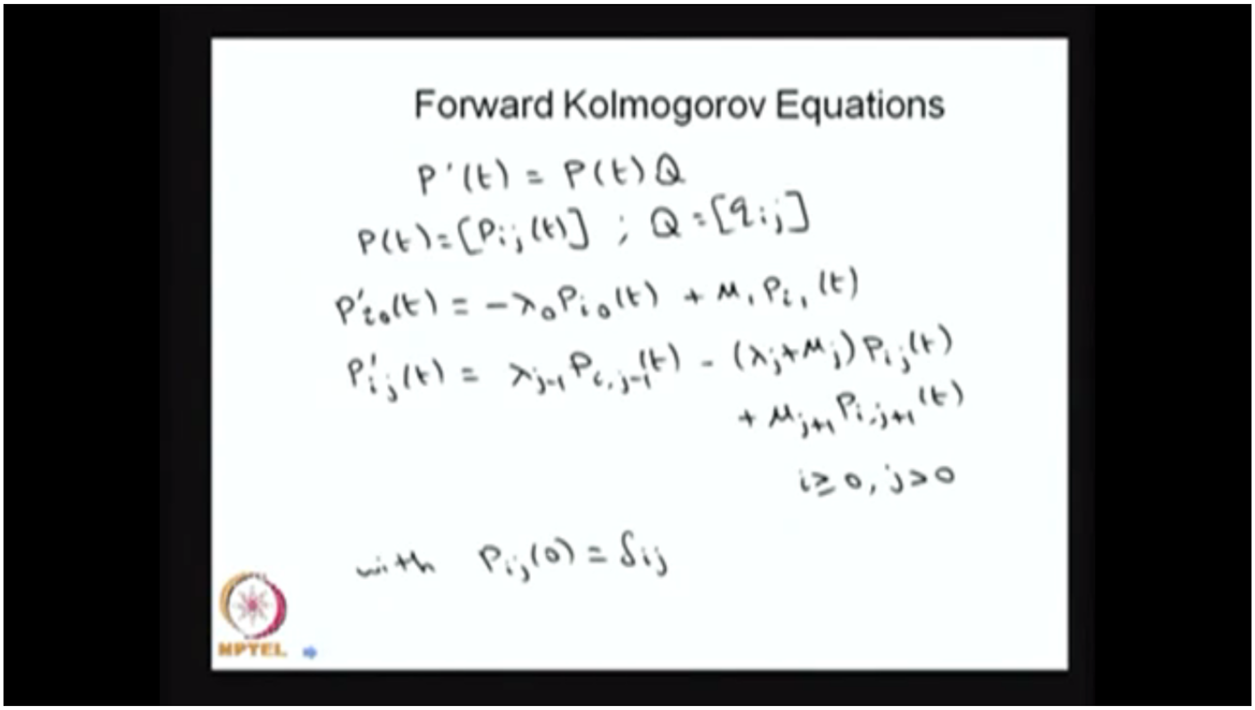
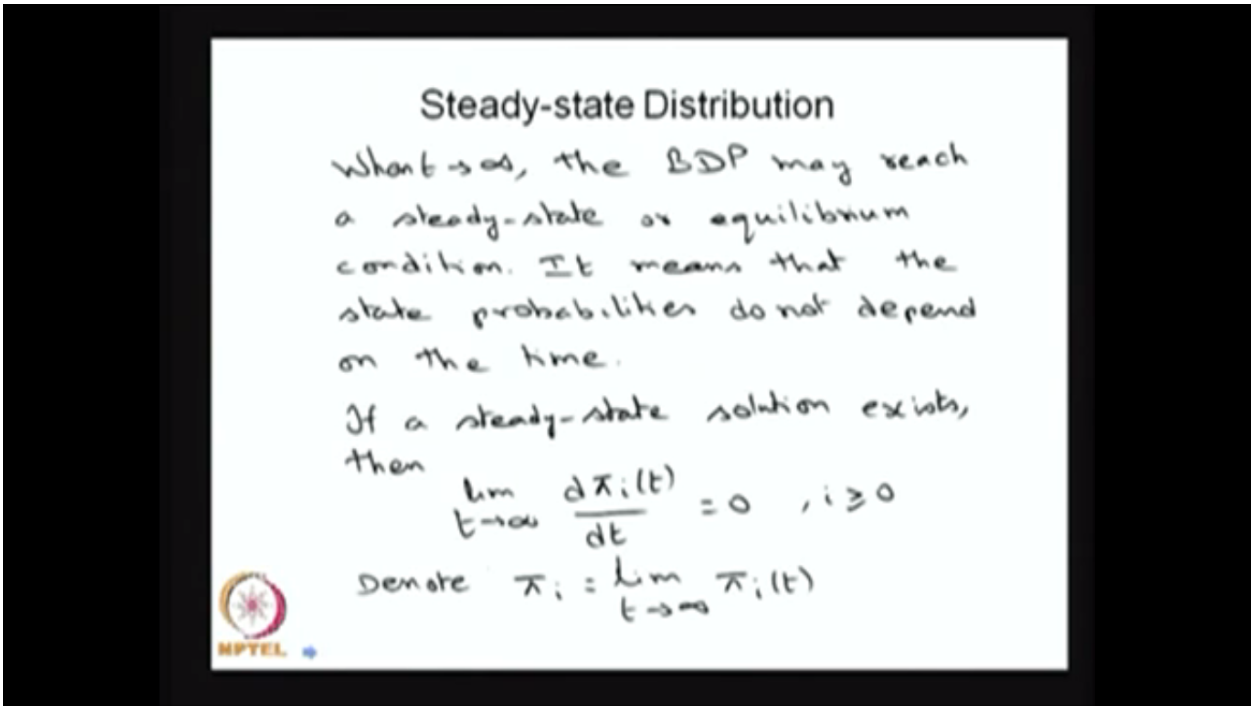


