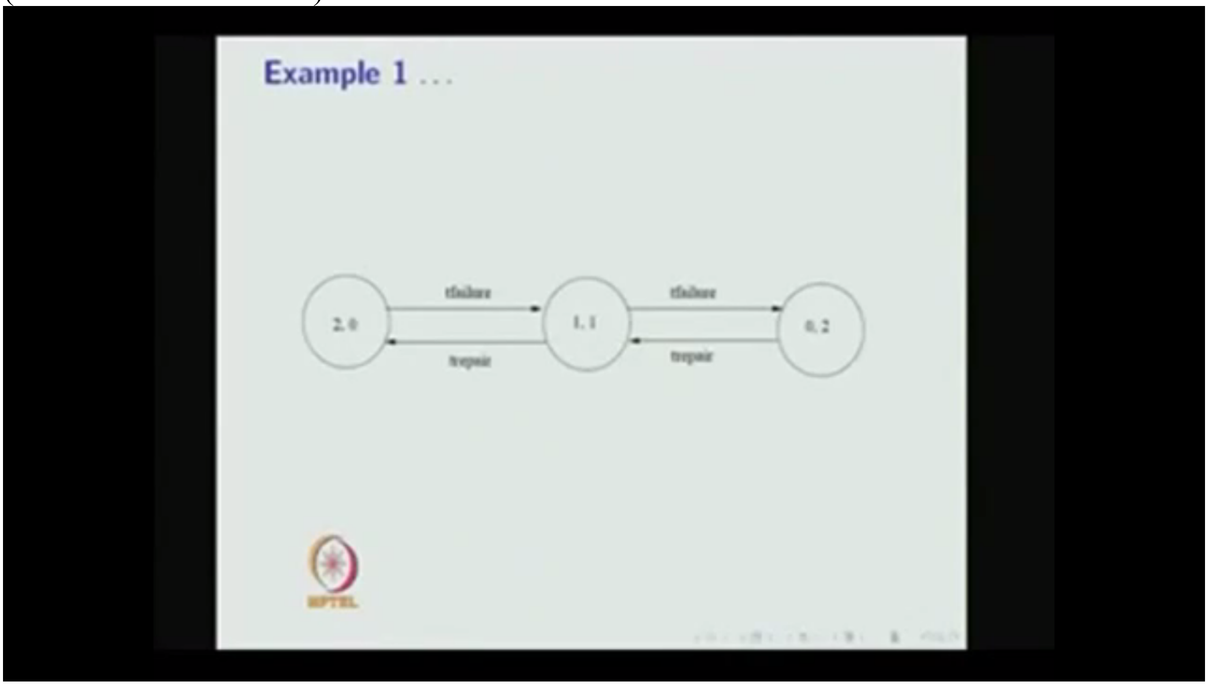


Consider this example, the example which we have considered earlier, Example 1.

