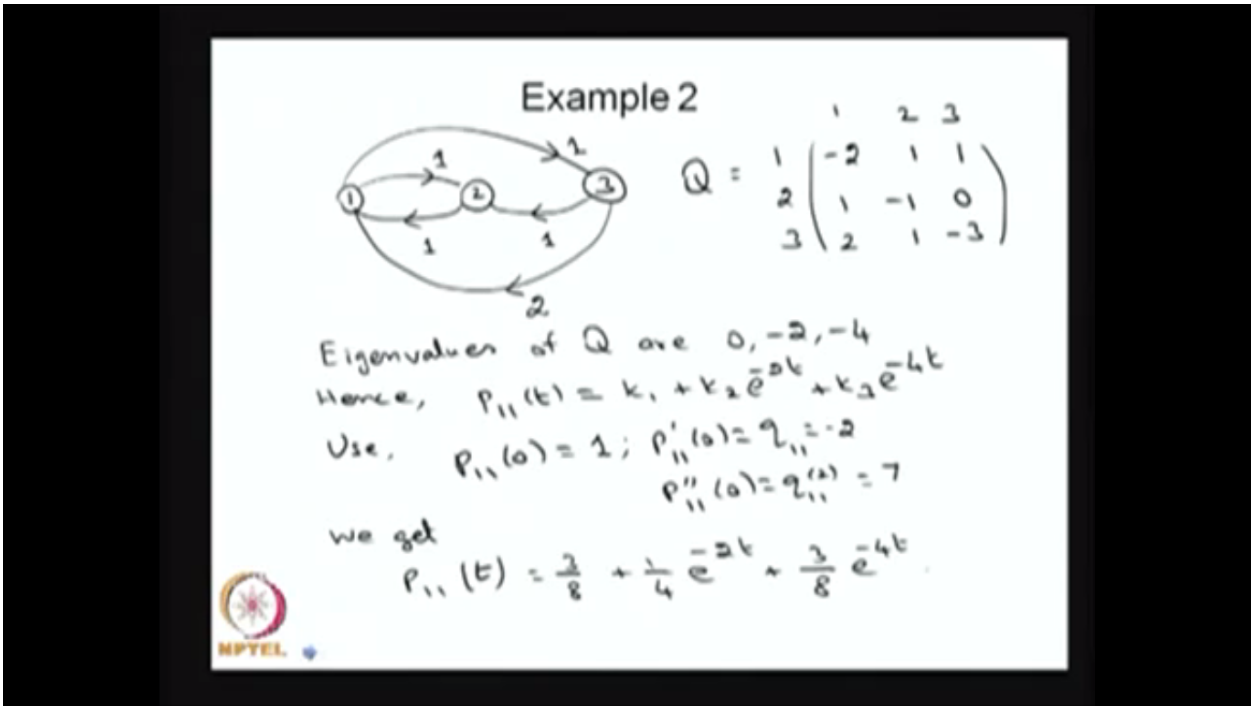


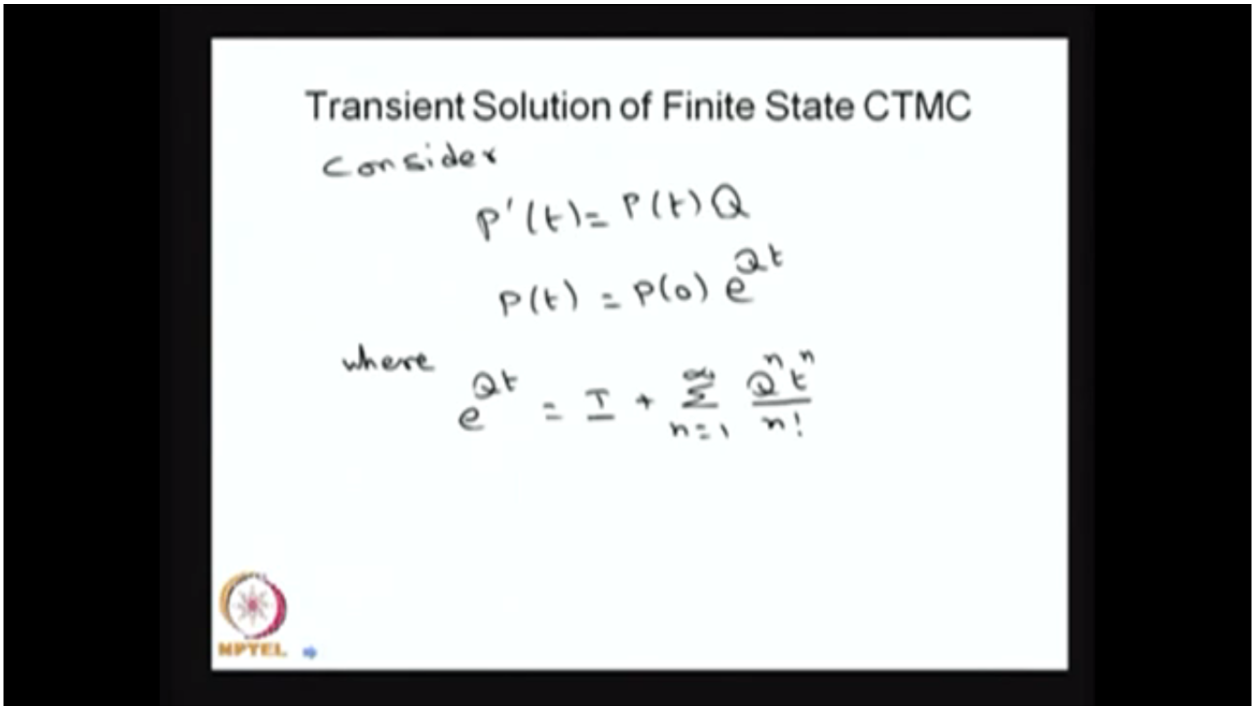
I'm going to do one more example. This has three states and this is a state transition diagram and the values are nothing but the rates in which the system is moving from one state to other states. So that's the difference between the state transition diagram of DTMC and the CTMC. So this is the rate in which the system is moving from one state to another state and the some arcs are not there that means there is no way the system is moving from the state two to three in a small interval of time whereas all the other possibilities are I have given.

