Slide 1: This is stochastic process model 5, continuous-time <Markov chain. Lecture 6, queuing networks. In the last five lectures we started with the definition of continuous-time Markov chain, then Kolmogorov differential equation, Chapman Kolmogorov equation, infinitesimal generator matrix in the lecture 1. Lecture 2 we discussed the birth-death process. Lecture 3 we have discussed the special case of birth-death process, Poisson process, then the fourth lecture we have discussed the application of continuous-time Markov chain in queuing models, that is the first special case… special Markovian queuing model that is MM1Q. Then in the fifth lecture, we have discussed the simple Markovian queuing models other than MM1Q. Now we are moving into the sixth lecture that is for the application of continuous time Markov chain in queuing networks. So basically the queuing network is nothing but a network consists… consisting of several interconnecting queues. Either you can say network of queues or queuing network.

Slide 2: In today's lecture we are going to cover the queuing network as the application of continuous time Markov chain. In the last two lectures we have discussed the application of CTMC in particular birth-death process, but now we are moving into the more general setup of CTMC that is coming in the queuing networks, that means the underlying stochastic process of the queuing network is a more general continuous-time Markov chain, whereas all the simple Markovian queues, the underlying stochastic process is the birth-death process. The study of queuing networks is going to be a very vast area, so we are going to cover here only very simple queuing networks. The first one is Tandem Queuing Network, then we are going to discuss the Open Queuing Network or you can say Open Jackson Queuing Network and also we are going to discuss the Closed Jackson Queuing Network, then followed by that we are going to discuss the application of CTMC in performance analysis of wireless network system. If time permits I will do it in this lecture or I will go into the next lecture. So now we are going to discuss the queuing network. Queuing network is nothing but a network consisting of several interconnecting queues.

Slide 3: So we have discussed already the queues in the fourth lecture. I am giving three different examples for the queuing network. So you can say this as the first queue or we can say the queuing network as the node one, which consists of separator one queue, where the arrival comes waiting for the service after service is over then it is moved into the second queue, and this is the separate queuing… queues in which the arrival rate for the service, after the service is over, that it move into the next queue, like that it has many queues. So this is the one simple example of queuing networks and this queuing network is called a Tandem Queuing Network, because the output of first queue is the input for the second queue and there is no feedback. The output of the second queue is the input for the third queue and so on. Like that it is a system in which it has some finite number of queues in series. Therefore this is called the Tandem Queuing Network.

Second type this also consisting of many queues, interconnecting queues. Whereas here the output of the first queue split sum probability, the summation of this probability and this probability is 1, and move it into the input for the second queue as well as third queue, if you label this as the node 1, node 2, node 3, node 4, node 5, therefore this is a five nodes queuing network. The output of second queue and the third queue is the input for the third queue whereas after the service completion of the third queue with some probability the jobs or customers can move in again come to the third queue. Therefore this these type of, is called the queues with the feedback. We call it as queues with feedback. Then the other proportion, it moved into the input for the fifth queue. After the service is completed it grows away from the system. So this type of queuing networks in which the arrivals comes from the outside, from source and it departure from the system, there is a possibility, the customers who finish the service in the fourth queue it can depart from the system. So this is the example for the open queuing Network. The third type, here also we have four queues or four nodes, there is no output of queues from the output from the system, as well as there is no input from the outside the domain, that means there is a constant number of customers or jobs will be moving from one node to the other nodes with these probabilities and these probabilities are called the routing probabilities in which after the service is completed in the second queue the customers or jobs move into the third queue, with some probability it will move into the fourth queue, therefore this probability is called the routing probability of jobs on customers moving from one node to the other nodes with some probability. This type is called a Closed Queuing Network.

Slide 4: So even though I have started with some three examples and these three examples are related to the different types of queuing networks, but before I move into the types of queuing networks, let me give the characteristics of queuing networks. So what are all the minimum information is needed or is provided to study the queuing networks. It consists of nodes and interconnected, so the interconnected are given by the information routing probability matrix.

Slide 5: Suppose in this example if the node 2, after the service is over, the packets or the customers or jobs are moving into the node 3, then the routing probability is P2 3. So P is the matrix, routing probability matrix and the second row third column element it's corresponding to the routing probability of customers or jobs or packets moving from node 2 to node 3 after the service is completed. If that particular probability is 0, that means that routing probability there is no path from node 2 to node 3.

Slide 6: So we need a routing probability matrix that matrix order is number of nodes in the queuing networks and the entries are the probabilities values, therefore it will be either... it will be greater than or equal to 0 and lies between 0 to 1. Obviously this routing probability matrix will be a stochastic matrix. The row sum is 1. All the entities lies between 0 to 1. Now what are all the information is needed in the each node or in the each queuing system inside the queuing networks. You have to provide the arrival process, arrival pattern. If it is inter or arrival time is a random variable then what is the distribution of that? Then you have to provide number of servers in each node, whether it is only one server or multi server or infinite servers. Then what is the time taken for the service, whether it is a deterministic or probabilistic, if it is probabilistic then what is the distribution of service type? For each node you have to provide. If it is one server then what is the service time distribution, if it is more than one servers then whether they are the identical servers or non-identical and what is the distribution of each server's service time? Next what is the capacity of the queuing system in each queues, in each… the whole queuing networks what is the maximum number of jobs or customers are allowed as a waiting space… in the waiting space, whether infinite capacity system or finite capacity system, if it is a finite capacity stone then if the customers are coming from the previous some other node and if the system is full then it will be blocked. So therefore we should consider, what is the capacity of the each queues also? What is the scheduling policy, in what policy the customers are getting served in each queue, so scheduling policy, whether it is a first-come first-served or last come first served or priority based or random order. So we should know what is the service… what is a scheduling policy for the each node? So if you provide these five information for each node and the routing probability matrix as well as what type of customers are coming into the whole queuing networks, either only one type of customers are moving inside the different queues or more than one type of customers are in the system. Our interest in these course, how the CTMCs used in the or CTMCs playing a role in a queuing network, therefore I am going to discuss a very simple queuing network, in which you can able to map the underlying stochastic process with the CTMC therefore we can get some of the performance measures through the knowledge of CTMC.

Slide 7: Now I am discussing the type of queuing networks, even though I have explained the type of queuing networks through these three examples, like that you can frame many more queuing networks, but in this course I am going to cover only these three types of queuing networks, therefore I have given these three examples. So through these three examples I am going to classify the queuing networks as the… majorly in two parts, that is the OPEN QUEUING NETWORK and the Closed Queuing Network. What is the meaning of Open Queuing Networks? The jobs or customers arrive from external sources, routed to other nodes and eventually depart. Therefore there is a source and there is a sync also, the external source and it has eventually departed from the queuing networks also therefore that type of a queuing network is called the Open Queuing Networks. The Close To Queuing Networks has a fixed number of population, some constant capital K number of jobs or customer circulate continuously and they never leave, so that type of queuing network is called the Closed Queuing Network. That means always if you count how many customers in the system, system means how many customers in each node, if you sum it up, then that total number is going to be always a capital K, that is the population size. Therefore no one will leave the system and no one depart from the system also. So it's a constant number of customers always there in the system at the different queues. You can always convert to the Closed Queuing Network into the Open Queuing Network by including one source and a sync, you can convert the Closed Queuing Network into Open Queuing Network. So in general we can classify the queuing networks as the Open Queuing Network and Closed Queuing Network.