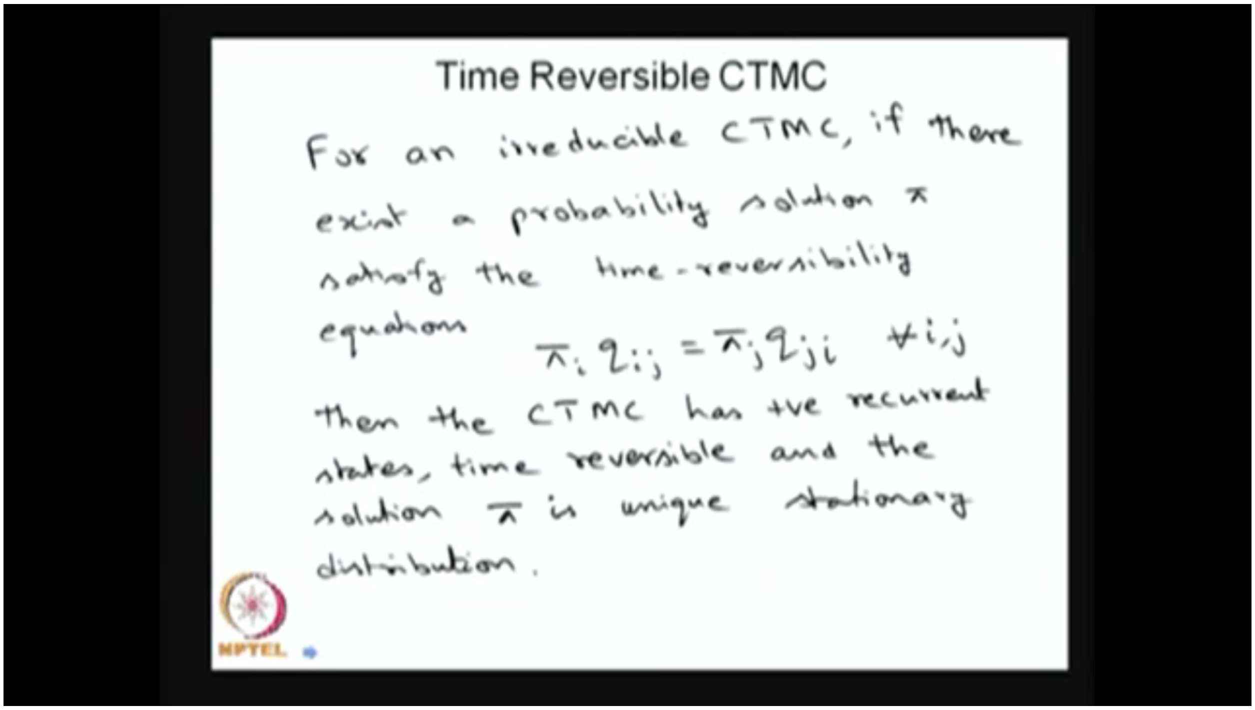
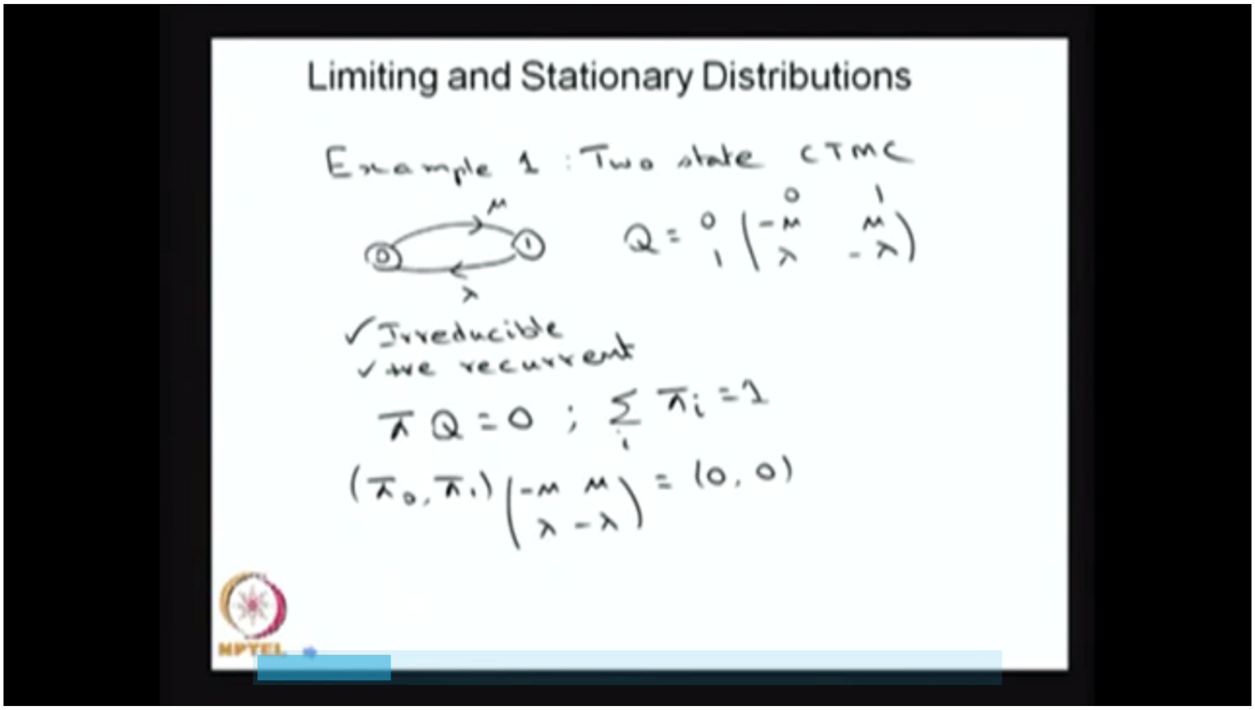
