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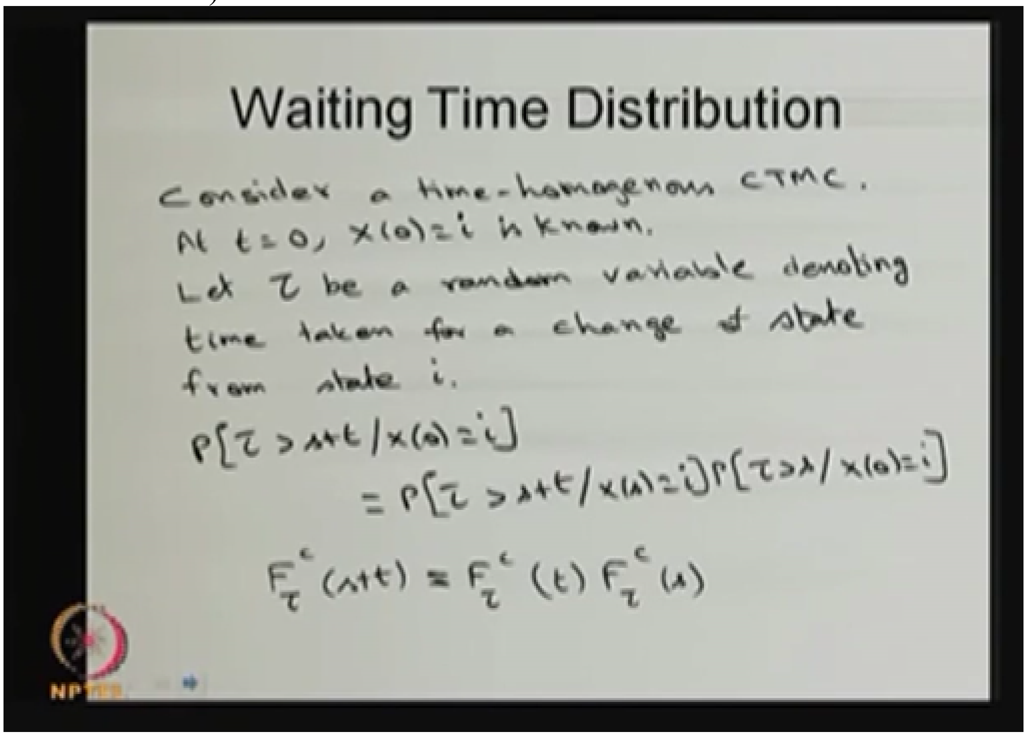

So before going to the PIJ you see the sample path of,

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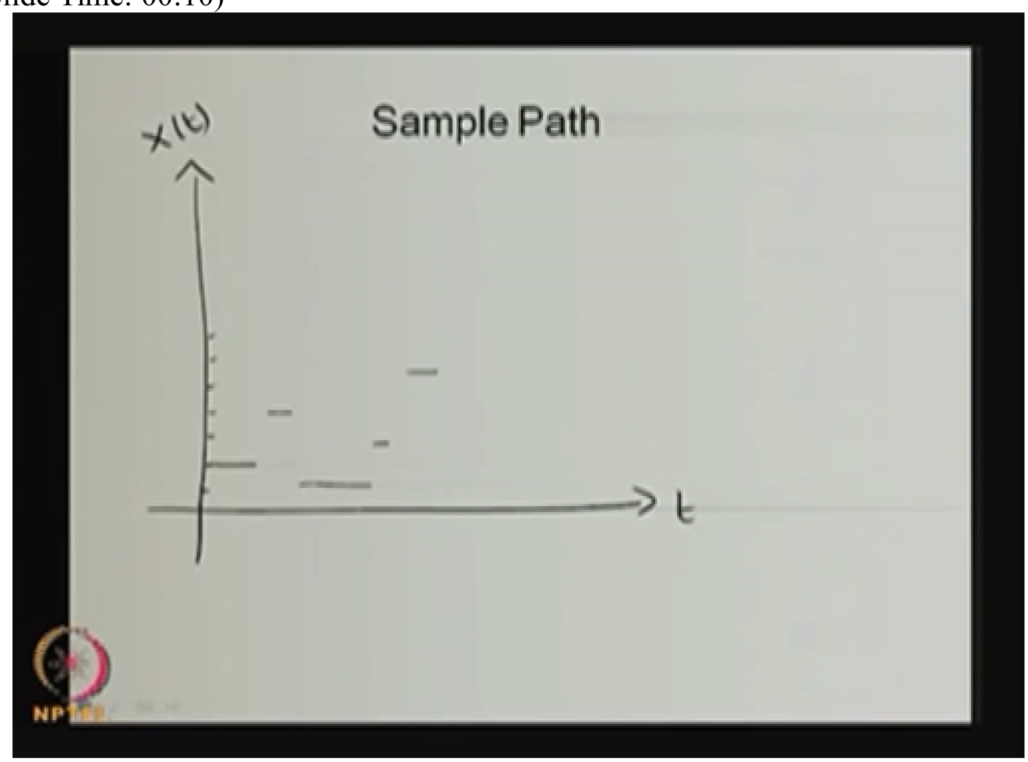

