Slide 1: So the Jackson theorem says in steady state, the number of customers in different nodes are independent… the number of customers in different nodes are independent, that means the behavior of the queuing system is a consisting of behavior of many independent nodes. The queue I behaves as if the arrival stream is Poisson.

Slide 2: So not only each node behaves independently… not only each node behaves independently, as if it behaves the arrival is going to be a Poisson for each node. Therefore this point it is a Poisson, whereas this point is not a Poisson arrival process, but still in steady state the number of customers in this queue… number of customers in this queue, this queue and this queue, all the queue size are independent as well as in steady state the arrival process for the each node behaves as if… as if Poisson process, but they are not in general Poisson process. So in steady state whenever you have open queuing network with these assumptions, in steady state this behaves as a independent queuing. Therefore the joint probability of N1 customer in the first queue, N2 customers in the second queue similarly and so on till that NK customers in the Kth node. Since each nodes behaves independently, so the number of customers in different queues are independent and arrival or poison and already you made assumption, it is infinite capacity each queue, as well as only one server in each queue, therefore the arrival follows the Poisson, service is exponential, only one server, infinite capacity that means even though it is open queuing network with the feedback, each node behaves as if MM1 queue in steady state. That means, as a time dependent, the system may depend on the size of the other number of customers in the other nodes, but in steady state this behaves independently and each one behaves like MM1 infinity queue.

Slide 3: Therefore you can get the joint distribution of any customers in Ith node, that is the product of NI customers in Ith node. So if you make a product, that is going to be the joint distribution, because a joint distribution is going to be the product of unusual probabilities, if each random variable is independent, therefore you can use that logic to use… to get the joint probability as the product form solution. And hero Rho Is are nothing but lambda Is divided by mu Is and which is… has to be less than 1, I forgot right. Each Rho I has to be less than 1, it has to be stable. Each queue, queuing system has to be stable. So in steady state you have a product from solution, where Rho Is are lambda I divided by mu I, you have to find out lambda is by solving the system of K equations. Lambda Is are the unknown, R Is are given, routing probabilities are given, so using that sulfur lambda Is from this K equations. So once you know the lambda Is check whether lambda Is divided by mu Is less than 1 then the stationary distribution exists, using the Jackson theorem the joint distribution is the.. joint distribution that is the stationary distribution is of the product form solution.

Slide 4: Assuming that each queue behaves as the MM1 queue. So for any general K queues, not Tandem queue, open queuing network with the feedback you can get the average measures also. The way we have calculated for the two queues model or the Tandem queue model, the same logic can be used for the open queuing networks with the feedback. We are getting average number, then sojourn time in each node, then mean waiting time in node… each node by subtracting the average service time for each node. You have K nodes, so therefore you are getting the… these measures for each node. Once you know the result for the each node, you can find out the total sojourn time by using the Little's formula, because the Little's formula is valid. Oh here the external arrival rate is lambda so that we have to add, suppose here you have to find out the lambda.

Slide 5: And for the open queuing network the lambda you are to compute by adding all the external arrival rates R1 plus R3 plus R4, that is going to be lambda in this example. So if you had all the external arrival rates to different queue, different nodes that summation is going to be the total arrival rate to the system, because to apply the Little's from you think, you consider the whole thing as a one system in which R1 plus R3 plus R4, all are independent therefore the summation is going to be the arrival rate for the system. So the lambda on that example R1 plus r2 plus… sorry 1 plus R3 plus R4.

Slide 6: Remarks for the open queuing network. The networks behaves as if it were composed of independent MM1 queues in steady state that is important, I forgot to write that, in steady state. In time dependent this is not the case that I am going to… I have written as a fourth remark. In steady state this network's behaves as if it is composed of independent MM1 queue. So the equilibrium queuing length distribution in a Jackson network is of product form, so this solution is called a… this solution is called the product form solution. For open queuing network with the feedback that equilibrium queuing length distribution is of product form. This, the previous result can be extended to the multi server model also. In that model we have taken it as a single server infinite capacity, you can think of each queue is MMC infinity also instead of MM1 infinity, in steady state this solution is valid with the MMC infinity logic. Whereas the time dependent, queuing length process are not independent. Therefore the product form solution won't work and the time dependent scenario is completely different and with the steady state or the equilibrium queuing let the distribution, you cannot discuss the behavior of a time dependent. The time dependent queuing length distributions are not independent for the each queues. Now I am moving into the closed queuing network. So here comparing with the Open Queuing Network, here we have a fixed number of population is moving around the queues. No one leaves and no one enter the system also therefore you keep some K customers in the system. In this example, I have four nodes, and instead of either you can have a one server or more than one server is also allowed with the infinite capacity queuing system. You make the assumption, the service time is exponential distribution for each queue and all the server's are identical.

Slide 7: Here also you can get the product form solution and the Joint Distribution of the system size that is same as the product of Rho Is power NI divided by DIs of NI for the K nodes. The small k is the K nodes in the system and the capital K is the total number of population. So here this Ks are nothing but the normalizing constant and the Rho Is in terms of alpha I divided by mu I and alpha you can calculate by solving this equation, where P is the routing probability matrix, you solve alpha times P is equal to alpha and the summation of alpha I is equal to 1, using that you will get alpha and substitute alpha is here, therefore you will get Rho Is, then you substitute DIs here based on the number of servers in the each node is 1 or more than 1, accordingly you can use these. And once you know the DI and NI substitute here this product form solution will give joint distribution of system size in steady state, and this result is given by Gordon and Newell for the closed queuing network.

Slide 8: As a remark this equilibrium solution is also product form and the routing probability matrix is a stochastic matrix. And suppose you assume P is irreducible then the solving alpha P is equal to alpha and the summation of alpha is equal to 1 that is nothing but… alphas are nothing but the stationary distribution, whenever P is irreducible. Not P is irreducible, the underlying DTMC is irreducible with the… assuming that the underlying DTMC is irreducible, then the alpha is nothing but the unique stationary distribution and this is valid for the cyclic queue also. And the toughness is, how to compute A of K, where K is the number of customers in the whole queuing network. So you need a efficient and stable competition or algorithm for calculating this normalizing constant A of K. Here is a mistake, assuming that the DTMC is irreducible… the DTMC is irreducible, alpha is a unique stationary distribution.

Slide 9: So now we will move into the summary. So in this lecture we have discussed the applications of CTMC in Tandem, open and decay queuing networks. We have discussed the stationary distribution and other performance measures for the Tandem open and closed queuing networks with only the product form solutions. We didn't discuss the non product form solution, we have only discussed the product form solution. And the application of CTMC in the performance analysis of wireless network system that I will discuss in the next lecture. Also I am going to discuss the simulation of a simple Markovian queuing networks in the next lecture. These are all the reference books. Thanks.