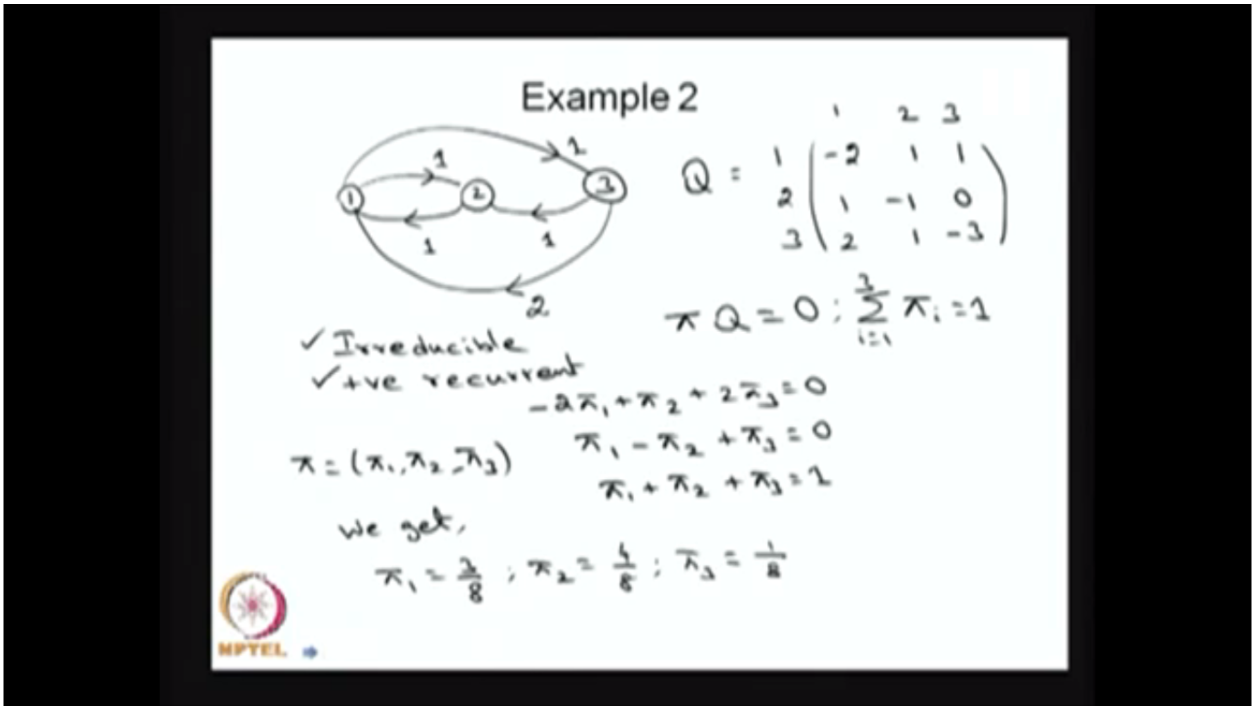