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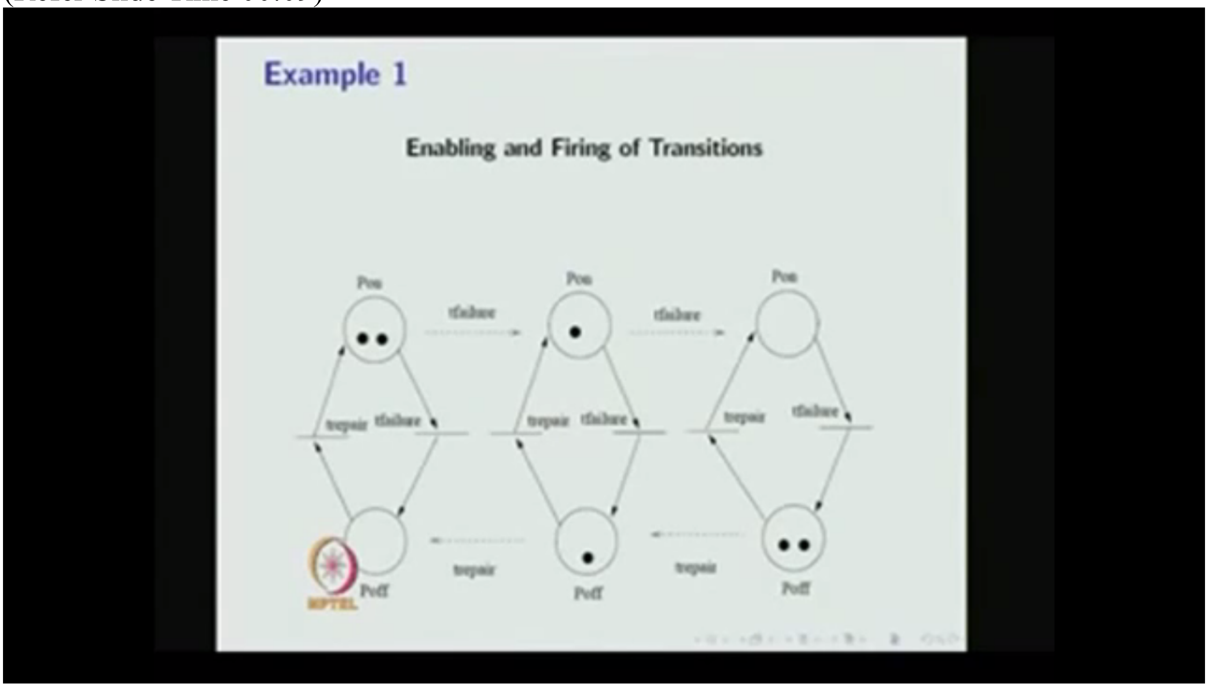

That means we have two places, place Pon, the other place Poff. Initially, two tokens in the place Pon, no token in the place Poff. Therefore, the marking will be a 2 tuple. Number of tokens in each place form a marking. Therefore, the first place is Pon; second place is Poff. With that assumption, suppose, suppose the first place is Pon; second place is Poff. Then the number of tokens at time 0 is (2, 0). After tfailure firings, the marking will be (1, 1). Again the tfailure firing, the marking will be (0, 2). From (1, 1), if trepair fires, then the marking will be 2, 0. From (0, 2), if trepair fires, then the marking will be (1, 1).

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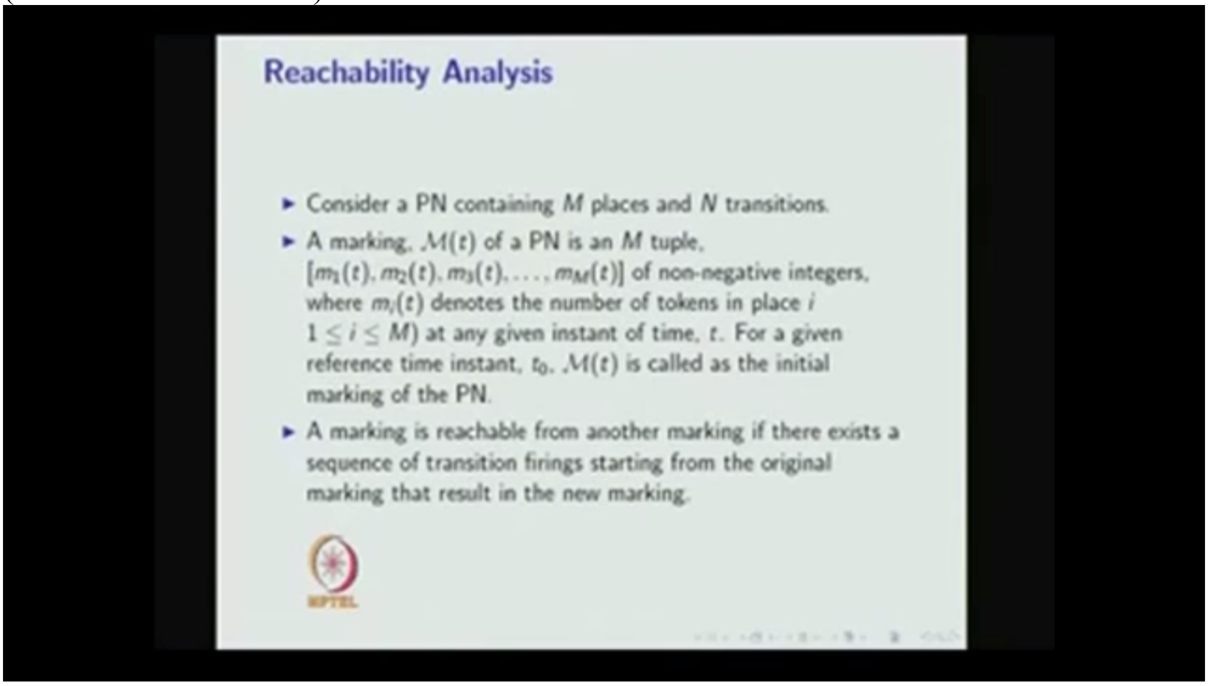