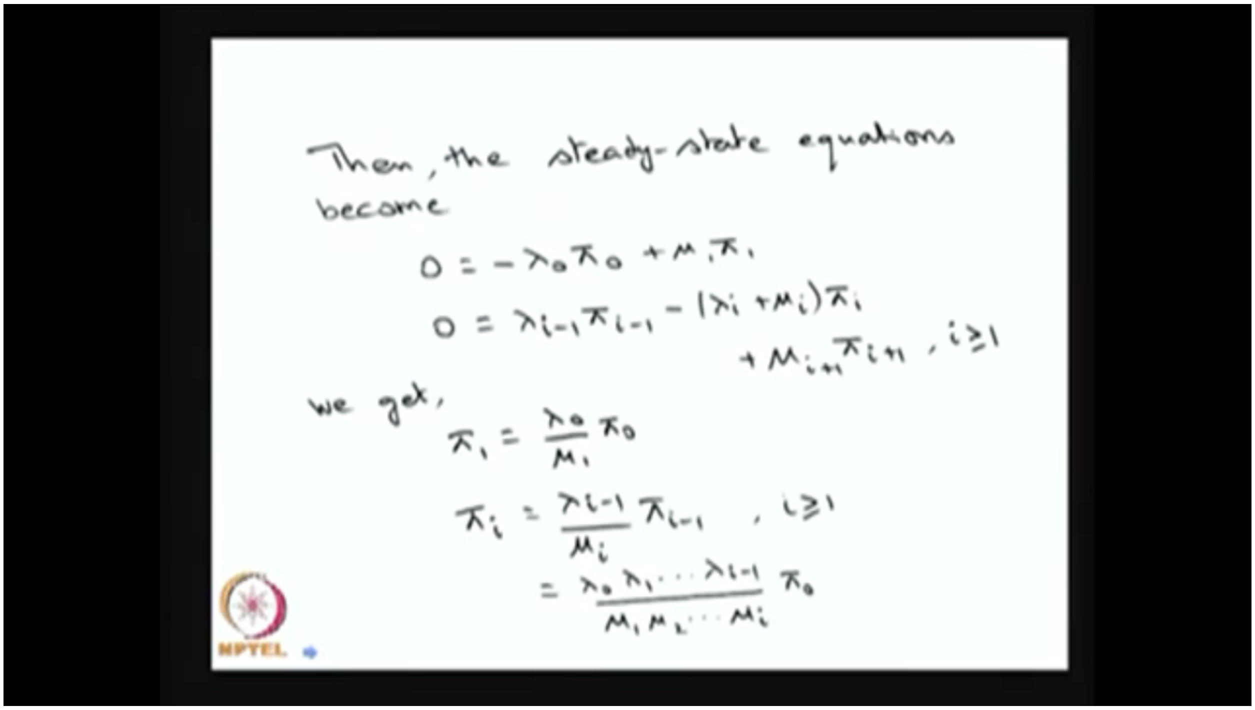
We are discussing the forward Kolmogorov equation for a special case of continuous time Markov chain that is a birth-death process. For a birth-death process the Q matrix is a tri-diagonal matrix therefore you will have the equations from the forward Kolmogorov equation you will have only two terms in the right-hand side for the first equation and you will have only three terms the diagonal element and two of diagonal elements therefore the second equation one can – the first equation one can discuss first the P dash of i,0 that is nothing but the system is not moved from the state 0 moving from the state 0 that rate is lambda naught therefore not moving minus lambda naught times the probability or the system can come from the state 1 with the rate of mu1 therefore mu1 times Pi, 1 of t. For all other equations either the system comes from the previous state with the rate lambda j minus 1 or it comes from the forward 1 state with the rate mu j plus 1 or not moving anywhere. So these are all the possibilities. Therefore with these three possibilities you have a three terms in the right hand side and that is the net rate for any strategy. So if you solve this equation with this initial condition Kolmogorov Delta ij you will have the solution of a Pij.