The way I have explained the time reversible concept in the DTMC, the CTMC also has the time reversible concept. So the time reversibility equation is a Pi is equal to Pi a times a Qij is equal to Pij times Qji. The Q's are nothing but the rates and the Pi's are nothing but the probability values. So if Pi's exist this is stationary probability or stationary distribution exist then if the stationary distribution exists as well as the time reversibility is satisfied by CTMC then that CTMC is a positive recurrent and you can say that it is a time reversible and the solution Pi can be Pi is nothing but the stationary distribution. So this result says for a irreducible CTMC if there exists a probability solution Pi satisfies the time reversibility equation this is a time reversibility equation where Q's are rates Pi's are the probability solution. If it is satisfied by the irreducible CTMC the time reversible equation then that CTMC has a positive recurrent states and that CTMC is called a time reversible as well as the Pi is called the stationary distribution.

So initially we have not taken as a stationary distribution. Some probability solution satisfies the time reversibility equations and it is a irreducible CTMC. Then that CTMC as a positive recurrent states and Pi is nothing but the unique stationary distribution. So the usage of this concept is whenever any CTMC is first it is irreducible and satisfies the time reversibility equation of this form then you don't want to solve Piq's is equal to 0 and the summation of Pi's is equal to 1 to get the stationary distribution instead of that use this time reversibility equation instead of solving Piq's equal to 0 and then use a summation of a Pi's is equal to 1 to get the one unknown. That means use the time reversibility equation repeatedly recursively and to get all these in terms of one unknown either P naught or Pi one whatever it is. Then use the summation of Pi's is equal to 1 to get to find the that unknown instead of solving Piq's equal to 0. So whenever it is model is irreducible and time reversibility equations are satisfied then you can conclude all the states are positive recurrent and you can find Pi the stationary distribution in easily instead of solving Piq's equal to 0.