Hence, the marking is the M tuple, number of tokens in the place i at any given instant of time and the marking is reachable whenever some sequence of a transition fires and the reachability graph is a directed graph whose nodes are the markings in the reachability set with directed arcs between the markings representing the marking-to-marking transitions.

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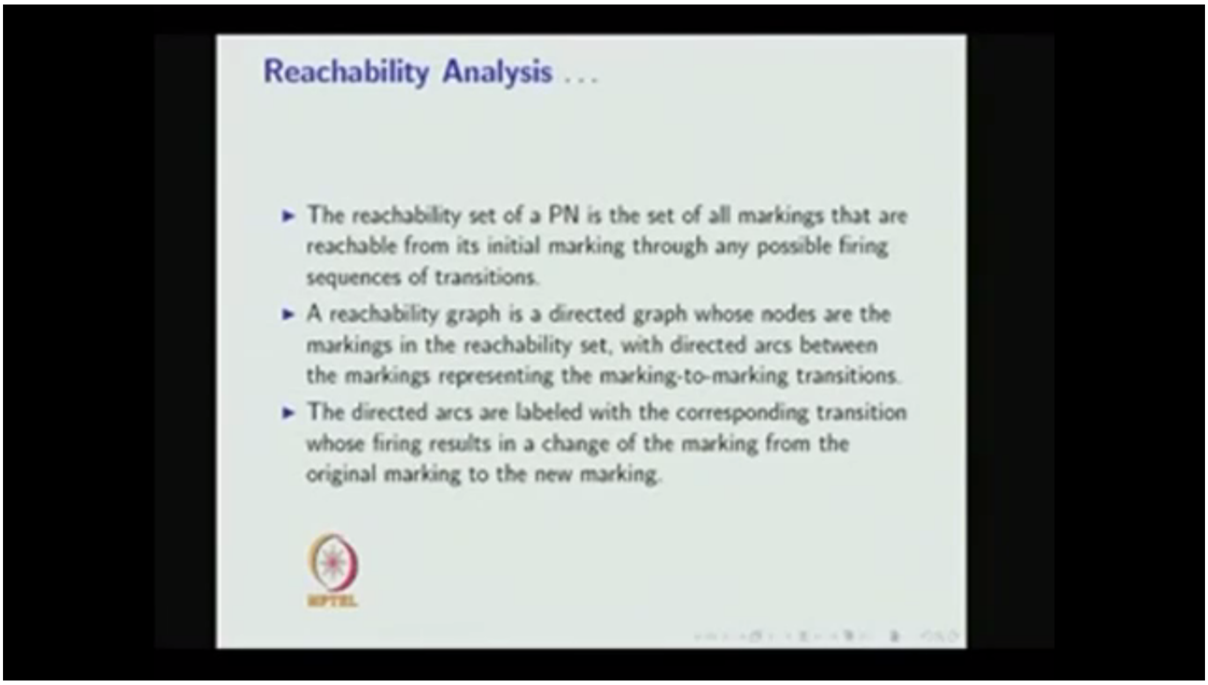


