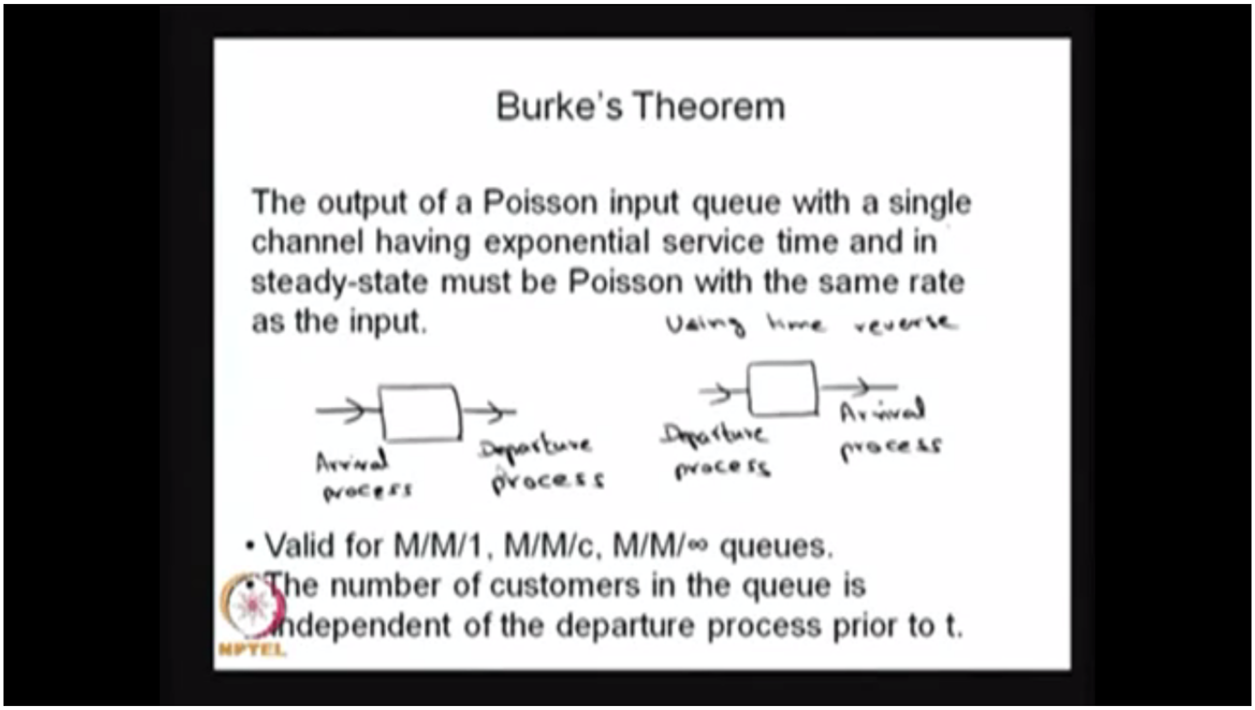
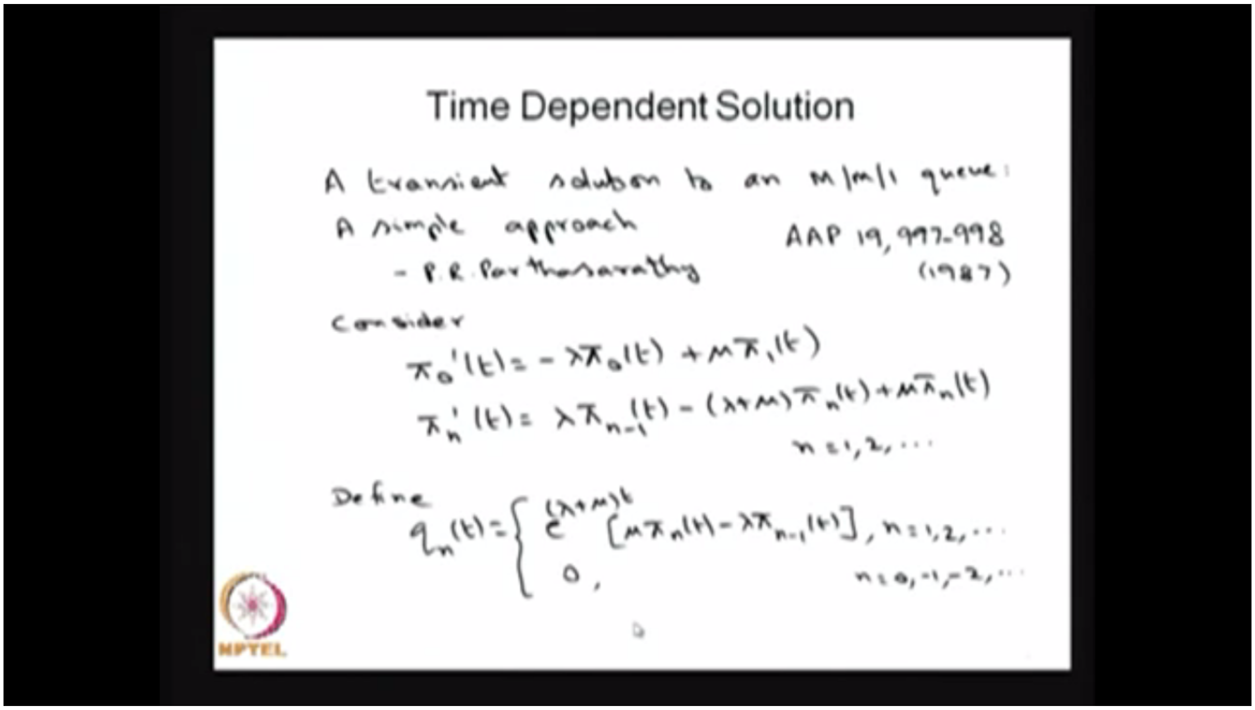
