Slide 1: Now I am moving into the last part of module 5 continuous-time Markov chain, in this I am going to discuss the simulation of queuing systems. So the model 5 continuous-time Markov chain started with the definition and the properties and so on, then I discussed the birth-death process, then I discussed the application of a birth-death process in simple Markovian queuing models. Then also I have discussed the queuing networks, that is also multi-dimensional continuous-time Markov chain as application. And finally I have given few practical application in cellular networks for the performance analysis. Now I am going to discuss the discrete event simulation of simple Markovian queuing systems.

Slide 2: So in this queuing Network modeling lab one can do discrete event simulation and the discrete event simulation for the queuing network involves Markovian queues, we can do some experiment over the Markovian queues and you can do the discrete event simulation for the non Markovian queues, and one can do the discrete event simulation for the queuing network also, and finally one can do the fluid queues also, one can simulate. But since I have discussed only continuous-time Markov chain till now, I am going to do a discrete event simulation for the Markovian queues. So that is in the first three experiments. The other three experiments are related to the non Markovian queues. The first experiment consists of simulation of MM1 queue, single server queue and MMC finite server queues and the infinite server queue. So in the… some time I have done the discrete event simulation of MM1 queue so let me go for MMC finite server queue.

Slide 3: So in these you need, the input of arrival rate, input of the departure rate, and the number of servers. So multi server infinite capacity model, so suppose you choose the arrival rate is 2 and the service rate is 3, and the number of servers are 2, that means it is a MM2 infinity model with the arrival rate 2 and the service rate is 3, so one can start the discrete event simulation by clicking the start, then you will get the window of. So this is the sample path over the time, what is the size. So at this time point one arrival comes then two then service is completed and so on. So this is the sample path over the time. And here you can get the performance measures. So whatever I have discussed, the performance measures of a steady-state distribution and all other performance measures, one can get it with the theoretical results in the third column, whereas the second column gives the running time till the discrete event simulation runs for 15 time unit, and what is the result for the mean number of system and so on, and this will converge to this value for T tends to infinity. So make sure that this arrival and departure rates are satisfying the condition so that the steady-state distribution exists, therefore as T tends to infinity this will… reaches the steady-state theoretical result. If you change the value of the arrival rate and the departure rate, something else, then there is a possibility if the conditions for the stationary distributions are not satisfied then still the runtime results you will get, but that won't be converged to the vertical and also the steady state results won't be possible. It will be a dash here, so as long as the steady-state distribution, those conditions are satisfied, then you will have the results in the third column, otherwise, you won't have the results here. So now you can see the throughput, till this much time, you are getting 2.0, whereas the steady-state throughput is to... throughput is nothing but the number of customers served per unit time. Like that one can get the throughput utilization, average response time or mean sojourn time and mean waiting time in the queue. So using Little's formula, used to find out this quantity, so this quantity you can get it from the discrete event simulation at any time as well as the theoretical result and this is the mean number of customers in the system, that is EN and the mean number of customers in the queue that is EQ, which we have got it. So with this result, one can see the discrete event simulation as well as, as T tends to infinity, what is the theoretical result if the conditions are... conditions for the stationary distribution is satisfied. And here this information is, how many calls are entered, how many customers enter into the system, and how many are served, and how many customers are blocked. Here there is no blocking because it is a infinite capacity system. And we are considering the retrial orbit and so on, therefore here it says number of orbit customers is 0, but this is not necessary. This is irrelevant information in the MMC queuing model.

Slide 4: Now let me go back and do the live simulation of MM infinitely so here you have to provide only the arrival rate and the service rate because the number of servers are infinite, it's a self service system. You can cross-check the theoretical results are correct and so on. So here I am getting the steady state probability... steady-state result based on the theory, whereas this is a run time results. There is no blocking probability, because the system infinite capacity. Mean number of customers in the queue that is 0 and here also 0, because it is infinite server therefore who... customers who enter into the system immediately will get the service. Therefore the queue, average number of customers in the queue that is 0 and the average time spending in the queue that is also 0, that's correct. There is no blocking and utilization is, what is the probability that the servers are utilized. Now let me go to the next experiment, that is second experiment for the finite capacity queuing model, whereas the first experiment is the infinite capacity queuing model. So in this, we have, we need to give the arrival rate, service rate and the number of servers, you can go for the multi server MMC N model, you can give 1 also, you can give a infinite service also one can give. So here the number of, sorry the number of servers suppose we fix at 3, and the capacity is 4, you can do the simulation. So the blocking probability in the run time there is at this time, it does not cross the number 4, therefore the blocking probability 0 in the run time result, whereas the theoretical steady-state result is the blocking probability is 0.005, so whenever the system touches 5, then you will have the blocking probability in the run time. So this is a discrete event simulation. See number of customers blocked is till now 0, therefore you are getting the blocking probability 0 at this run time. Suppose you run it again, run it again, let's see, now also it does not crosses the system size by 2, so maybe I can reset, with the number of servers I can put 2 and the capacity is 3. So the blocking probability is this much, the capacity of the system. So there is no arrival after crossing, therefore still you are getting the blocking probability as 0, whereas the steady-state theoretical result says 0.037. So one can simulate with the different parameters and you can see the sample path, so this is the sample path for MM 2 3 queuing system, with the arrival rate 2 and the service rate is 3. Here now we are getting the blocking probability because the system crosses, it touches capacity 3, therefore some calls are blocked, some customers are blocked, therefore the blocking, so you can find out from here. Number of customers 44 entered and 4 are blocked, therefore that ratio will be the blocking probability, because it's a discrete event simulation.