Hence, in the same example, the markings are (2, 0), (1, 1), (0, 2). The marking can be reachable from (2, 0) to (1, 1) by the transition tfailure fires. The marking (1, 1) is reachable to the marking (0, 2) with the tfailure firing. The marking (0, 2) is reachable to the state, to the marking (1, 1) by trepair fires. Similarly, (1, 1) to (2, 0) by firing trepair transition. So this is called a reachability graph.

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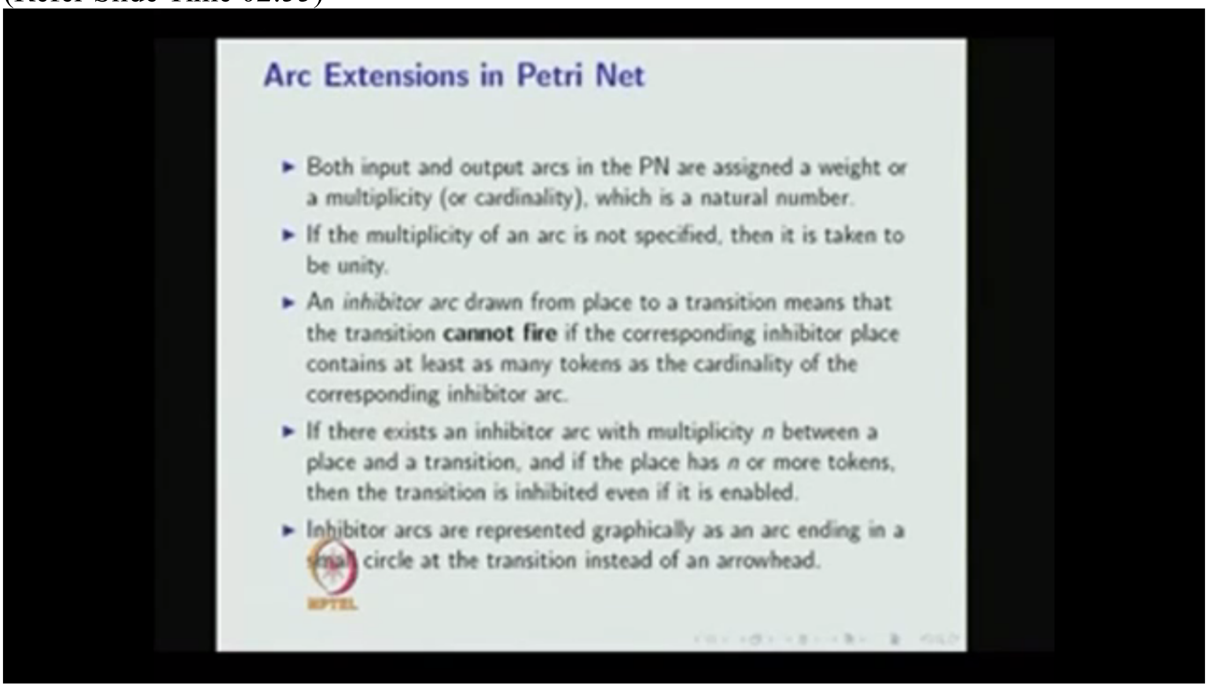


Now we are moving extensions, arc extensions in Petri Net. Till now we have considered very simple Petri Nets. Now we are going for arc extensions. Both input and output arcs in the Petri Net are assigned a weight or multiplicity (or cardinality), which is a natural number. If the multiplicity of an arc is not specified, then it is taken to be unity. So in the previous examples, we have considered as a multiplicity is unity.