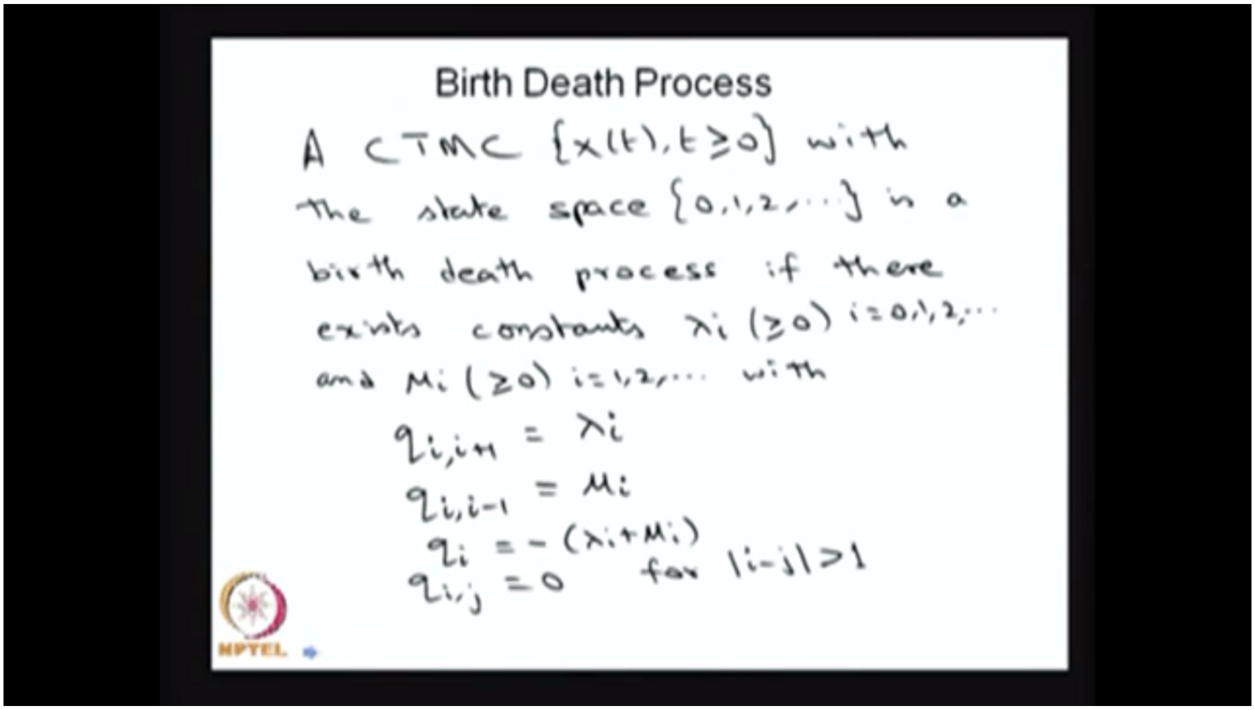


I'm going to give one simple example what is a limiting and stationary distribution. Take the two state CTMC and do we know that Q matrix and you can verify whether this is going to be a reducible [Indiscernible] [00:03:28] positive recurrent since it is a finite state model and both the states are communicating each other. Therefore, it is a reducible positive recurrent states. So you can solve Piq's equal to 0 and the summation of Pi is equal to 1 so Pi times Q Pi is the vector Q is the matrix and again 0 therefore if I take the first equation I will get minus mu times Pi naught plus lambda times Pi1 equal to 0 by taking the first equation minus mu Pi naught plus lambda times Pi1 that is equal to 0. From this I can get the Pi1 in terms of Pi naught since it is a homogeneous equation I have to use one non-homogeneous or normalizing condition summation of Pi's equal to 1 so using that I will get Pi naught is equal to lambda divided by lambda plus mu. Once I know Pi naught then Pi1 is equal to mu divided by lambda plus mu. So this is the stationary distribution as well as the limiting distribution because it satisfies both the conditions.