So the corresponding Q matrix it's a 3 cross 3 matrix and you can make out all the row sums are going to be 0 and the diagonal elements are minus of sum of other values in the same rows and other than the diagonal elements the values are greater than or equal to 0.

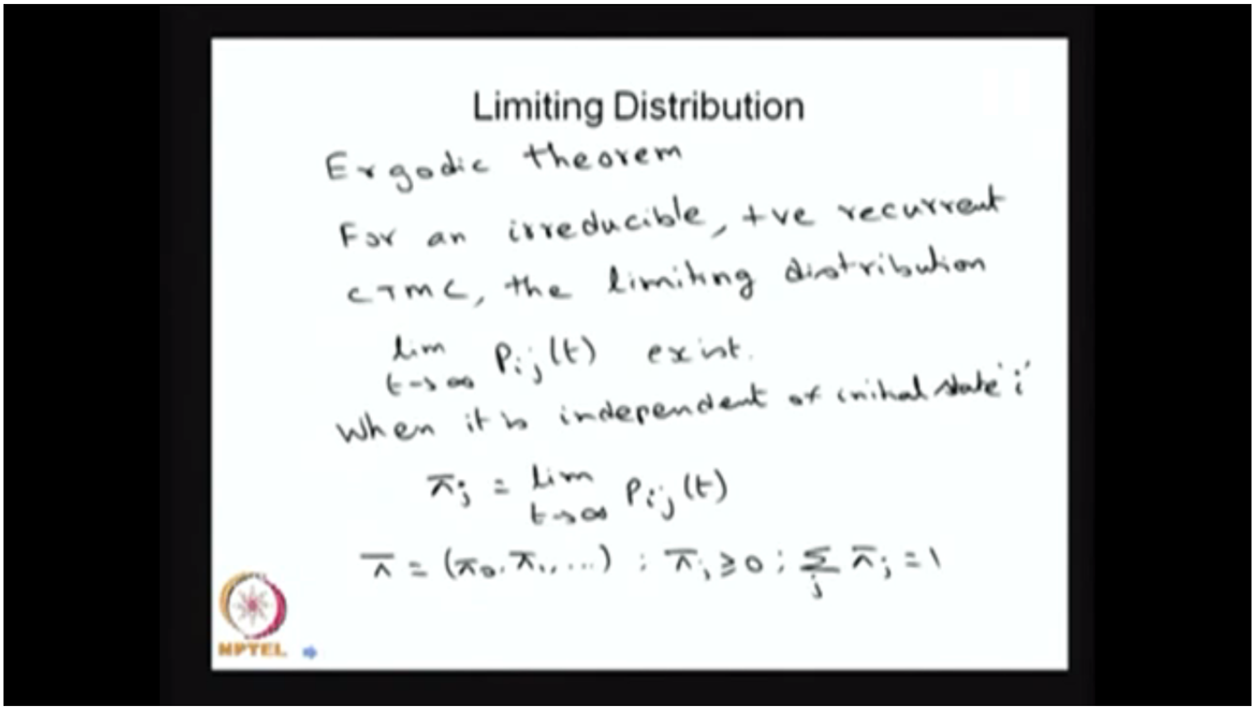
My interest is to find out the time dependent solution for this example also. I can make a forward Kolmogorov equation P dash of t is equal to P of t times Q it's a 3 cross 3 matrix therefore I have a 3 equations and I have one equation I can have a summation of probability is equal to 1 and I can start with the initial condition, the system being in the state 1 at time 0 that probability is 1. I can start with that and I can solve those three equations with the initial condition and I can get the solution that's one way.