The next extension in the Petri Net is inhibitor arc. An inhibitor arc drawn from place to the transition means that a transition cannot fire if the corresponding inhibitor place contains at least as many tokens as the cardinality of the corresponding inhibitor arc.

Usually, input arc and output arcs makes a transition enabling and firing, removing the tokens, depositing the tokens with the multiplicity or cardinality, but the inhibitor arc drawn from place to the transition means that the transition cannot fire if the corresponding inhibitor place contains at least as many tokens as the cardinality of corresponding inhibitor arc.

If there exists an inhibitor arc with the multiple n between a place and a transition, and if the place has n or more tokens, then the transition is inhibited even if it is enabled. The transition can enable based on the conditions through the input arcs, but if there is an inhibitor arc, then it may be inhibited based on the number of tokens in the corresponding input place.

Input arcs are represented graphically as an arc ending in a small circle at the transition instead of arrowhead. Usually, input arcs as well as output arcs drawn with the arrowhead, but inhibitor arcs are represented graphically as an arc ending in a small circle. We will see the example in this slide.

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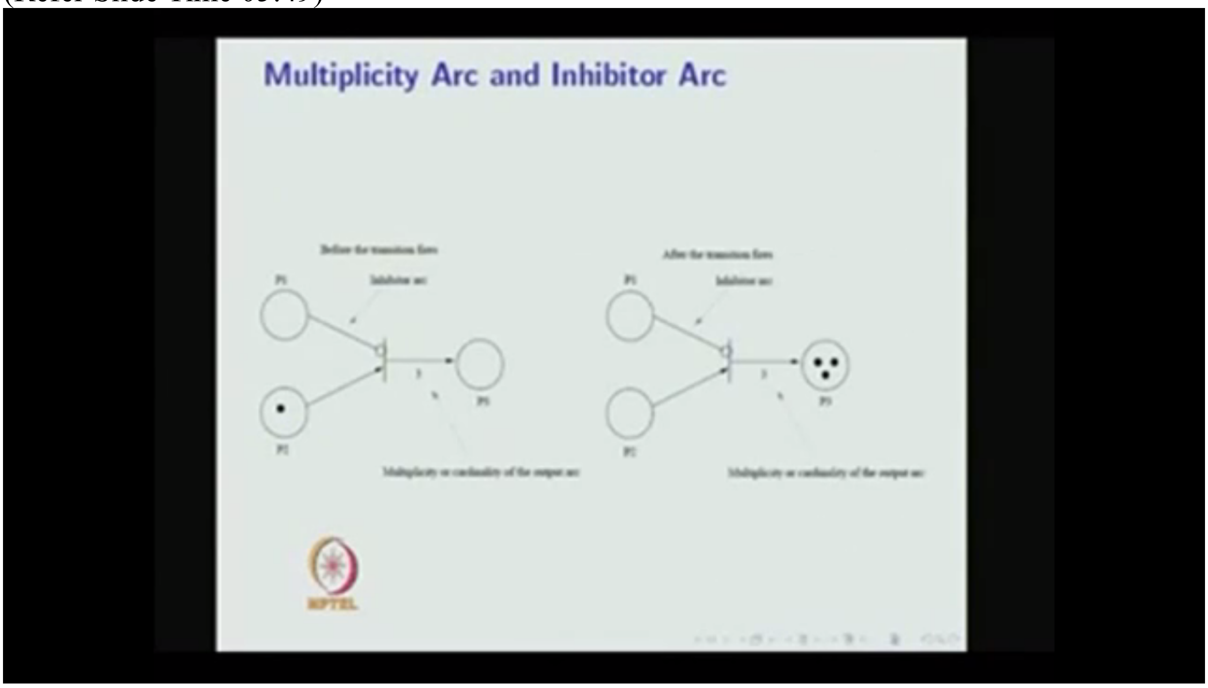


