Slide 1: Since we land up the underlying stochastic process as a continuous time Markov chain, now we can find out the stationary distribution, that is our interest. Solve by queue is equal to 0 and the summation of pi I is equal to 1, now the pi Is are not just 1 pi I, it is a 2 index term. So by solving this equation that means you write the balance equation then use the summation of probability is equal to 1. You need a condition so as long as the Rho… Rho 1 is a lambda divided by mu 1 that is less than 1, as well as the Rho 2, that is a lambda divided by mu 2 that is also less than 1, then the stationary distribution system and you can find out the stationary distribution probability as a probability that N1 customer in the first node and the N2 customer in the second node, that is nothing but N1 customers in the first node, that probability distribution is 1 minus Rho 1 times Rho 1 power N1 and the N2 customers in the second node that is nothing but 1 minus Rho 2 times Rho 2 power N2. So this is sort of a product the probability of N 1 customer in the first node and the probability that N2 customer in the second node together that is same as what is the probability that N1 customers in the first node multiplied by what is the probability that N2 customers in the second node. And this form is called the product form solution. So for Tandem queuing Network, the stationary distributions are of the product form. This together probability is the product of individual MM1 infinity stationary distribution probabilities, therefore this is called the product form. So this exists as long as Rho 1 is less than 1 and Rho 2 is less than 1. if it is great than or equal to 1 then the stable… the system is not stable, equivalently stationary distribution does not exist.

Slide 2: Now I can extend that these two queues queuing network into K queues for K nodes. So there also my interest is to find out the stationary distribution and the probability of the N1 customer in the first node, N2 customer in the second node, and the NK customer in there and so on, till NK in the Kth node, that joint distribution is same as product of individual distribution of NI customers in Ith. So this is a product form, product of I is equal to 1 to K and this result is nothing but a stationary distribution of MM1 infinity queue. So this exists as long as all the Rhos is less than 1 and since the arrival is lambda… mean arrival rate is lambda, therefore all the Rhos are lambda divided by the service rates for the corresponding queue.

Slide 3: So once we know the number of customers in, what is the distribution of number of customers in the each node, for the K node Tandem queuing network, you can find out what is the total number of customers in the whole queuing networks. That means you have to sum it up all the average number of customers in each node, if you sum it up, that will be the total number of customers in the whole queuing network. And since this… each note is going to behave like MM1 queuing system, therefore you can use the average number of customers in MM1 queuing system that result, that is a lambda I divided by mu I minus lambda I, whereas here all the lambdas Is are lambda, because it is a Tandem queue. And if you sum it up that is going to be the total number of customers in the system. Once you know the total number of customers in the system, system is the whole queuing network, using Little's formula you can find out the average time spent in the system. Some books they say average response time, average switch on time. So all those things are the same. So the average switch on time using Little's formula you can get it after substituting expected number in the system divided by the arrival rate, because the Little's formula is applied to the queuing system in the sense, the arrival rate to the system is lambda and the total number of customers in the system, so that the whole queue, all that series of queuing, all the queues, that are whole Tandem queue, that you treat it as the one system and it satisfies the… all the Little's law conditions, that is first-come first-served, when the arrival rate is lambda and after the service is over the system… the customers leave the system and so on. So you can apply the Little's formula and get the average time spent in the system for the whole system, not for the individual queues.

Slide 4: Now I am moving into the open queuing network with the feedback. So I am just giving one simple scenario of four nodes queuing network, which with the feedback also. So this is a open queuing network and here the assumptions are for each node the service time is exponentially distributed with the mean 1 divided by mu I or the parameter mu I. Only one server in each node and the infinite capacity… only one server in each node and the infinite capacity of the… for each node also. The external arrivals, external arrival process to the node I that is the Poisson process with the rate lambda Is, that means for the node 1 there is external arrival that is R1, there is no external arrival to the node 2, whereas there is external arrival from… for the node 3 that is R3, that arrival is a Poisson process, and this arrival is also Poisson process with the parameter R4. And I have not supplied the routing probabilities… I have not supplied the routing probabilities, so after the service is over in the first node with some probability, it moved into the again node 1, with some probability it goes to node 2, as well as node 3 in this partition, I can multiply this and this, then with some probability it goes away from the system. So the summation of these plus these and this probability has to be 1. Sometimes we won't draw this arc, outgoing arc, so there are two possibilities here, either this arc probability is 0 or non 0, whenever the summation of other than the going out, if that probability is not equal to 1, that means 1 minus of summation of probability, that is 1 minus of the summation of probability is greater than 0, then that is the probability that the system is after, the service is over, the customers who finishes the service in the node 1, leave the system, whenever the summation of arcs from the node I to all other nodes, if that summation of probability is less than 1 then the 1 minus of that summation of probability, that is the probability in which after the service is over the customers leave the system. So that's the way you can make out the probability for these, otherwise we need to supply the routing probability matrix. So here I am concluding with the open queuing network with the feedback and assumptions are infinite capacity in each node, and the single server and arrival… external arrival process is the Poisson process with the rate RIs. Whereas the actual or the total arrival rate that is different from R1, because the external arrival rate is the Poisson process with the R1, but there are some customers after finishing the service in the first node they are again coming back. Therefore the total or actual arrival stream into the node 1, that is different from R1. So that we are going to calculate later.