Since it is a finite state CTMC there are many ways to get the time dependent solution. Basically, we are to solve the system of our difference or differential equations with the initial condition. Here, I am using the Eigenvalue method that means I find the Eigenvalues for the Q matrix therefore use Eigenvalue and Eigenvector concept and you get the P 11 of t with the unknowns K1, K2, K3 and to find the unknowns of K1, K2, K3 use the initial condition. Here I am using the initial condition as well as the Q matrix of values the Q11 that means the element corresponding to the 1, 1 that is nothing but the P dash of 1,1 of 0.

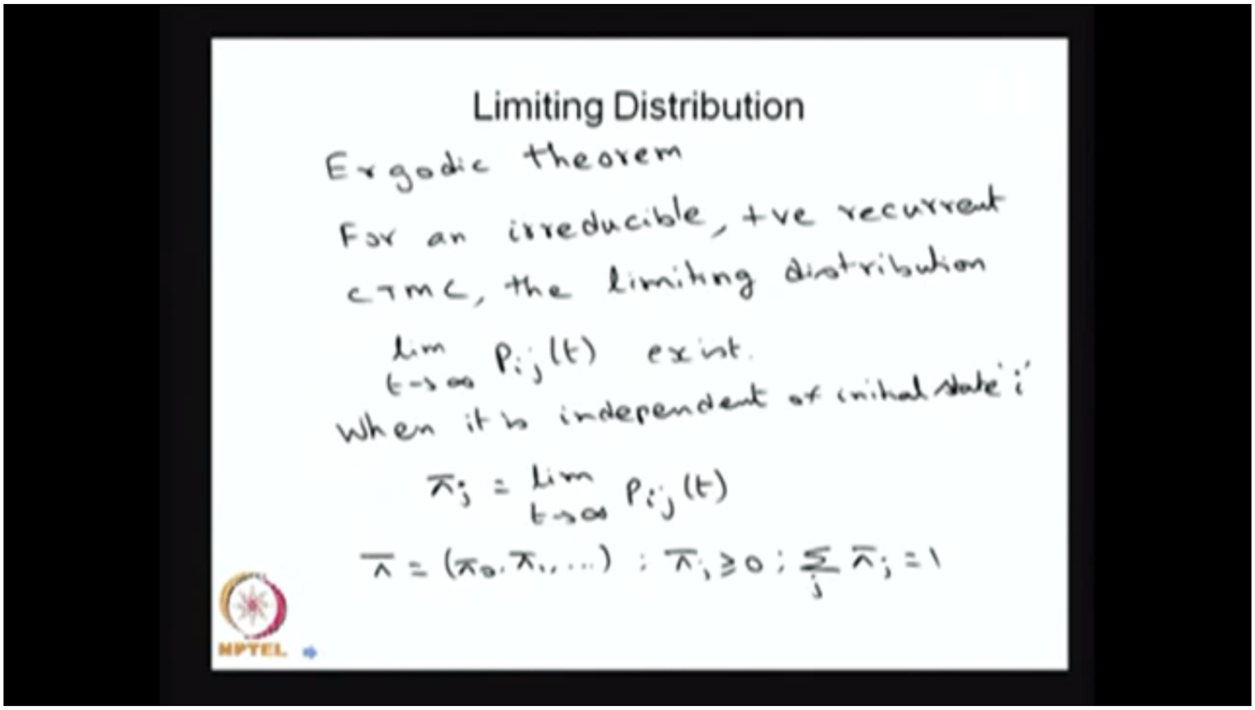
Similarly if I go for Q square matrix and Q11 of 2 the element in the 1, 1 in the Q square matrix that is nothing but a P double dash of 1,1 0 therefore now I can use these three initial conditions to get the unknowns value K1 and K2, K3, K1,K2 and K3, so once I know the K1, K2, K3 I can substitute that for the P11 of t is equal to this much. Similarly I can go for finding the P1,2 of t and P1,3 of t I don't want P1,3 in the same way because once I know the P1,1 of t and P1,2,t so 1,3,t is nothing but 1 minus of that those two probabilities because the summation of probability is equal to 1. So this is the other way of getting the time dependent solution, the transition probability of system being in the state J given that it was in the state I at time 0.