In this example, we have three places P1, P2, P3. We have one transition, one input arc, one output arc, one inhibitor arc. Here whenever the number is written next to the arcs, that means the multiplicity or cardinality of the output arc is number 3. If there is no number, natural number written next to the arcs, that means the default multiplicity is 1. Here also the default multiplicity is 1. That means if one or more token in the place P1, even the transition is enabled by the condition through the input arcs for this place input place, this transition may not fire if one or more tokens in the place P1. So here no token in the place P1 whereas one token in the place P2, no token in the place P3.

Hence, the transition enables and then fires by removing one token in the place P2 and one token deposited in the place P3. Only the tokens will be removed from all the input places which are connected, the places connected with the input arcs to the transition and the tokens will be deposited to the all the output places, which are connected from transition to places through output arcs.

So in this example, after the transition fires, no token in the place P1, no token in the place P2 and three tokens in the place P3 because the multiplicity of the output arc is 3. So even though one token is removed from the place P2, because the multiplicity of the output arc is 3, hence three tokens will be -- multiplicity is 3. Therefore, three tokens will be deposited at the same time in the place P3.

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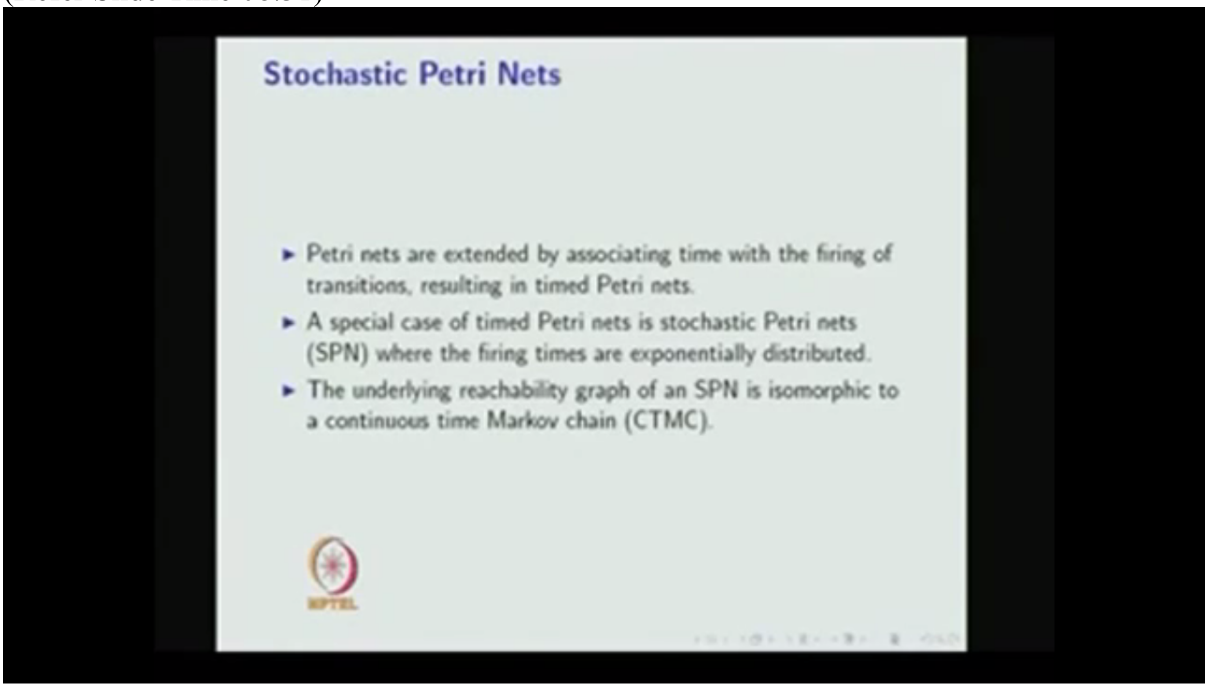


Now we are moving into the extension of Petri Nets into stochastic Petri Nets. Petri nets are extended by associating time with the firing of transitions resulting in timed Petri Net.