Here I am discussing the steady state distribution. The way I have discussed the limiting distribution that is a limit t tends to infinity probability of I,j of t exists and then it is called the limiting distribution and the stationary distribution is nothing but for the DTMC Pip is equal to P summation of Pi is equal to 1 for the CTMC Piq is equal to 0 and the summation of Pi i is equal to 1. That is going to be a straight distribution stationary distribution.

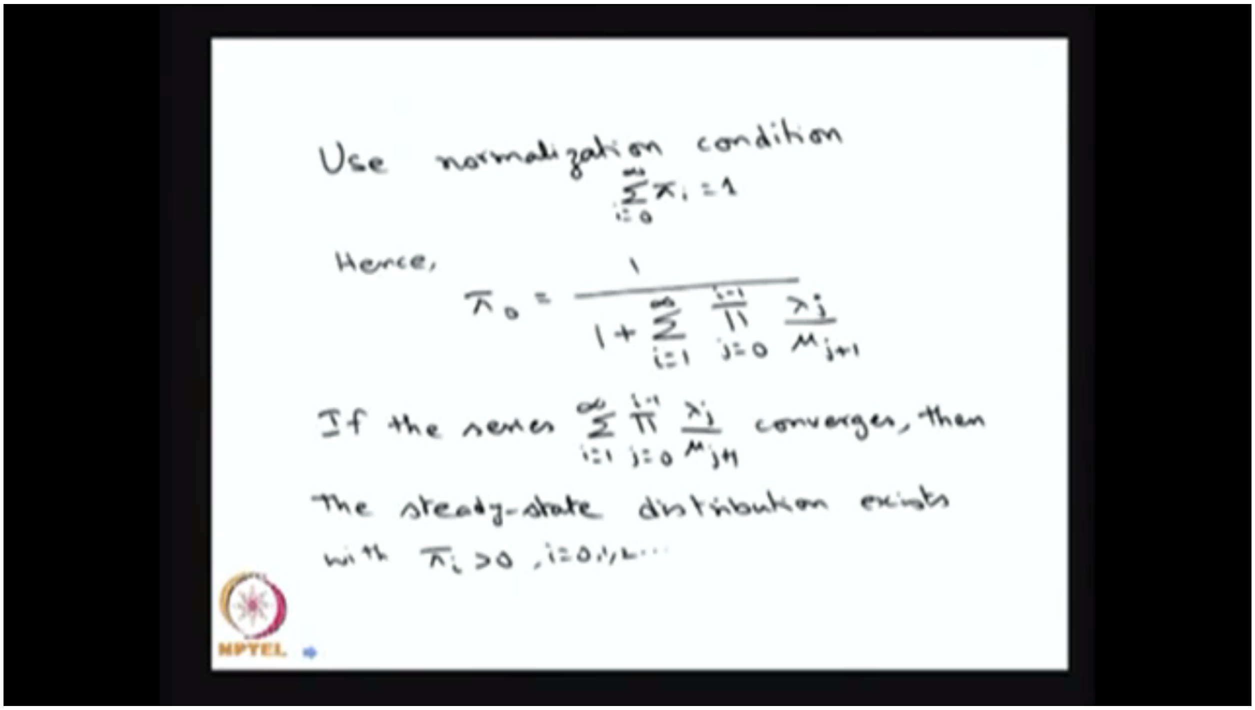
Now I am discussing the steady-state distribution that is nothing but when t tends to infinity the birth-death process may reach steady state or equilibrium condition. That means the state probabilities does not depend on time. That is the meaning of a steady state distribution. As t tends to infinity whenever we say the birth-death process reaches a steady state or equilibrium that state probability does not depend on time. That means if steady state solution exists since the time dependent – since the state probabilities does not depend on time t the derivative of the time dependent state probability at time t that derivative at t tends to infinity becomes 0 if the steady state solution exists. Since the state probabilities does not depend on time t as a t tends to infinity I can write as the Pi's limit t tends to infinity of Pi of t. So this is different from the way we discuss earlier the conditional probability Pij of t but using Pij of t one can find out what is Pi I of t that is nothing but the Pi I of t that I have given in the first lecture for the CTMC the Pi I of t that is nothing but what is the probability that the system will be in the state I at time. That is same as what is the probability that the system will be in the state i given that it was in the state some K at time 0 multiplied by what is the probability that it was in the state K at time. That is nothing but summation of K and this is nothing but the transition probability and this is nothing but the initial probability vector element. So using Pki of t or Pij of t that's a conditional probability one can get the conditional probability this is nothing but the distribution of X of t. So this is a probability mass function, probability mass at time state I. So now what I am defining whenever the steady state distribution exists that means it is independent of time t therefore as t tends to infinity the Pi of t can be written as the Pi. And whenever the steady state solution exists

I can use a limit t tends to infinity the derivative of a Pi of t that is going to be 0. Therefore I am going to use these two to get the steady state probabilities for the birth-death process.