Slide 5: Now let me go back and go to the experiment three, experiment three we have a retrial model with the bulk arrival and bulk service so now it's no more birth-death process simulation, this is a non birth death process because you have a bulk arrival and the bulk service was possible and the retrial, therefore let me show the simulation. You need some more information. So what is the arrival rate you have to supply, suppose if it is a bulk arrival then you should say what is the distribution, whether the bulk arrival comes in a bulk, some constant number or it comes in some distribution. You can choose it's geometry distributed and the parameter… parameter for the bulk arrival parameter, you can choose some 0.5, then the departure rate, you can choose the departure rate, it can be a bulk departure or bulk. So either you can choose bulk arrival or bulk departure or you can choose both also. The number of servers in the system, suppose the two servers in the system and the capacity of system is 4, or you can choose the infinite capacity of the system also and if you need orbit, then you have to click for orbit and if you are changing the queuing discipline, the first-come first-served, last come first-served or random order you can choose the queuing discipline also, according to the discrete event simulation goes. So this is not birth-death process, this is a continuous time Markov chain simulation, with a different queuing discipline also one can go for it. Now once you start the simulation, yeah since it is a bulk… we have made a bulk arrival, where as we made it… we didn't click for the departure… bulk departure, therefore the customers keep going by one by one, whenever the service is over, but since it is a bulk arrival with the arrival distribution is the number of arrivals that is a geometrically distributed, therefore it just jump with the bulk arrival, whereas the departure is I1 by 1. And here is the performance measures, is the runtime performance measures, and we didn't… till now we didn't supply the theoretic… steady-state or theoretical results for this model. and these are all the different results in the… over the runtime and similarly one can go with the bulk departure and you can change the queuing discipline also. So this is a discrete event simulation sample path for this scenario, so I can reset, and if I don't want the bulk arrival, and if I choose the capacity of the system is 4, so this will be a no bulk arrival, no bulk departure, therefore this is MM2 4 system, with the first come first served and I can go for instead of first come first out, I can go for the last come first out also. The customers last entered, he is getting the service first, so this is a steady state and this is a time dependent result. And I can change again this into… that means it is two servers, infinite capacity model with the first come first. Still this is in the testing phase so some of the things should be removed, some of the things has to be edited, so still it is in the testing phase. So this is the performance measures for the MM2 infinity model, so that means in the… in these experiment also you can remove the bulk arrival and the bulk departure part and so on, you can try the simplest… simple Markovian queuing model also, one can do the discrete event simulation of that. And since it is a infinite capacity model the blocking probability is 0 and the steady-state throughput is 2, one can find out from the formula also. Whereas the runtime is this, so if… if this discrete event simulations runs for a longer time, then this will converge to, like that you can discuss the all other results also, all the results will be converges to the steady-state theoretical probability, the theoretical results.

Slide 6: So with this let me complete the discrete event simulation of a simple Markovian queuing model, because we are discussing this under the title of, application of continuous time Markov chain. Therefore I discussed only the Markovian queues, there are some non Markovian queues and so on, so we… I am not discussing the non Markovian queues at this stage, after I discuss the renewal theorem and Markov regenerative process and semi Markov process and so on, I will be discussing the non Markovian queuing systems also. So here we have discussed only the simple Markovian queuing system that is nothing but the applications of continuous-time Markov chain.

Slide 7: As the summary, the last seven lectures we have discussed the continuous-time Markov chain from the definition and properties and few simple continuous-time Markov chain starting with the Poisson process, birth-death process, Pu birth process, Pu death process, then application of CTMC in the queuing model, starting with the MM1 queue and there simple Markovian queues, then we discussed few queuing network, which as the product form solution, then in the last, this lecture we have discussed the application of, applications of CTMC in 2G, second generation cellular networks as well as the third generation cellular networks and application of CTMC in the 2G networks is the birth-death process, whereas the applications of CTMC in 3G cellular networks is the Quasi birth-death process. Even though I have not discussed in detail the complete modeling one can see it from that paper, my intention here is to explain the Quasi birth-death process through the applications. And we discuss the stationary distribution and all other performance measures for the birth-death process comes in the second generation networks and the Quasi birth-death process in the third generation networks and finally I have given the discrete event simulation for simple Markovian queuing systems only, there are some more non Markovian queuing systems, so that I will be discussing in the other models. The references are Gross and Harris book and Medhi book, Kishor Trivedi's book, thanks.