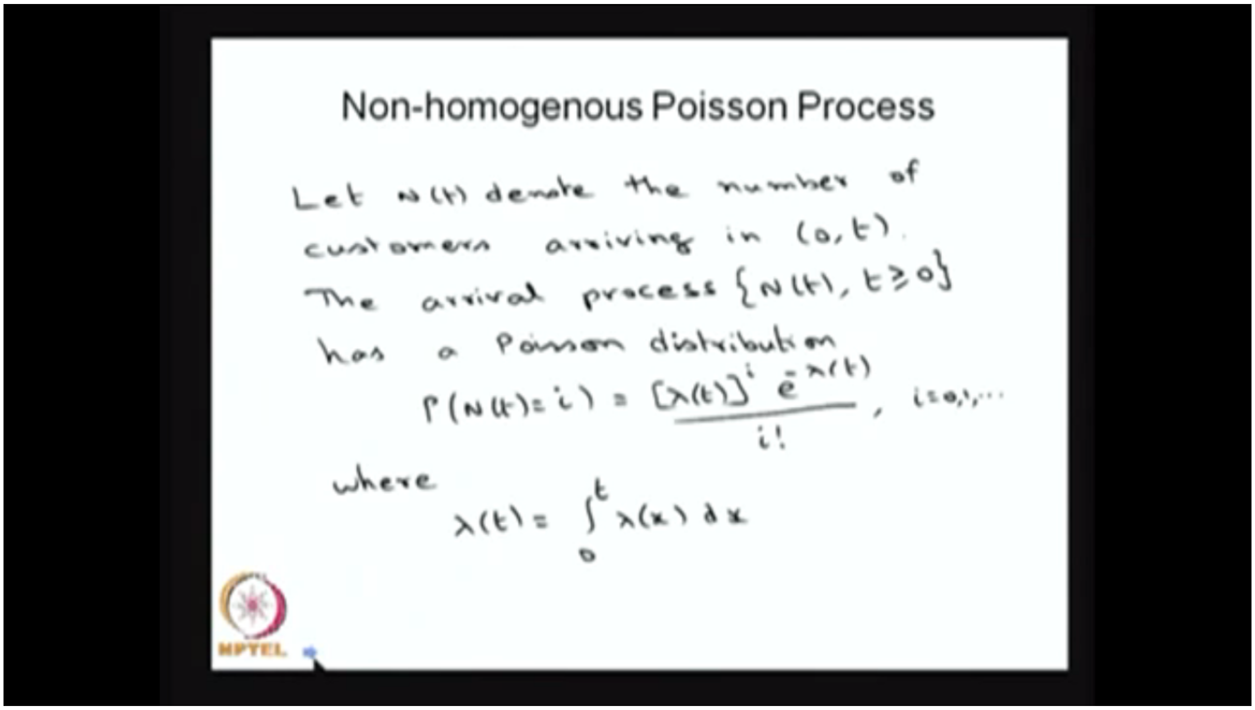
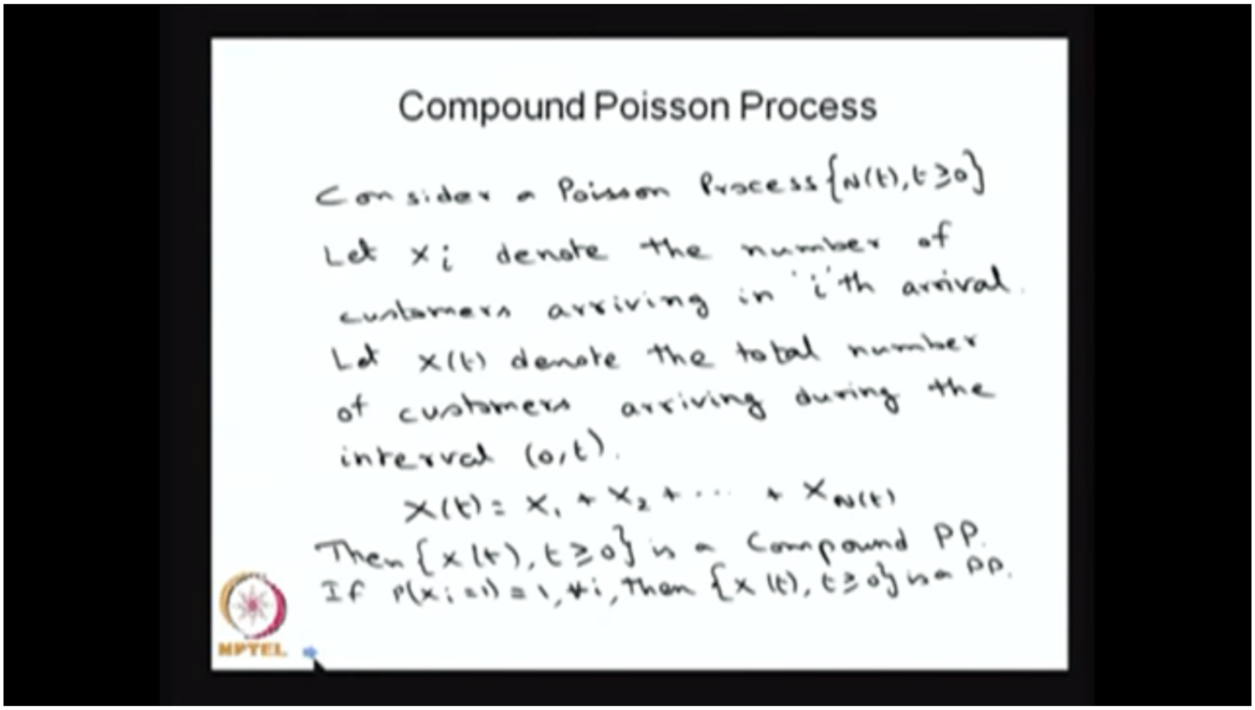