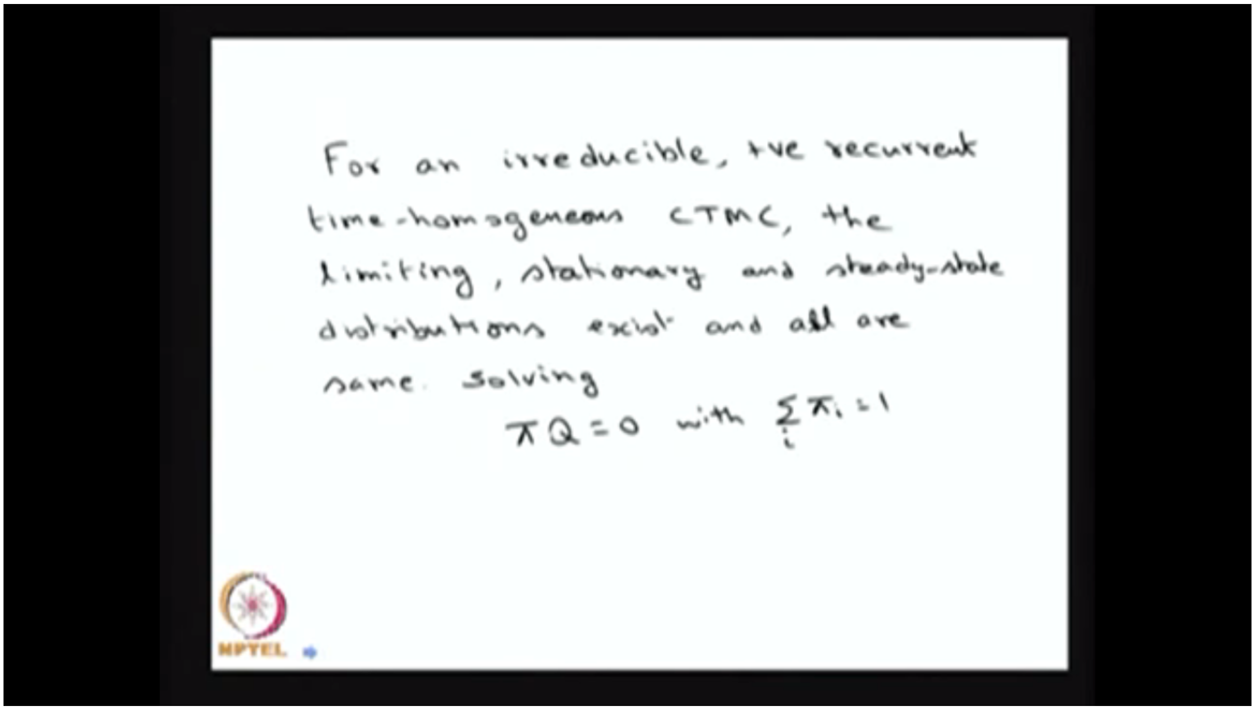


Since as t tends to infinity the derivative of Pij of t is equal to 0 therefore all the left-hand side in the forward Kolmogorov equation that is going to be 0. the right-hand side you will have as a t tends to infinity the Pi of t that can be written as the Pi0 and the Pi1. So the way we write the conditional probability Pij with the Kolmogorov forward equation you can write the similar equation further and condition probability Pi's also. So now I am putting the left-hand side zeros because of this condition limit t tends to infinity the derivative is equal to 0 and the right hand side I am using as t tends to infinity this probability is nothing but the Pi's therefore it is going to be minus lambda naught times Pi naught plus mu 1 times Pi 1 and all other equation has the three terms and this is homogeneous equation and you need one normalizing condition.