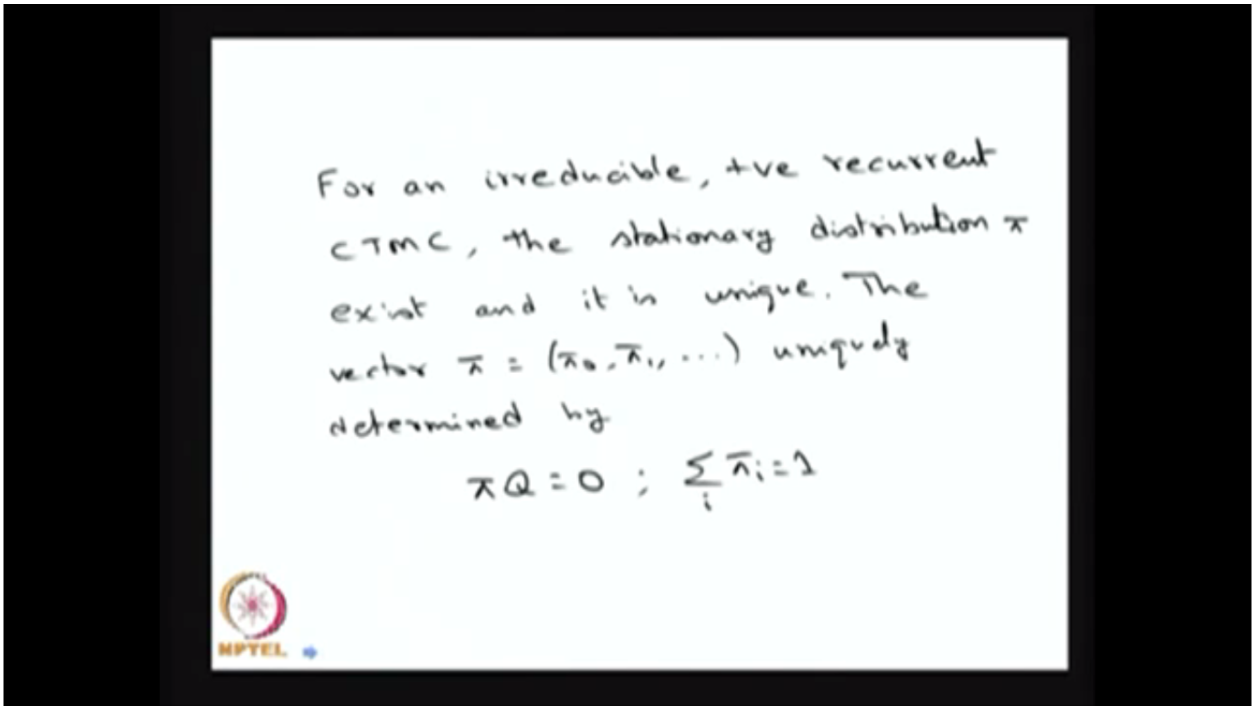
Suppose the CTMC has the finite state space then I can use the exponential matrix also to get that time dependent solution that's what I have given this way. So start with the forward equation. Therefore the solution is going to be P of t is equal to P of 0 e power Q of t. P of t is the matrix. P of 0 is the matrix. e power Qt that is also again going to be a matrix exponential matrix. Therefore I am writing at e power Qt is nothing but Q is the matrix and the t is the real value so great or equal to 0 therefore e power Qt is going to be the I matrix. I matrix is nothing but the identical matrix of order whatever the state space number plus the summation n equal to 1 to infinity of Q power n times or t power n divided by n factorial so that the whole thing is going to be the exponential matrix and using that you can get the P of t. that I am not going detailed for how to compute this e power Qt and so on but whenever you have a CTMC the finite space through this method also one can get the time dependent solution. So with this I have completed the examples for the CTMC to find out the time dependent or transient probabilities.