Slide 5: So I am going for the routing probability with the notation P of I,J P I,J denotes the probability that the job living queue I goes to the queue J. Whenever this probability is a summation of probability that Rho sum is less than 1, then the probability of a job leaving the system, leaves the system after the queue I, that is going to be 1 minus of summation J is equal to 1 to K, we have K… K is the total number of queues in the open queuing network with the feedback and the P IJ is the routing probability of jobs going from the node I to node J. Either you can say node or queue. The actual or the total arrival rate that lambda I denotes the arrival rate of a job to the queue I, that can be computed using this formula. So the lambda I is equal to RI plus what are all the different rates lambda J and this is the packets are or the jobs are moving from the queue J to queue I. So if you multiply this routing probability and the arrival rate, that summation plus the external arrival, that rate RI, that will give the arrival rate of job to the node queue, where I is running from 1 to K. Let me give a one simpler situation, how one can calculate the arrival rate for any queue. Suppose you think of the external arrival rate is R1 and after the service is over with the probability P… with the problem P, the customers who finishes service in the first node and come back, with the probability 1 minus P… with the probability 1 minus P, it leaves the first node. Therefore our interest is to find out what is the even rate for the node 1 with the simple only one feedback. So here suppose I make it R1 is lambda, that is the external arrival rate, so the arrival rate to the node 1, that is lambda 1 that is same as the external arrival rate, plus what are all the possibilities in which the node 1 is build up. So that with the probability P, so this is a P times, suppose lambda 1 is arrival rate and lambda 1 times P, that is going to be the proportion, the P is the proportion in which it is coming back and we can use the Burke's theorem, the departure process is the Poisson process. If they arrival rate is lambda 1, then the lambda 1 times P, using the Poisson process split, one Poisson process can be splitted into… suppose this is the departure process, the departure process is splitted into two Poisson process lambda 1P and the lambda 1 times 1 minus P. So the lambda 1 times P, that is feeded again into the mode 1, therefore that will give lambda 1 times P, so the lambda 1 is equal to lambda plus lambda 1 times P, that is the way. So here the arrival rate lambda I is equal to RI plus the summation of lambda J P J I, that is the same thing I am applying for the I is equal to 1, so the lambda 1 is equal to lambda that is the external arrival rate plus what are all the ways you have a input arc to the node 1. So this point the departure process is a Poisson process using the Burke's. Theorem the, suppose you make it arrival rate is lambda 1, therefore the departure process is the Poisson process with the parameter lambda 1 and the Poisson stream is splitted into 2 P, 2 with the 1 Poisson stream in the proportion P and the another Poisson stream with the proportion 1 minus P, therefore you have a 2 Poisson stream with the parameters lambda 1 times P and lambda 1 times 1 minus Patient, this two are the two independent Poisson streams and this Poisson stream is faded again into the node 1. Therefore… therefore this is the only one input for the node 1, therefore you will have a lambda 1 is equal to lambda plus lambda 1 times P. Like that if you have many K nodes, you have to write the equation for all the K nodes, then you have to solve for lambda Is, then you get the, what is the arrival rate for the node I. So here I have only one equation, so I can get the lambda 1 is going to be lambda divided by 1 minus P, by solving this equation lambda 1 is equal to lambda plus lambda 1 times P, therefore lambda 1 is going to be lambda divided by 1 minus P, this is known to me, lambda is known to me and P is known to me, therefore using these two I can get lambda 1, that is the arrival rate for the node 1. So this is a very simplest example, but for any open queuing network with the feedback by framing these equations, K equations, by solving you can get the lambda Is. The product form solution is valid for the stationary distribution that's what is given as a Jackson theorem.