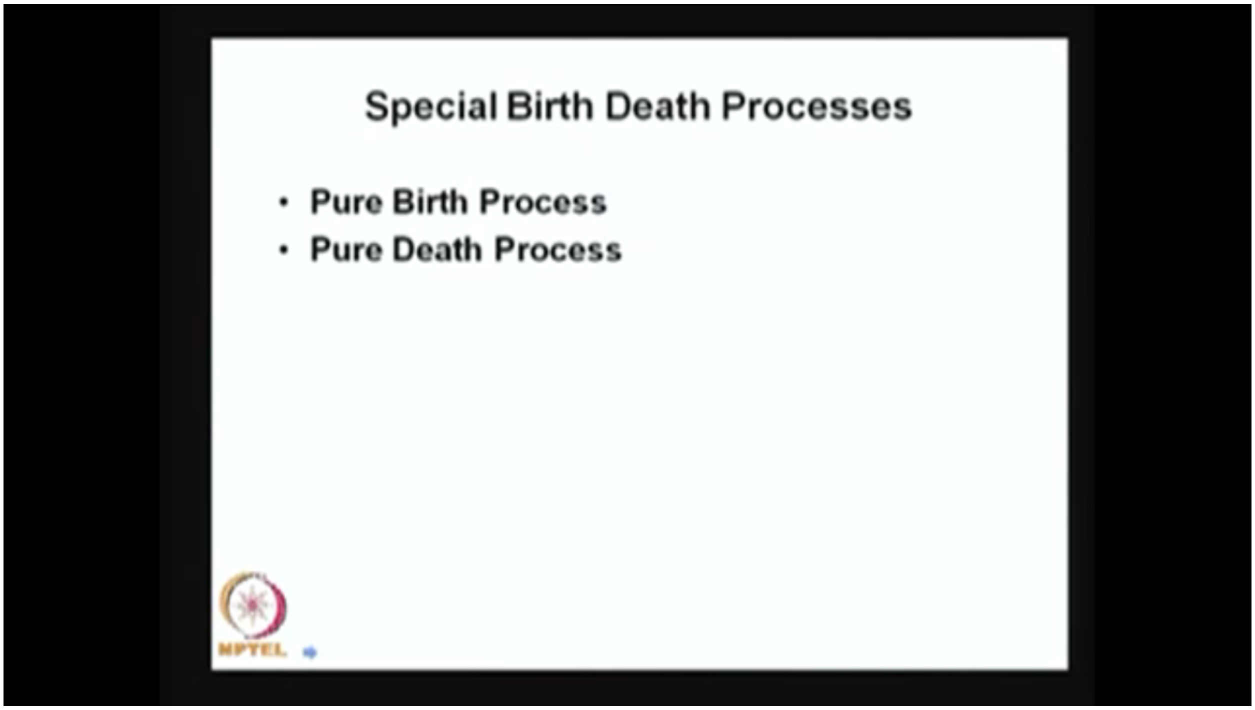
So from this homogeneous equation I can get recursively Pi's in terms of Pi naught. So from the first equation I can get a Pi 1 in terms of Pi naught and the second equation I can get Pi 2 in terms of first Pi 1, then I can get a Pi 1 in terms of Pi naught therefore recursively I can get a Pi's in terms of Pi naught for all I greater or equal to 1.



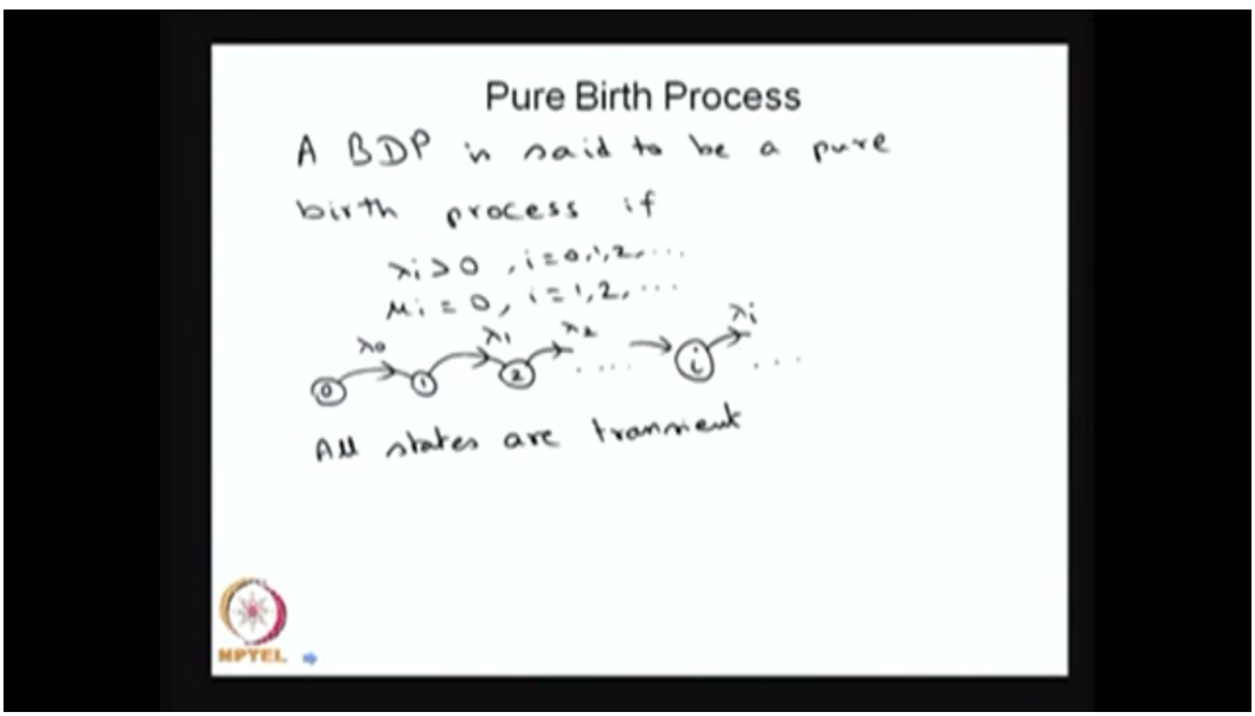
Now I can use a normalizing condition summation of Pi's equal to 1 therefore I will get a Pi naught is equal to 1 divided by summation of this many terms in the product form. Since we need a steady state probabilities and all the Pi's are in terms of Pi naught as long as the denominator is converges you will have a Pi 0 is greater than 0. once Pi 0 is greater than 0 then we get all the Pi's the summation of Pi is equal to 1. So whenever these series converges then I will have a steady state distribution with the positive probability and the summation of probability is going to be 1. So this is the condition for a steady state distribution for a birth-death process because we started with the birth-death process a forward Kolmogorov equation using these two conditions we have simplified into this form and use a normalizing condition and get the Pi naught as long as the summation is or the series is converges then we will have the steady state. If the series diverges that means by substituting the values for the lambdas [Indiscernible] [00:08:08] and if the series our denominator series diverges then the Pi naught is going to be 0 in turn all the Pi's are equal to 0 that for the steady state distribution won't exist if the denominator series diverges. I'm going to give one simple result for an irreducible positive recurrent time homogeneous CTMC we know that the limiting distribution exists, stationary distribution exists.