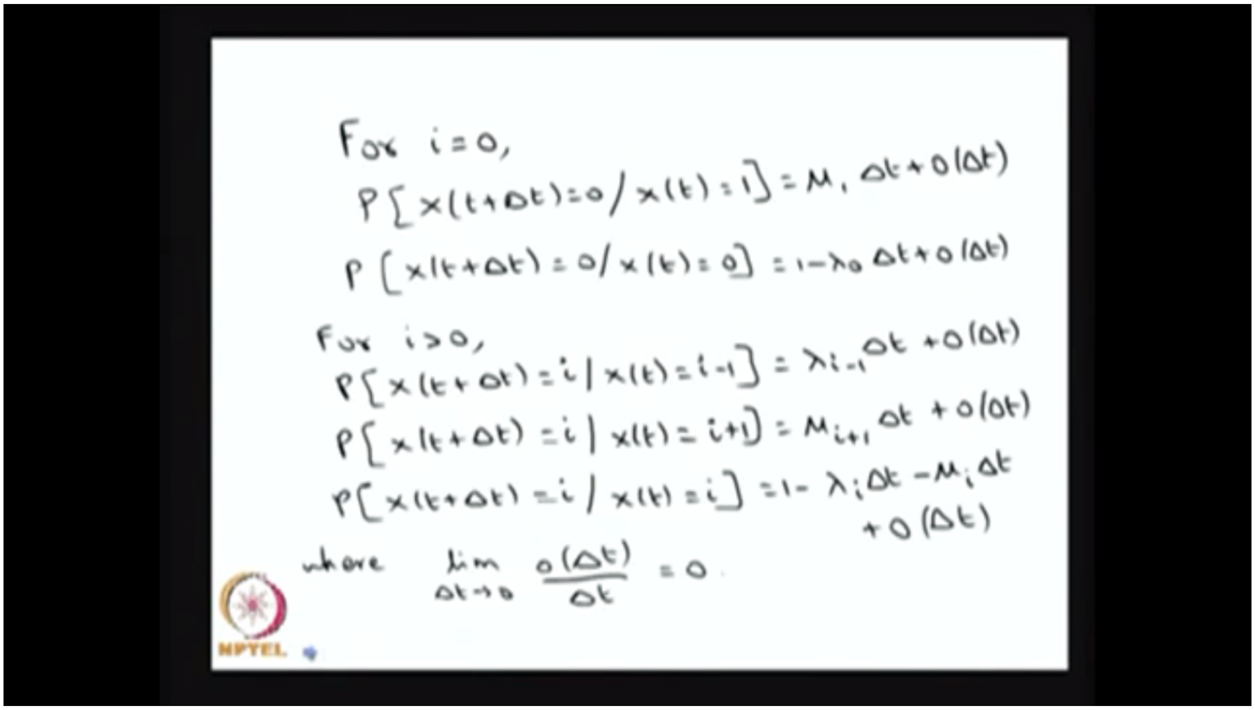


Take the second example. second example also finite state model. All the states are communicating with all other states. Therefore it is a irreducible. Since it is a finite state model you won't have a null recurrent. It's a positive recurrent model. So I can solve Piq is equal to 0 and the summation of Pi is equal to 1 so there are three equations. So I take the first two equation and one normalizing equation and solve these three equations I can get Pi1, Pi2, Pi3. You can verify that the summation is going to be 1. So this is the limiting distribution as well as the stationary distribution because the model is the irreducible positive recurrent model. So this limiting distribution and the stationary distributions are one and the same.

Instead of solving Piq is equal to 0 you can use the time reversibility but before that you should verify whether the time reversibility equations are satisfied by this model. If this model satisfies the time reversibility equation for all i,j then you can conclude it is a time reversible Markov chain and so on but example 1 is a time reversible Markov chain whereas that example 2 is not a time reversible Markov chain you can verify it.