A special case of timed Petri nets is stochastic Petri nets. In other words, SPN where the firing times are exponential distributed, exponential distribution. So whenever the firing times of all transitions are exponential distribution, then the corresponding timed Petri Nets are called stochastic Petri Nets.

Every Petri Net one can get the reachability graph. The underlying reachability graph of an stochastic Petri net is isomorphic to a continuous-time Markov chain. For a stochastic Petri Net, the underlying reachability graph is isomorphic to a continuous-time Markov chain.

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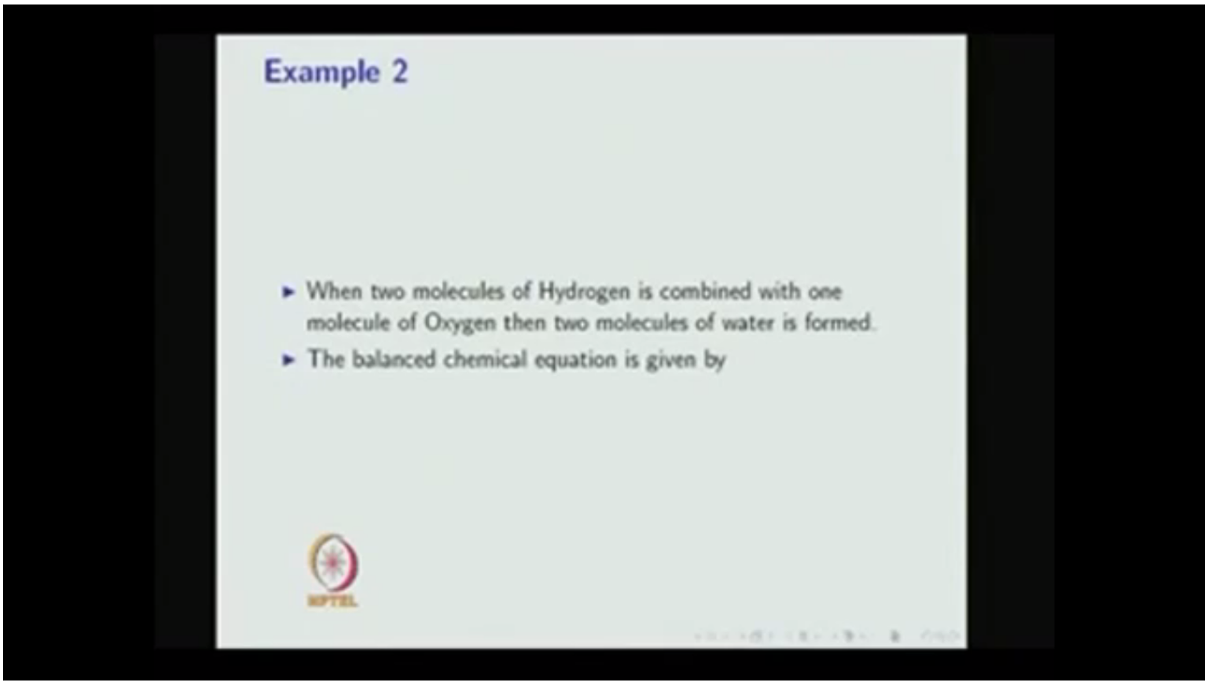


Let's see through the examples. When two molecules of Hydrogen is combined with one molecule of Oxygen, then two molecules of water is formed. The balanced equation is written, the balanced -- the balanced chemical equation is given by $2H_2 + O_2$ gives 2 times $H_2O$. So this is the balanced chemical equation.