Next I'm going to give some more process related to the Poisson process. The first one is the non-homogeneous Poisson process. Let the N(t) denote the number of customers arriving in the interval 0 to t. The arrival process has a Poisson distribution but here the change instead of the mean arrival rate is a constant, mean arrival rate is a constant lambda but here it is a function of t. lambda t is the cumulative rate till time t that is a change from the Poisson process.

Then this stochastic process is called a non-homogeneous Poisson process. Instead of a mean arrival rate is a constant here the lambda t is a function of t therefore this stochastic process is called a non-homogeneous Poisson process.

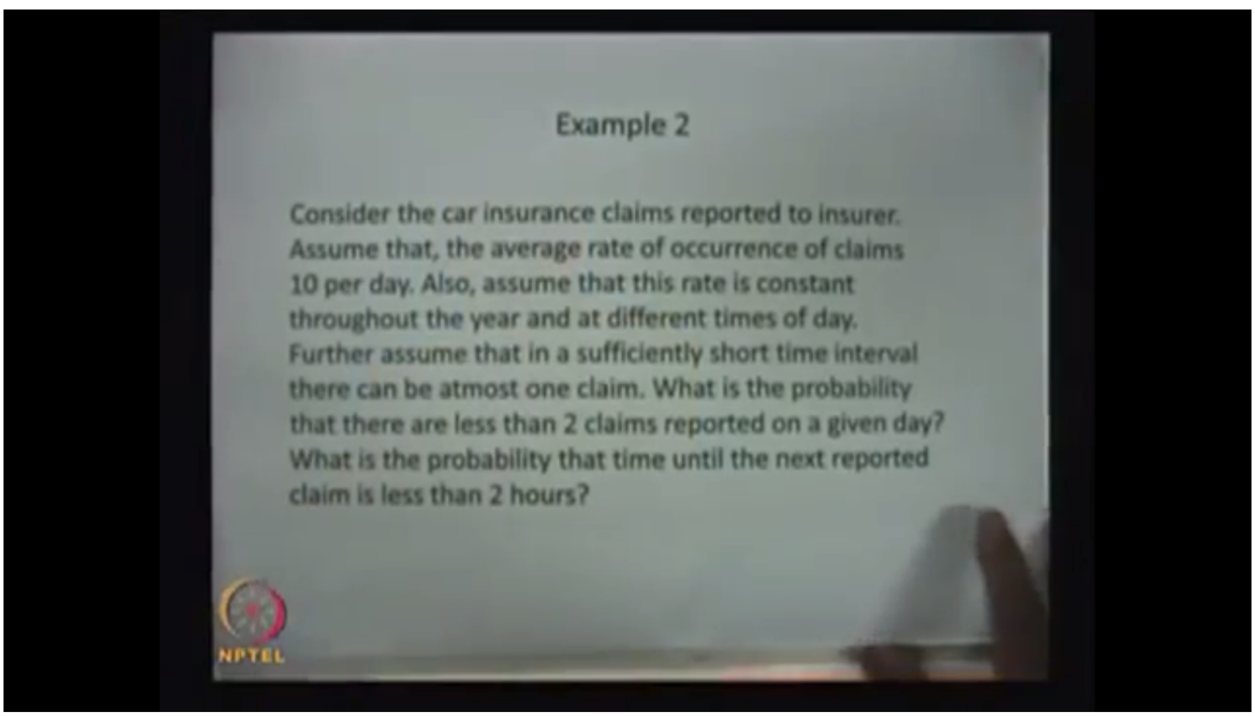


Second one compound Poisson process. Using Poisson process one can develop little complicated stochastic processes related to the arrival that is called a compound Poisson process. Consider a Poisson process N of t. Then you define a random variable Xi denote the number of customers arriving at the ith that time point of arrival. X1 denotes how many arrivals takes place at the time of first arrival, first arrival time point and X2 will be what is the second time of arrival how many arrivals take place. Therefore I'm making a new random variable X of t that involves the t that denotes the total number of customers arriving during the interval 0 to t, that means that it is going to be how many arrival takes place in the first time point X1, how many arrival takes place at the second time of arrival that is X2 and so on plus X of t. X of X N of t. here Nt is a random variable and how many arrival takes place that is Xi's altogether that is going to be the total number of arrivals. The Xi's are independent and identically distributed random variables with some distribution function G independent of the Poisson process N of t.

So this is nothing but a random sum because these are all the random variable and how many random variables you are going to add that depends on the value of N of t over the t. Therefore this is a random sum of Xi's with N of t. Obviously these two are independent Xi's are independent of N of t and since it is a number of our customers arrvial during the ith time point therefore Xi's are discrete random variable and N of t is also Poisson process therefore X of t is going to be a discrete state continuous time stochastic process and we are using a Poisson process to get this stochastic process therefore it is called a compound Poisson process.