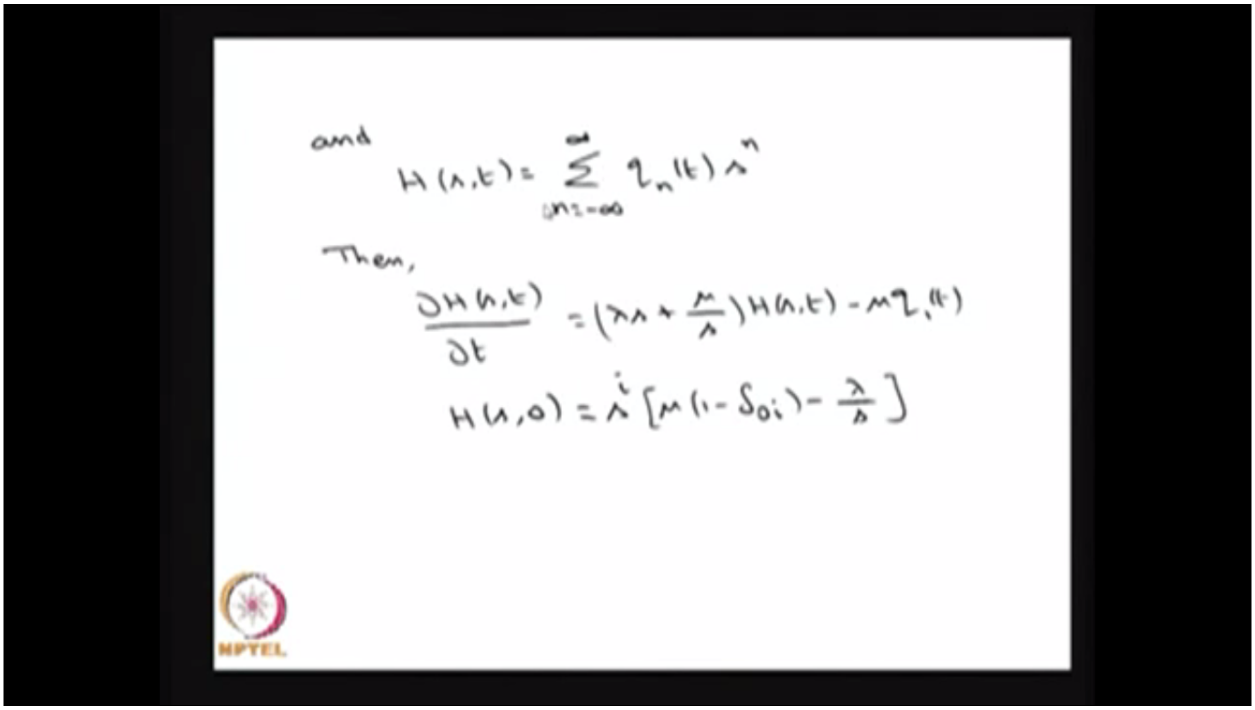