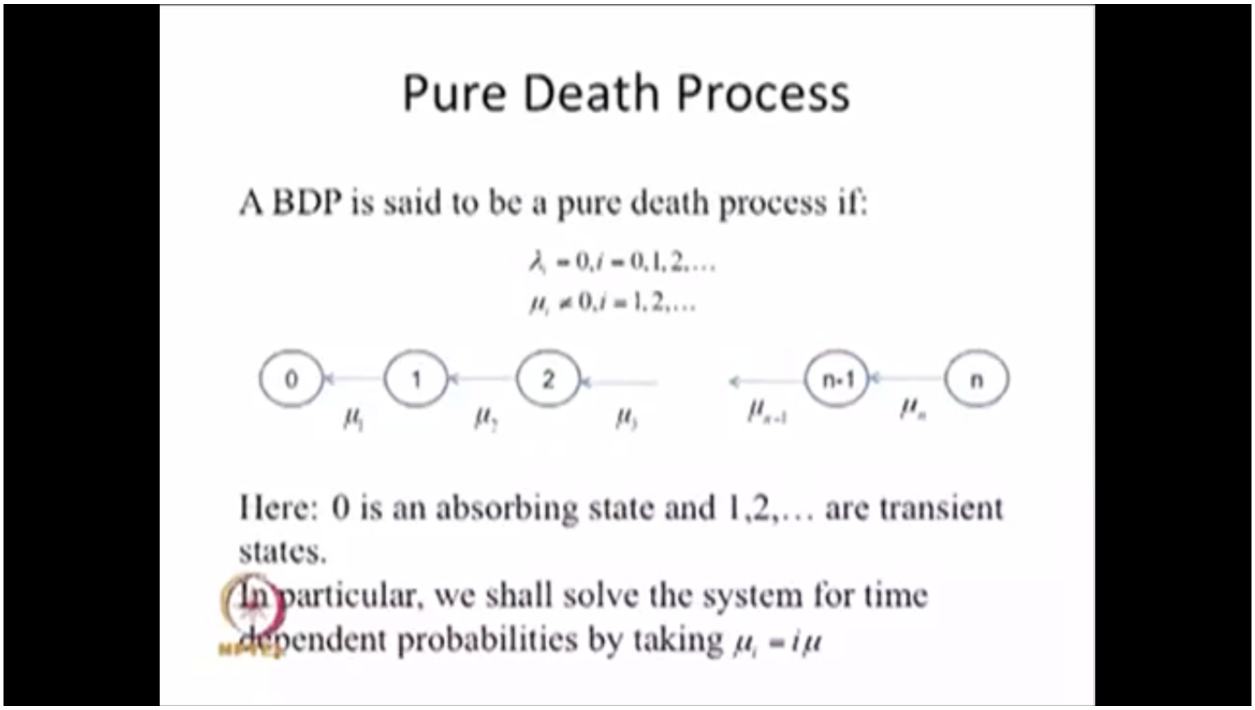
Now I'm including the steady-state distribution also exists I have given for a steady-state distribution for the birth-death process not for the CTMC but here I am giving the result for the CTMC. All the three distributions exist and all are going to be same. Whenever the CTMS is a time homogeneous irreducible positive recurrent all these three distributions are seen and the one can evolve it – one can solve these two equations by Q is equal to 0 and with the summation of Pi is equal to 1 you can get the limiting distribution, stationary distribution, or steady state or equilibrium distribution.