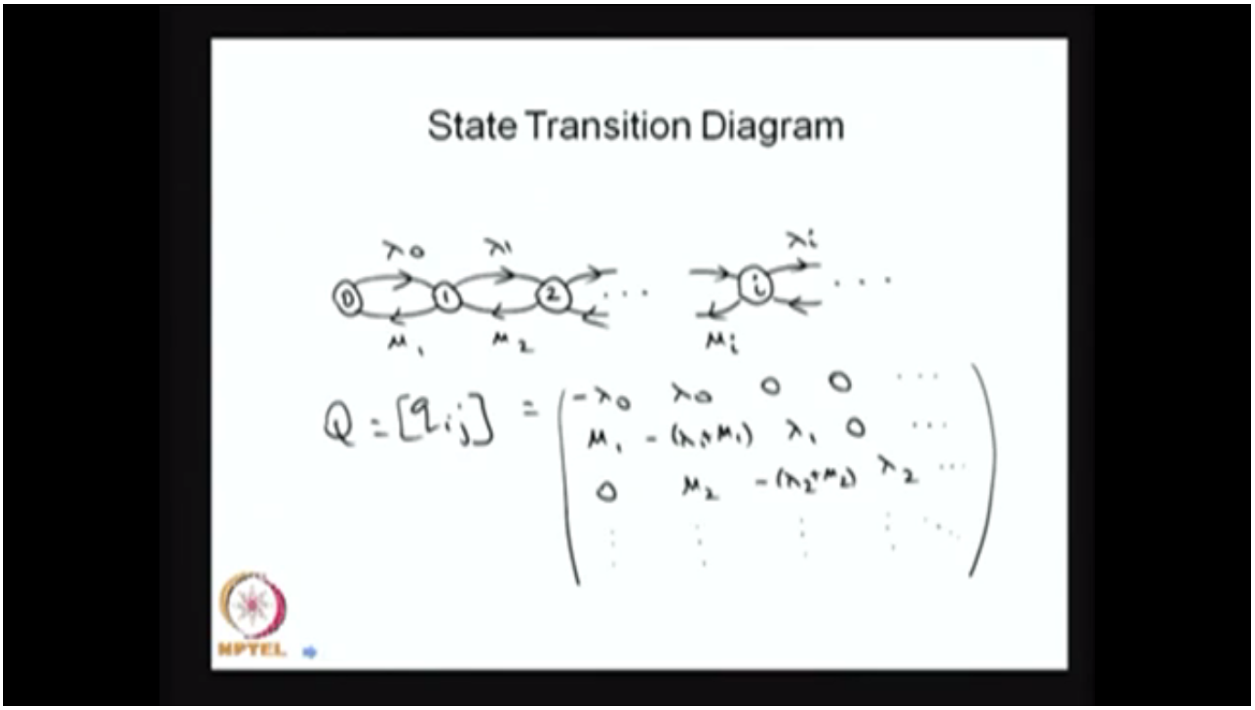


Now I'm moving into the special case of a continuous time Markov chain that is a birth-death process. This is a very important time homogeneous continuous time Markov chain because many of the scenario can be mapped with the birth-death process either the finite state or infinite state. Let me first give the definition of a birth-death process. I started with a continuous-time Markov chain it's a time homogeneous continuous time Markov chain with the state space a countably infinite. It can be a finite also. That CTMC is going to be call it as a birth-death process if there exists a constant lambda mui's such that and these are all nothing but the infinite decimal generator matrix elements and this is i to i plus 1 that rate is always lambdai and the rate in which the system is moving from the state i to i minus 1 that rate is a mui and the diagonal elements are minus of lambda i plus mui whereas all the other rates the system is moving from the state i to j other than i to i plus 1, i to i minus 1 and i to i and all other rates are always 0 absolute of i minus j is greater than 1. That means you will have the infinitesimal generator matrix in which you have only diagonal matter tri-diagonal matrix and all other elements are going to be 0.



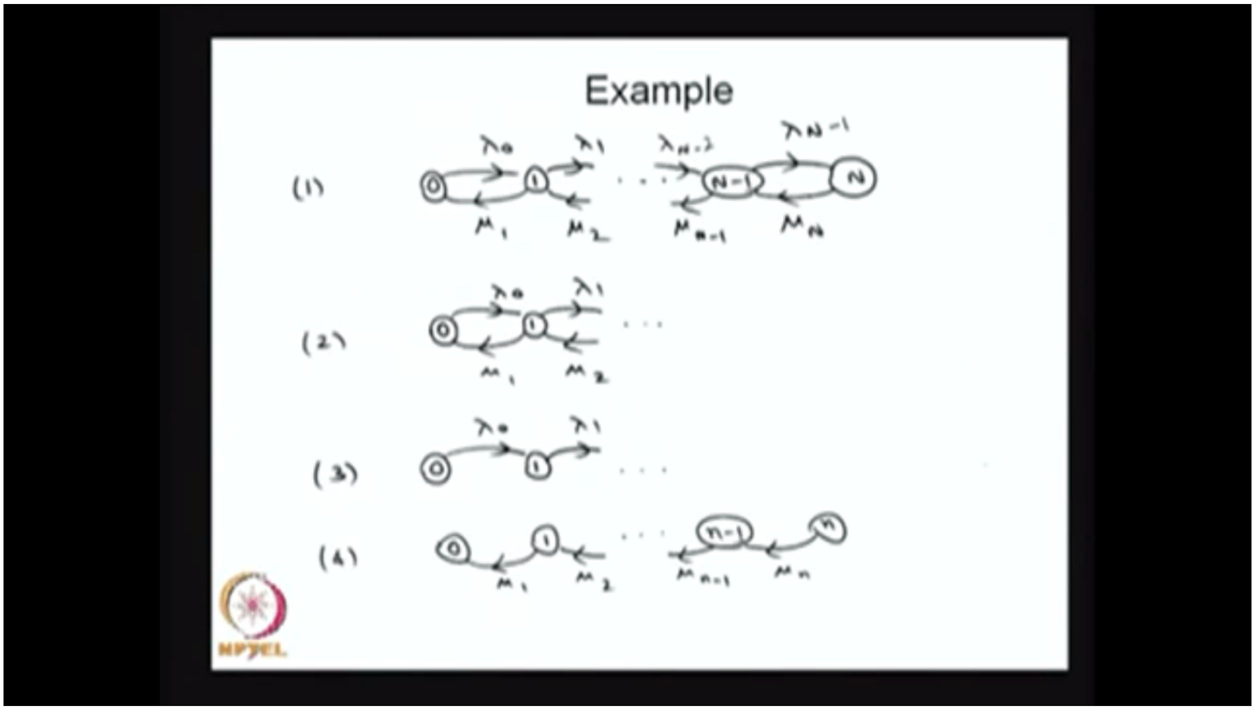
I can write down the condition so that it land up at the rates are going to be only lambda i's ans mu i's so on not all other rates are going to be 0. So if I start with i's equal to 0 the system is moving from the state 1 to 0 in the interval of a delta t because it's a time homogeneous model so this is nothing but this probability the system is moving from the state 1 to 0 in the interval of delta t that is nothing but the rate is mu 1 times delta t plus order of delta t.