Here I am giving the concept of output process. The arrivals follow the Poisson process for the MM1 queue and the service is exponentially distributed. It is independent of the arrival process and customers leave the system you know the question is what is the distribution of the departure process. That means what is the inter-departure time. After first customer leaves how much time it takes for the second customer leave the system. Then the third customer how much how much time it takes for the inter-departure time. And therefore what is the distribution of the departure process. That is given by the Burke's theorem. The output of a Poisson input queue with a single channel having exponential service time and in steady state must be a Poisson with the same rate as the input. So whenever you have a system in which the arrival follows a Poisson and whenever the system has a single channel and the service time is exponentially distributed in a longer run the departure process is also going to be a Poisson process and the rate will be the same rate as the arrival process.

So this can be proved but here I am giving the interpretation using the time reverse process because in a steady-state this model is going to satisfy the time reversal therefore the stationary distribution exists and if you make a this MM1 queuing model the underlying birth-death process that's why it is a time reversibility equation therefore using the time reverse you can conclude the departure process you can reverse it and that is going to be independent of the arrival process and this is also going to be again Poisson process. So using the time reverse concept one can prove the departure process is independent of the arrival process and departure process is also Poisson process with their same rate as the arrival rate and even though I said it is a single channel having exponential service time and this is valid for MM1 queue, the multi-server Markovian queue as well as infinite server Markovian queuing. So all those models can be combined with the single channel having exponential serve string whether it is a single server or multi server are infinite server gives a result [Indiscernible] [00:02:55]. And the next result is the number of customers in the queue is independent of the departure process prior to it. That's also satisfies.



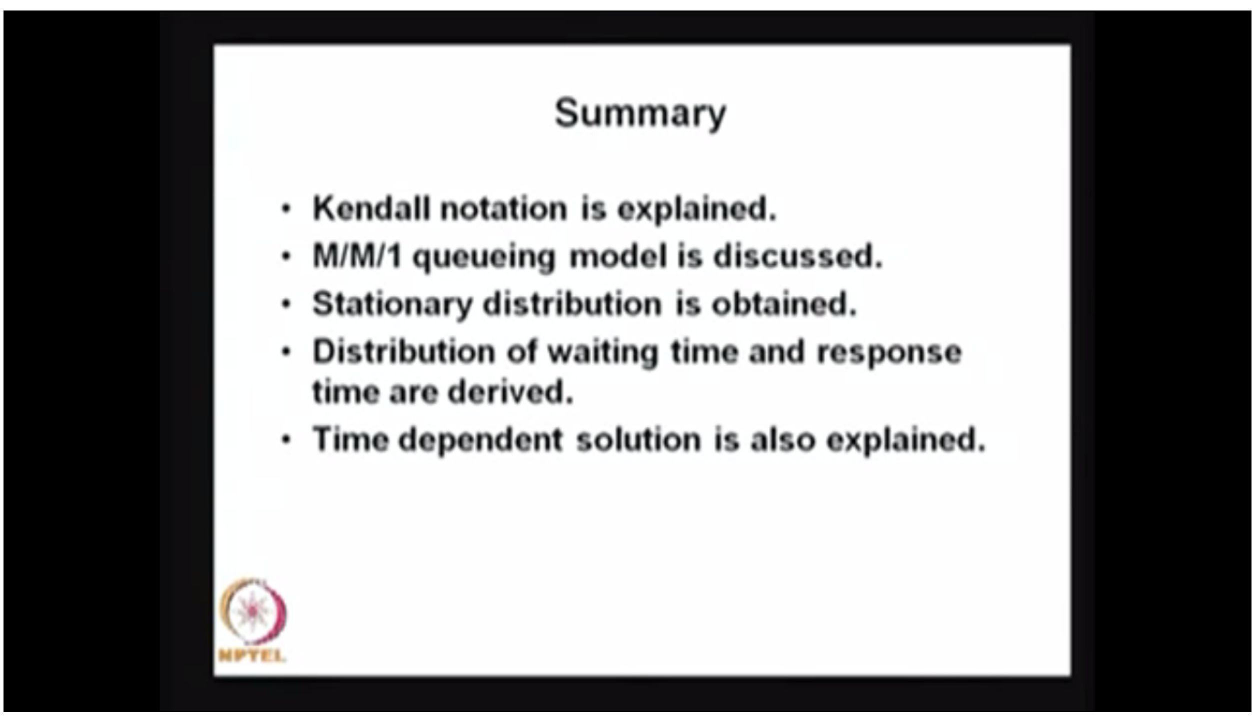
Now we are giving the time dependent solution of a MM1 queue. There are many more methods to find out the time dependent solution for a MM1 queue. It started with the spectral method and the combinatorial method and also the difference equation method. Like that there are many more methods in the literature to find out the time dependent solution and here I am presenting the time dependent solution by P.R. Parthasarathy and this work is appeared in the advanced applied probability volume number 19, 1987. So in this paper he has considered the system of difference differential equation there is nothing but the forward Kolmogorov equation and making a simple function Qn of t that is a difference of Pi n's with the manipulation e power lambda plus mu of t.