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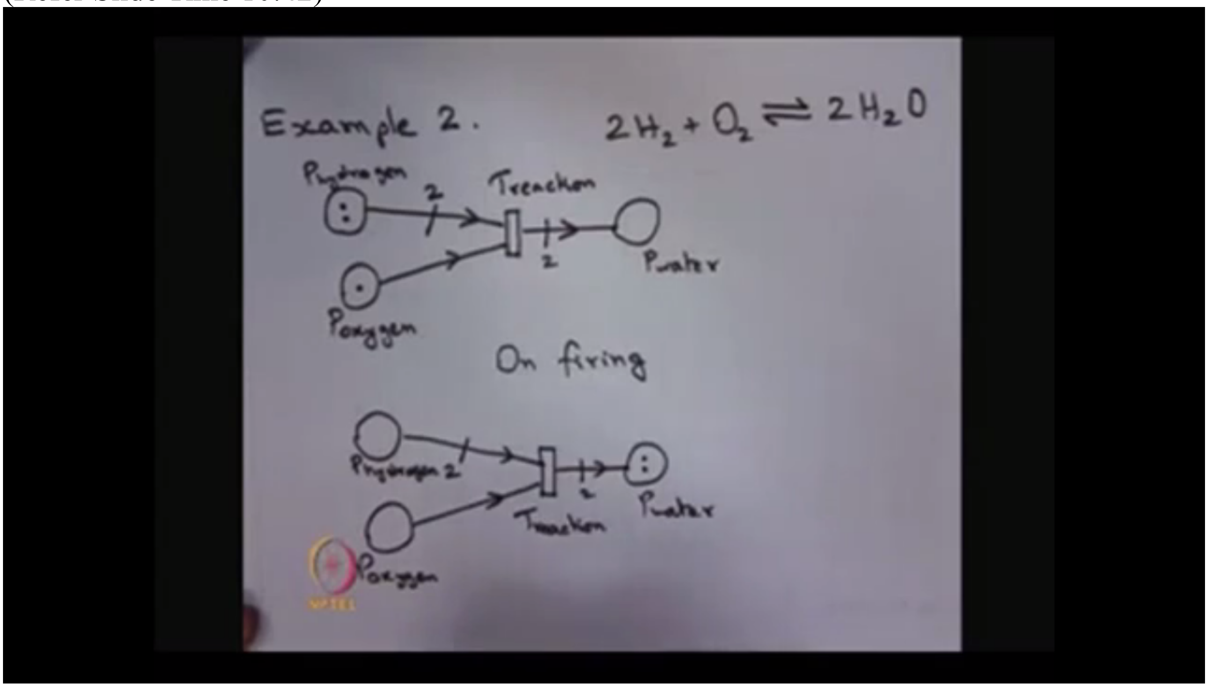


For this scenario, we have made three places $P_{hydrogen}$, $P_{oxygen}$, $P_{water}$ and we have one transition that is called a $T_{reaction}$. The multiplicity of a input arc from the place $P_{hydrogen}$ to the T transition that is 2 whereas the multiplicity of the arc from $P_{oxygen}$ to the $T_{reaction}$ that is 1 whereas the multiplicity of output arc from the transition to the place $P_{water}$ that is 2.

On firing of the transition $T_{reaction}$, two tokens because the multiplicity is 2, two tokens will be removed from the place $P_{hydrogen}$. The multiplicity is 1. Therefore, one token will be removed from the place $P_{oxygen}$. Since it is a triangle bar that means the time the transition follows -- the time of a timed transition follows exponential distribution. So after the -- so two tokens will be removed from the place $P_{hydrogen}$, one token will be removed from the place $P_{oxygen}$.

On firing of the reaction $T_{reaction}$, two tokens will be deposited in the place $P_{water}$ because the multiplicity is 2 and here we made the assumption, the reaction time is exponential distribution. Hence we made a stochastic Petri Net and we make the model using stochastic Petri Net of this chemical reaction.