Slide 1: The Tandem Queuing Network is a special case of open queuing network in which all the nodes or the queues are interconnected in series. Let me start with the very simplest Tandem Queuing Networks with two nodes or two clearing system, connected in series. We are going to relate the queuing network with the stochastic process and so on, therefore let me start with the random variable X of T and Y of T, X of T denotes number of jobs or customers in the node 1 at anytime T, node 1 or first queue both are one and the same. Y T, the another random variable, that denotes the number of jobs in the second queue at time T. Therefore that together X of T and Y of T, you can think of a vector Z of T is equal to X of T to Y of T, that vector is a random vector at anytime T. And if you collect over the T, then that is going to be a stochastic process. So this stochastic process instead of one random variable it has a two random variables X of T and V of T, therefore it's a vector. So that is a stochastic process. X of T, Y of T for T greater than or equal to 0, that is a stochastic process, since X of T and Y of T are the number of jobs in the first queue and second queue, and you are observing over the time how many customers in the first node and how many customers in the second node at any time T, therefore this is the discrete state continuous time stochastic process. Also you assume that infinite capacity in both the nodes, both the queues. That means after the service is over in the first node it will be immediately arrived into the second node. And if no customer in the system at the time, then he will get the service immediately, the service will be started immediately, otherwise the customer who comes after completing the service in the first node, he has to wait till a service start. So I am making one by one assumption, so that the underlying stochastic process is going to be a continuous-time Markov chain, that's my objective. So for that now I am making a assumption, the first assumption they inter arrival time of customers entering into the first node, that is exponentially distributed with the parameter lambda or the arrival process into the first queue is a Poisson process with the parameter lambda. So the population is infinite, the customers or jobs or packets, for example packets for the telecommunication system or any communication system, so the customers are entering into the node one, that process is a Poisson process with the parameter lambda. Now I am making the assumption for service, I make the assumption for the service time that is also exponentially distribution with the parameter mu 1 for the first node and mu 2 for the second node, other than the enter-arrival... inter arrivals are exponential distribution, I made the other assumptions that is the service time for the first node and the service time for the second node both are exponentially distributed with the parameters mu 1 and mu 2 respectively. So ultimately I want this has to behave as a MM1 queue and this also has to behave as a MM1 queue independently, therefore I make all the assumptions the inter arrival times are independent with the service for the first node similarly this one is the arrival for the second node is independent of the service of the second node and so on. So in that assumption each queue in this gaming network is behave as MM1 queue and the departure of the first order that will be the input for the second node. And here you can use the Burke's theorem, this is MM1 queue model, therefore using the Burke's theorem you can control the departure process is also, Poisson process, because the arrival process is Poisson and the service rate is U1, using the Burke's theorem you can conclude the departure process also, Poisson process with the parameter of the same parameter of arrival process. Therefore here the arrival process for the node 2 is also Poisson process with the same parameter lambda. Committee is independent of the service, not only that, the departure process is independent of the number of customers in the system and so on and the inter arrival therefore you will have a two independent MM1 queue using the Burke's theorem. Now this will separately act as I a MM1 queue because the arrival process is a Poisson with the parameter lambda and already we made the assumptions, arrivals are independent with the services exponentially distributed with the parameter mu 2. After the service is over the customers leave the system. Therefore this is a separate MM1 queue, because we made an infinite capacity in both the queues. Therefore the two queues at Tandem queue, the underlying stochastic process X of T, Y of T, going to be the continuous time Markov chain… continuous time Markov chain.

Slide 2: So now I am going to formulate the CTMC from the two queues Tandem queuing Network. The transitions due to arrival or departure of jobs in each queue based on this I can make a straight of the system X of T, Y of T and that is going to be a continuous-time Markov chain, this is not going to be a birth-death process, because you have two random variables X of T and Y of T and each one independently a MM1 queue, MM1 queue is the underlying stochastic process for the MM1 queue, that is a birth-death process, whereas here you have together X of T, Y of T, therefore this is not going to be a birth-death process, it will be a general continuous-time Markov chain. So once we identify this as a continuous time Markov chain, because the Markov property is satisfied by the stochastic process X of T, Y of T, we can go for drawing the state transition diagram.

Slide 3: It is Markov in nature because the two queues act independently and are themselves MM1 queuing system which satisfies the Markov property. The first index is for the number of customers in the first node and second index is for the number of customers in the second node. So 0, 0 means in both a queues, no one in the system. If one arrival comes into the first node, arrival cannot come into the second node, only the arrival come to the first node, the inter arrival is exponential distribution, therefore the rate of moving from 0, 0 to 1, 0 that is lambda. Similarly there is a possibility of one more customer entering into the system when one customer in the first queue. So it will be a parameter lambda, therefore the rate will be lambda for the X moving from 0, 0 to 1, 0, 1, 0 to 2, 0, and so on. Whereas after one customer already in the first node, the server in the first node, who would have completed the service before the one more arrival into the first node, therefore the service is exponential distribution in the parameter mu 1 therefore this system goes from 1, 0 to 0, 1, that means the customer was under the service, that service is over, therefore he moved into the second queue. Now the first node has 0 customer the second node has 1 customer in the system and he service will start once he enter into the second node. This so... there... this rate will give mu 1. Now the situations are either one more arrival, one arrival to the first node or the customer who is in the second node, he would have completed his service. So that service is over there for death rate is mu 2. Then the system goes to 0, 0 or it will go to 1, 1 with the rate lambda 1, sorry lambda. So the same way one can discuss for 1, 1 also, that means one customer in the first node one customer in the second node, so the one possibility is, the first customers service is over therefore it will be 0 2, with the rate mu 1 or one more arrival takes place therefore it will be 2, 1 with the rate lambda or the second service would have been finished. So death rate is mu 2, therefore it is 1, 1 to 1, 0. So these are all the possibilities, the system can move from 1, 1 to other states 0, 2, 1, 2 or 1, 0 and so on. So this is the way you can visualize the different transition arcs and the corresponding rates. So this is a state transition diagram for two queues, Tandem Queuing Network. Obviously from this diagram itself you can say that this is not a birth-death process. If it is a birth-death process and the system has to move forward one step or backward one step, no other moves. So here you have a two dimensional Markov chain, therefore this is not birth-death process. It's a continuous time Markov chain.