So once you use this definition, once you convert this system of difference equation with the Qn of t by making a proper generating function that is of the form n is equal to minus infinity to infinity Qn of t times S power n. Therefore this is sort of a generating function in terms of Qn of t where Qn of t is for n is equal to 1 to infinity this is of difference of mu times Pin minus lambda times Pin minus 1 multiplication e power this function and for n is equal to 0 minus 1,2 and so on 0 therefore you have a generating function so you can convert the whole difference differential equation in terms of Pin into the 1 partial differential equation with the initial condition also changes because if you assume [Indiscernible] [00:05:15] customers in the system at time 0 and this is going to be initial condition for a function H of s,t at t equal to 0. So now the question is you have to solve this equation with this initial condition this Pde using this initial condition. So use sum identity of modified Bessel function one can get the solution by N of t in terms of Pi naught where Pi naught you can get it in terms of Q1. where all the Qn's satisfies this equation that is in terms of the modified Bessel function. So one can see the complete solution in this paper but here I am giving the very simple approach of getting the time dependent solution for the MM1 queue by changing this system of differential equations into one Pde with the initial condition and solve that Pd and obtaining the in's and Pi naught in terms of a modified Bessel function. Before I go to the summary let me give the simulation of a MM1 queue.

So this is a queuing network modeling lab. So from in this queuing network modeling lab one can simulate the queuing network models. So for in this I am going to explain how to simulate the MM1 queue and the first experiment that is nothing but a live simulation of a MM1 queue single server as well as you can simulate a multi server queue model and you can go for the infinite server model also. So here I am simulating the MM 1 queuing model. So to simulate the MM1 queueing model you need the information about the inter arrival time that is exponential distribution you need a parameter lambda, the value of lambda as well as you need a value of mu that musing but the service rate. So suppose you supply the arrival rate suppose the arrival rate is

2 and the departure rate is 5 the number of services it's a MM1 queue therefore it is already 1 is placed it's a number of servers. So you can start. So this is the way the system increases so this is the actual simulation goes with the insert time X-axis and Y is the number of customers in the system and here the information is how many customers entered till this time that is 15 customers entered and nobody is blocked because it is a MM1 queuing system therefore all the customers who are entering it will be queued and how many customers are served during this time and a number of customers in the orbit this is nothing to do with the MM1 queue this is for the retreading queues and now how many customers are in the system at this time and here this table gives the performance measures the one we have calculated the average number of customers in the system E of R. and and the average number of customers in the queue E of this is mean number of customers in the system that is e of n the mean number of customers in the queue E of q mean waiting time in the queue that is a mean waiting time that is E of quarter. Means sojourn time in the system. Sojourn time spending time or a response to him all are the same. The mean sojourn time in the system is nothing but E of R. So this is nothing but the E of R this is nothing but E of W. This is nothing but Eof q and this is nothing but the E of n. And the utilization is nothing but what is the probability that – so here I am giving the run time what is the average values till this time and what is the result is going to be in a longer run in a steady-state and the blocking probability is here 0 because the system is a infinite capacity model therefore there is no one blocked therefore the blocking probability is 0. So this is the way we can reset and we can give some other values and you can we can start again and we get the another simulation also and initially it gives the fixed steady-state results in the steady-state theoretical result and the runtime is nothing but what is the result of other over the time. With this let me complete the simulation.



So in the summary we have started with the Kendall notation and the MM1 queue is a discussed stationary distribution, waiting time distribution, response time distribution is discussed for the MM 1 queue and also the time dependent solution and I have given the simulation of MM1 queue also. These are all the reference books.