Now I'm moving into the limiting distribution. The way we discussed the limiting distribution for the CTMC, the same concept can be used for the CTMC also. The change is instead of the point step transition probability matrix here we have to use the infinities [Indiscernible] [00:05:47] generator matrix in a different way. So I'm first giving the Ergodic theorem. Whenever the CTMC is reducible that means all the states are communicating at all other states since all the states are communicating with all other states if one is of the particular type it's positive recurrent then all the other steps are going to be a positive recurrent. If one is going to get null recurrent then all the other states also going to be null recurrent. So here I'm making the assumption the CTMC is a reducible as well as all the states are positive recurrent. Then the limiting distribution always exists. Suppose it is independent of initial state. It may not be a independent of initial state suppose the same thing is independent of initial state then I can write that limiting probability is a Pij of t since it is independent of I, I can write it as the ij. Then I can form a vector and since it is a limiting distribution it's a probability distribution. Therefore the probabilities are – these probabilities are always greater or equal to 0 and the summation of probability is going to be 1. It won't be defective. It won't be less than 1. That's Ergodic theorem says whenever you have a irreducible CTMC with all the states are positive recurrent then as t tends to infinity the system has the distribution, limiting distribution. If it is independent of initial state then you can label with the Pij as a probability and this probability distribution satisfies it's a probability mass function that's why it satisfy the probability mass function conditions. That means whenever you have a dynamical system in which it is irreducible model and all the states are positive recurrent that means the mean recurrence time is going to be a finite value then that system is called it as an ergodic system or the ergodic concept can be used therefore as a t tends to infinity we can get the limiting distribution. If it is independent of initial state means whatever be the [Indiscernible] [00:08:15] you are going to do it for the discrete event simulation for the dynamical system that is ergodic, for ergodic system then the initial condition C does not matter to get the limiting distribution. Later we are going to give some few examples how to find out the limiting distribution.



I am explaining the stationary distribution also. The stationary distribution the way I have explained the DTMC sorry the way I have discussed the DTMC the CTMC also same. So I have a vector if the vector satisfies these three conditions probabilities therefore greater or equal to 0, summation is equal to 1 and there you should be able to solve this equation and get the price. It's a homogeneous equation. So you need a second condition to have the nonzero probabilities. So if you solve Piq is equal to 0 along with the summation of Pij is equal to 1 and if these Pij's exist then the CTMC has the stationary distribution. The similar way I have discussed the stationary distribution for the DTMC model also instead of Piq is equal to 0 we had a Pip is equal to Pi. So if any vector satisfies that Pip is equal to Pi and summation of Pi is equal to 1 and all the Pi's are greater or equal to 0 then that is going to be a stationary distribution for DTMC. In the same way if Pip is equal to 0 and the Pi summation of a Pij is equal to 1. Pij's are greater or equal to 0 if this is satisfied by any vector then that is going to be the stationary distribution for a time homogeneous CTMC. Every time we are discussing the default CTMC that is a time homogeneous CTMC.



The main result for the stationary distribution whenever you have a irreducible positive recurrent CTMC the stationary distribution exists and that is going to be unique. Whenever the CTMC is a positive recurrent as well as a irreducible. There is no need of a periodicity in the CTMC whereas the same stationary distribution the stationary distribution for the DTMC we have included one more condition that is a periodic but for the CTMC there is no periodicity for the state. Therefore, as long as the system has a – system is irreducible and a positive recurrent one then the stationary distribution exists and it is unique and by solving these equations you can get the unique stationary distribution.