One can deduce a Poisson process from a compound poisson process by substituting each Xi takes a value only one unit, that means the number of customers arriving at the ith time pointer is going to be only one that means if I make a P probability of Xi takes the value only one that probability is one for all i then I will have only one value possible till N of t. Then this is going to be a Poisson process. Suppose the probability of Xi is – suppose the Xi's are going to be a discrete random variable with the possible values 0, 1, 2 and so on then the X of t is going to be a Poisson process. I can make a simple sample path for the compound Poisson process. This is over the time and this is over the Nx of t. Suppose these are all the time points in which arrival time point so this is a first arrival time point, and this is a second arrival time point, and this is the third arrival time point. It can be anywhere in the continuous time. Therefore this called a discrete state continuous time stochastic process. So here I am relating with the random variable X1. This X2 random variable. This is X3. So till the first arrival, till the first time of arrival the number of customers in the system is 0. At the first time of arrival the X1 suppose you think you make the assumption X1 takes a value 3. X1 takes a value 3. Therefore this will be incremented by 3 till the second arrival. At the time of second arrival suppose you assume that this takes value 2 with some probability probability of X2 takes the value 2 is greater than 0 so you have assumed value 2. It can take any other value also. So it is incremented by 2 till it takes a third arrival the value is so this is 0, this is 3, then 3 plus 2, 5. At this time whatever be the number of arrival accordingly this can take some value.

So the difference between the compound Poisson process and the Poisson process in the Poisson process the increment will be only one unit increment over that time whenever the time at which the available occur. arrival tim epochs whereas here wherever the time of arrival time epochs the number of customers entered that need not be one. It can be more than or equal to one. So that is the way that job goes. Therefore this is called compound Poisson process. So we have seen two variations of Poisson process one is a non-homogeneous Poisson process, the other one is a compound Poisson process. So before I go to the -- complete let me give the solution for the first -- second example which I started that is this question the car insurance problem. We have discussed the two problem. The first problem is related to the bus-stand bus arrival issues and this is the car insurance problem.