the sample path of time homogeneous continuous time Markov chain. As I said the system is staying for some positive amount of time in any state before moving you to any other states, our interest is what is the distribution or what is the waiting time distribution of system being in any

state before moving into any other states, that is our interest to find out, so how you are going to find out that I'm going to explain, that is called the waiting time distribution,
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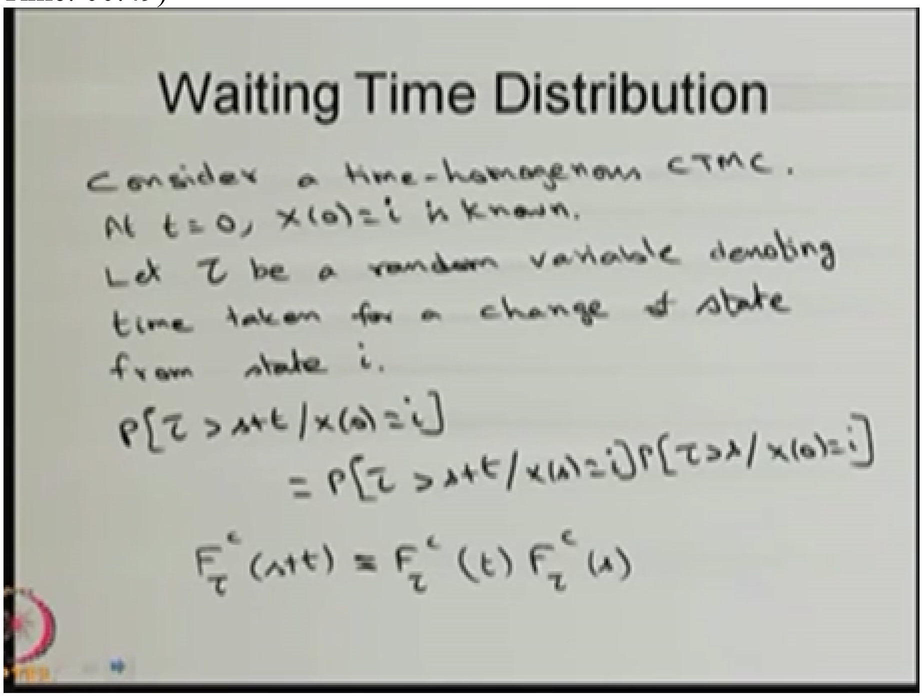

that means what is the distribution of a time spending in any state for a time homogeneous continuous time Markov chain before moving into any other states.

I assume that at time 0 the system was in the state I, that means X(0) = I that is known or the probability of X(0) = I that probability is 1. Let me make out the random variable tau that is the random variable denoting the time taken for a change of state from the state I, change of state means it doesn't matter which state it goes, my interest is to find out what is the waiting time distribution for the state I that I'm spending the state I.

For that let me make a simple graph,
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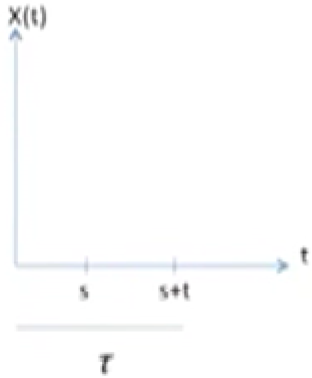

$$P[\tau > s + t \mid X(0) = i] = P[\tau > s + t \mid X(0) = i, \tau > i]$$

$$\times P[\tau > s \mid X(0) = i]$$

$$= P[\tau > s + t \mid X(s) = i]$$

$$\times P[\tau > s \mid X(0) = i]$$

$$F_\tau^c(s + t) = F_\tau^c(t) F_\tau^c(s)$$

so this is T and this is X(t), suppose you assume that the system was in the state I at the time point is 0, after sometime it moved into some other state, okay, at the time point S it was in the state I, at the time point T also it moved into some other state, so the tau here is nothing but the time spend in the state I from here to here, so that's a random variable.