As a special case of birth-death process I am going to discuss these two process in this lecture. Whenever we say the birth death process is a pure birth process that means all the death rates are going to be 0. We started in the birth death process with the only lambda i's are greater than 0 and the mu's are going to be 0 then it is going to be called as a pure birth process.



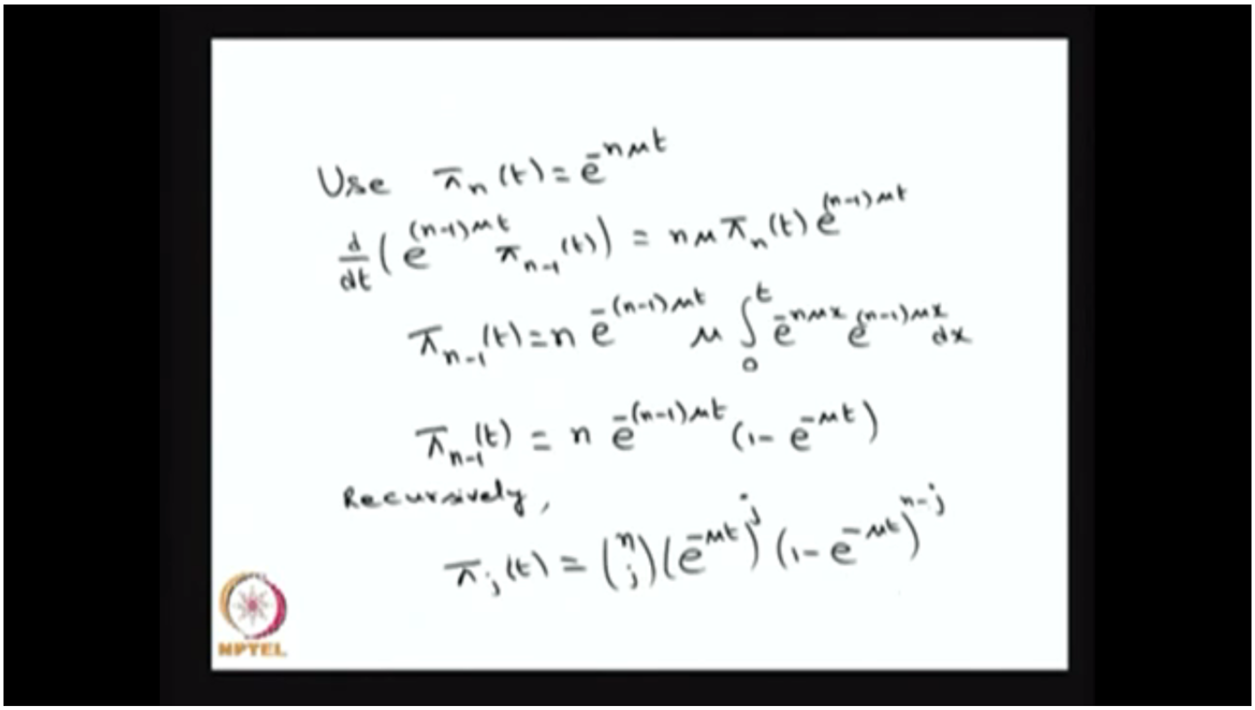
There is a one special case of pure birth process with the lambda i's are going to be constant that is lambda that's a Poisson process that I am going to discuss in the next lecture and in this pure birth process this lambda i's are the function of i. Here all the states are transient states.