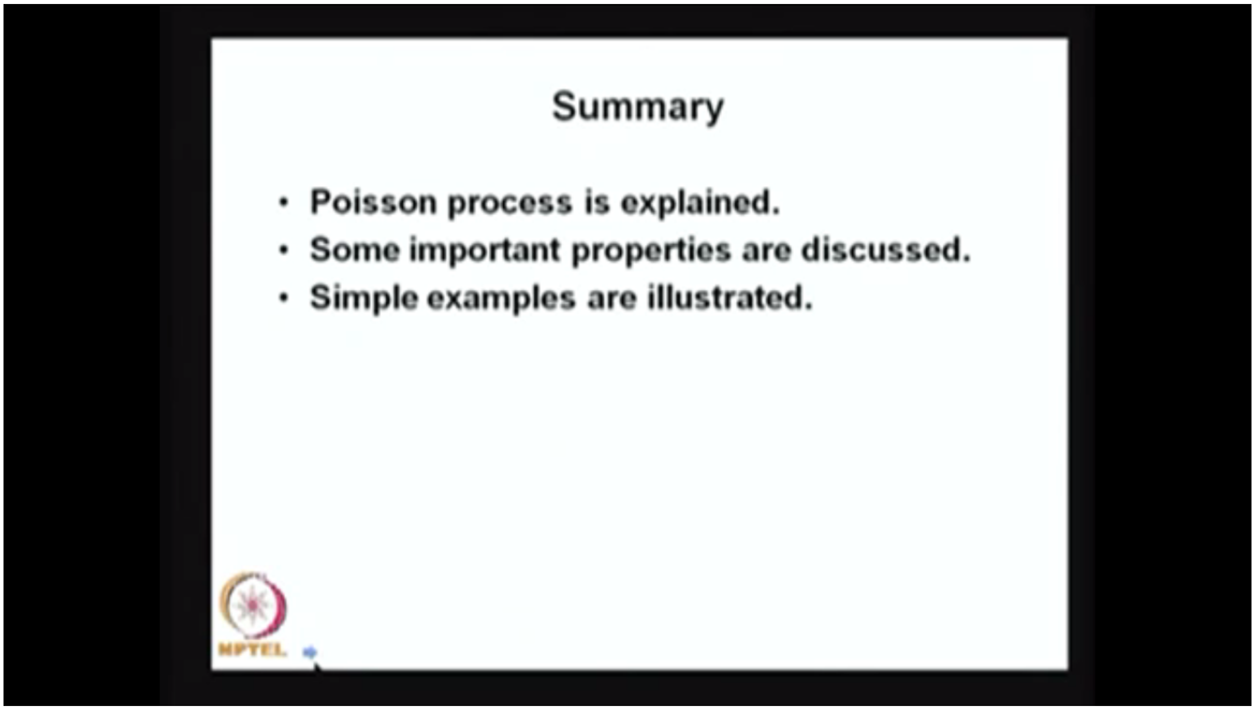


So in this problem we have not assumed the Poisson process but the problem is related to the Poisson process one can assume it is a form of Poisson process because you see the assumptions the average rate of occurrence of claims is 10 per day. Also the rate is a constant and in a very small interval of time at most one claim can happen. The questions are what is the probability that there are less than two claims reported on a given day. Since the increments are stationary so any day you can think of with the only the interval. What is the probability that the time until the next reported claim is less than two hours. So this is related to use the exponential distribution because the inter arrival times are exponential distribution.

So for the first question you can assume that the N of t is nothing but number of car insurance claims reported to the insurers that as a Poisson process. We can assume that N of t is a Poisson process based on the assumptions given in the problem. Once you assume that this is a Poisson process the question is what is the probability that there are less than two claims reported on a given day. So the given date two days you can shift into 0 to 2 days itself because of the increments are stationary. So the question is nothing but what is the probability that N of 1 is less than 2 in a given day, a day. So what is the probability that N of 1 is a less than 2. That is nothing but what is the probability that N of 1 equal to 0 or N of 1 equal to 1. Therefore the probability is added. So you substitute since it is a N of t is a Poisson process the probability mass function of a N of t is equal to K that is e power minus lambda here the lambda is a 10 per day 10 times t and a 10 times t power K by K factorial. So this is the probability mass function for the random variable N of t for fixed t. Therefore N of 0 N of 1 is equal to 0 that is nothing but e power minus 10. Here the t is 1, one day plus N of 1 is equal to 1 you substitute here therefore you will get 10 times e power minus 10. So the answer is 11 times e power minus 10. Numerically you can get what is the value. So the probability that there are two claims reported on a given day is 11 times e power minus 10.

The second question what is the probability that time until the next reported claim is less than two hours. So this is equivalent of the next reported claim is less than two hours that means that the residual time of the next claim that is going to happen the one claim is going to happen less than two hour. That means that you can use the inter-arrival time that is the exponential distribution with the parameter lambda. Here the lambda is 10 or 10 by 2 hours therefore you should convert the values so it is 10 divided by 24 claim can happen at any day throughout 24 hours therefore 10 per day therefore it is 10 divided by 24 per hour so the t is exponentially distributed with the parameter 10 by 24.

Now the question is what is the probability that the time until the next reported claim is less than two hours that means what is the probability that t is less than 2 that is nothing but since it is exponential distribution and you know the CDF of the random variable t. So the probability of t is less than 2 is nothing but 1 minus e power minus 2. So 2 times so 20 by 24. So the answer is a 1 minus e power minus 20 by 24 that is the probability that the next reported claim is going to be occur before two hours.



So with this I have completed the two examples also. So in this lecture we have discussed Poisson process and we have illustrated two examples for the Poisson process also. Some important properties also discussed in this. The next class I am going to discuss the applications of CTMC in queuing models. These are all the reference books. Thanks.