Similarly the system is moving from the state 0 to 0 from the time t to t plus delta t or during the interval delta t that is nothing but 1 minus lambda naught times delta t plus order of delta t. So these mu i's and lambda naught and so on these values are always going to be greater or equal to 0 strictly greater than 0 also. For i is greater than 0 the system is moving from the state i to i that is 1 minus lambdai times Delta t minus mui times delta t plus order of delta t whereas the system is moving from i plus 1 to i 1 step backward that is mui plus 1 delta t. The system is moving from the state i minus 1 to i for i is greater than 0 that's a forward one step move that is lambda times i minus 1 delta t plus order of delta t. This order of delta t it may be a function of delta t need not be the same as delta t tends to 0 these quantities are going to be 0 order of delta t divided by delta t is going to be 0. This is the way the system is moving from the one state to either one step forward or either one step backward or anywhere so these are all the only three possibilities with these probabilities. Therefore we land up the Q matrix is going to be the system is moving from the state i to i plus 1 forward one more that rate is lambda i's and the system is moving from the i to i minus one one step backward that is mui or the system being in the same state that rate is a minus lambda i plus mui. Therefore, there is no other move from the system from one state to all other states either one step forward or one step backward.



So this can be visualized in the state transition diagram. Since I started with the state space 0 to infinity that is a possibility you can have a label from some negative integers to the positive integers so you can always a transform into something. Therefore a default scenario or the simplest one I discussed from 0 to infinity. Therefore, you can visualize whatever be the label that can be transferred in a one to one fashion. So this is the rate in which the system is moving from the state 0 to 1 that rate is lambda naught. The system is moving from the state 1 to 2 that rate is lambda 1 or the system is moving from the state 1 to 0 that rate is mu1. Therefore the time spent in the state 1 before moving into any other states that is a minimum of the time spending in the state 1 before moving into the state 2 or the system time spending in the state 1 before moving into the state 0. So both are exponentially distributed with the parameters lambda1 and mu1 and the minimum of that time is the spending time or the waiting time in the state 1 that is going to be exponential distribution with the parameter lambda1 plus mu1 because both are independent. The time spending in the state 1 before moving into the state 2 and similarly the time spending in the state 1 before moving into the state 0 and both the random variables are independent that is the assumption therefore it is going to be exponentially distributed random variable – the time spending in the state 1 that is exponential distributed with the parameter lambda1 plus mu1. Like that you can discuss for all other states.

So whenever you have birth-death process the system either move one step forward or one step backward then it is called a birth-death process. Therefore, here these are lambda i's are called the system is moving from one state to forward one step therefore these lambda i's are called birth rates. The system is moving from one state to the previous one state and the corresponding rates mu i's, mu1, mu2, mu3 and so on and these rates are going to be call it as a death rates. So lambda i's are nothing but the birth rates that means the rate in which the system is moving from the state i to i plus 1 that depends on i therefore that rate is lambda i. The system is moving from the state i to i minus 1 that is related to the death by 1 that that is a function of i therefore that death rate is mui. So the lambda i's are the birth rates and the mui's are the death rates. Therefore suppose example the system moving from the state 2 to 1 the death rate will be mu suffix 2. So you can fill up the Q matrix. If you see the Q matrix a it's a tri-diagonal matrix.



So here I'm giving few examples of other birth-death process. The first example consists of – the first example is a finite state model. The birth rates are lambda naught lambda1 till lambda n minus 1. The death rates are mu1, mu2 and mun. it's a final state birth death process. The second example is the infinite state birth death process. The third example all the death rates are 0, that is also possible. The fourth example all the birth rates are 0 that's also possible but you can discuss the – one can discuss the state classification also. The first one it's a finite state model all the states are communicating with all other states therefore it is a irreducible positive recurrent birth death process. The second one say infinite state all the states are communicating with all other states it's a reducible but one cannot conclude without knowing the values about the lambda naughts and lambda i's and the mui's one cannot conclude it is a positive recurrent or null recurrent if the mean recurrence time that is going to be a finite one then you can conclude it is a positive recurrent otherwise it is null recurrent. So as such we cannot discuss now the positive recurrent or null recurrent but you can conclude it is a recurrent state. The third example the system is keep moving forward therefore all the states are transient states. It's not a irreducible. It's a reducible model all the states are transient states that means as t tends to infinity the system will mean some infinite state. So one cannot define infinite state therefore the limiting distribution won't exist in this situation.

The fourth example it's a finite model but all the states are not communicating with all other states therefore it is a not a irreducible. It's a reducible model. Whenever the system starts from some state other than 0 over the time the system is keep moving backward and once it reaches the state zero it will be forever. Therefore, state 0 is observing barrier.