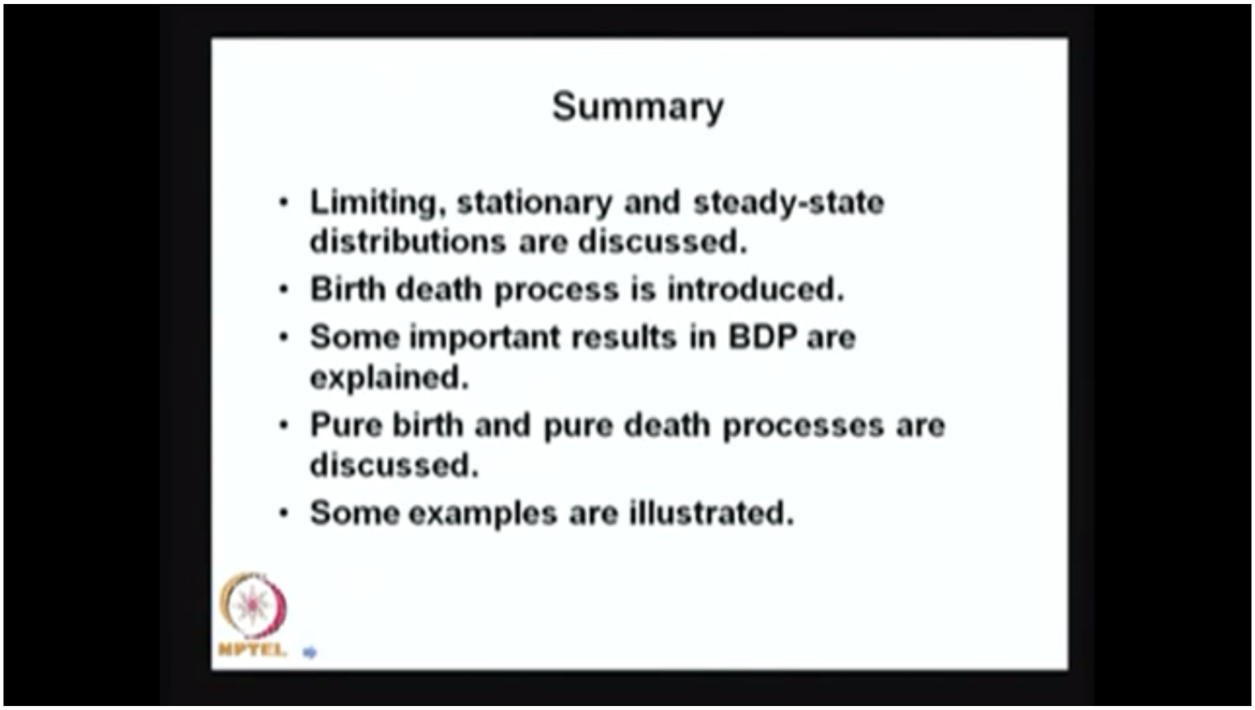
Here I am discussing the pure death process. A birth death process is said to be a pure death process if the birth rates are 0 and the death rates are non-zero. In particular we shall obtain the time dependent probabilities of a pure death process in which the death rates mu i's are equal to i times mu. As I have given the example as a fourth example in the birth death process this state zero is observing barrier. Therefore the state zero is a observing state and all other states are going to be transient state and here the limiting distribution exists and one can also find the time dependent probabilities for this model. Suppose you start with the assumption the system at time zero in the system is in the state n at time zero the systems in the state n at time zero. With that assumption I can frame the equation that is a Pi n dash of t is equal to minus n times mu of Pi n of t that means that the rate in which the system is moving in the state n that is nothing but not moving to the state n minus one with the rate n minus n times mu. Therefore, the equation for the state n that is Pi n dash of t that is equal to not moving from the state n therefore minus that outgoing rate that is n times mu being the state is n therefore Pi n of t. I can use the initial condition Pi n of 0 is equal to 1 so I will get Pi n of t.

For the second equation I had to go for what is the equation for the state n minus 1. So the Pi n minus 1 dash t that is nothing but either the system comes from the state n or not moving from the state n minus 1. Therefore system coming from the state n that is a n mu times the system being in the state n minus n minus 1 times mu Pi n minus 1 of t so we will have a two terms in the right hand side coming from the 1 forward state or not moving from the same state. So we will have two terms for j is equal to 1 to n minus 1.

For the last state that is the state 0 the systems come from the state 1. Since the state 0 is observing states there is no second term. So it is going to be mu times Pi n of t. So you know Pi n of t. Use the Pi n of t in the equation for n minus 1 and get the Pi n minus 1 like that you find out till Pi 1. Use the Pi 1 to get the Pi 0 of t. Use the recursive way.



So using the recursive way you will get the Pi j of t is equal to ncj combination ncj and e power minus mu times t power j this is survival probability of system being in the state and 1 minus e power minus mu of t n minus j. Suppose the system being in the state j that means from the state n this many combination would have come and the survival probability is e power minus mu times t that power so this is nothing but the probability P power j and 1 minus p over n minus j. Therefore this Pij follows a binomial distribution with the survival probability e power minus mu t being in the state j.



So for the few death process I have explained time dependent probabilities of being in the state j that's unconditional probability. So with this the summary of this lecture is I have discussed the limiting stationary and steady-state distributions. I have introduced a birth-death process. Some important results also discussed and at the end I have discussed the pure birth and pure death processes also.

In the next lecture I'm going to explain the important pure birth process that's a Poisson process. These are all the reference books. Thanks.