So what I am going to do, I'm going to find out what is the complement CDF for the random variable tau that is what is the probability of that? The tau greater than S+T given that X(0) = I, that is same as the probability of the tau is greater than S+T given that, I can introduce the one more condition tau is greater than S, then I can multiply by using the total term of probability, tau greater than S given that X(0) = I, that is same as, the first one I can rewrite as a probability of a tau greater than S+T given that X(s) = I, because X(0) = I as well as tau is greater than S, where tau is the time spent in the state I therefore I can make out X(s) = I by combining these two concept multiplied by the probability of tau greater than S given that X(0) = I.

That is same expression here, now the probability of tau greater than S+T given that X(s) = I that I can rewrite because this Markov chain is a time homogeneous Markov chain so that S to S+T that is same as the complement CDF of the random variable tau for the time T, because it is S to S+T, since it's Markov chain is a time homogenous only the length is matters that is the interval of length T, therefore this is nothing but the complement CDF for the random variable tau at the time point T multiplied by, this is nothing but 0 to S, so this is the complement CDF of the random variable tau the time point S.

Whereas the left hand side is the complement CDF for the random variable tau for the time point S+T, so what do we got the result is that the complement CDF of the unknown random variable tau at the time point S+T that is same as the product of complement CDF at the time point S and

T, so this is valid for all S and T, greater than 0, so we have to find out what is the random variable or what is the distribution going to satisfies these complement CDF at the time point S+T same as the product of complement CDF's at the time point S and T. If any distribution satisfies this complement CDF property then we can find out the distribution for the random variable tau.

So in this derivation we have used the time homogeneous property as well as the total probability rule, as well as we have used the Markov property therefore it land up the complement CDF satisfying these equation.

Now we have to find out what is the distribution going to satisfies these property.
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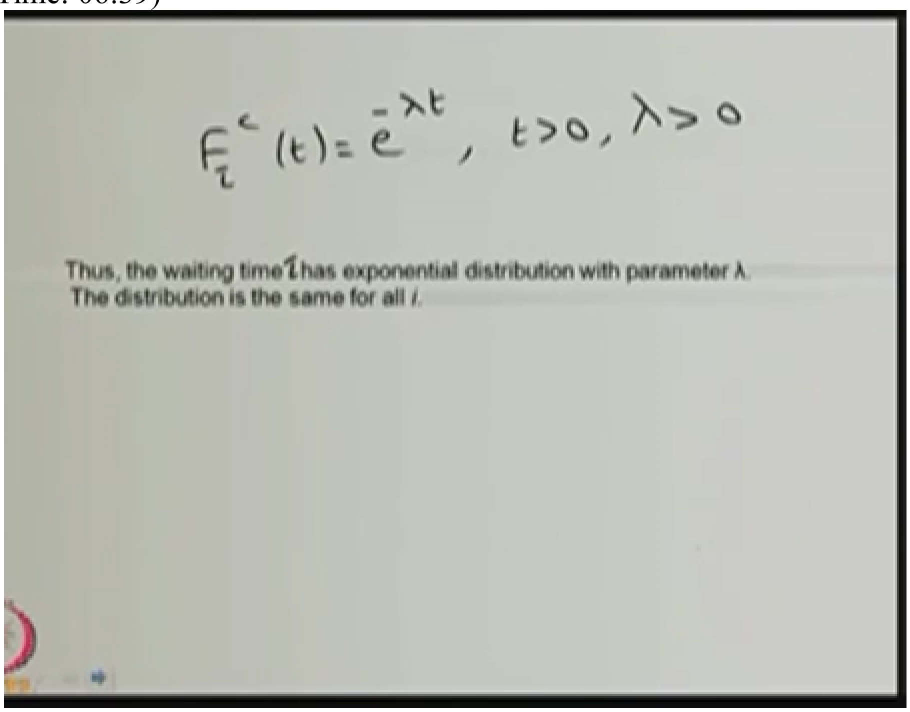

So if you substitute any function with the E power, any parameter lambda if the exponential of E power –lambda T the previous equation is going to be satisfied as long as the function is of the form E power –lambda T. For lambda is greater than 0 and T is greater than 0, since the complement CDF is E power –lambda T therefore the CDF of the unknown random variable tau that is 1-A power – lambda T for T greater than 0 for some lambda.

And you know that if the CDF of the random variable is 1 – E power –lambda T for T greater than 0, and lambda greater than 0, then that random variable is a exponential distributor random variable, so we can conclude the amount of time or the time taken by the system staying the state I, and that time is a exponentially distributor, that's a continuous random variable whose distribution is exponential distribution with the parameter lambda. Even we can specify lambda suffix I that means it is going to be a function of, it depends on the I, that means if the random variable is going to spend in some state and that is always exponential distribution with some parameter lambda and that parameter lambda may depend on the state I, that means if I go back to the sample path

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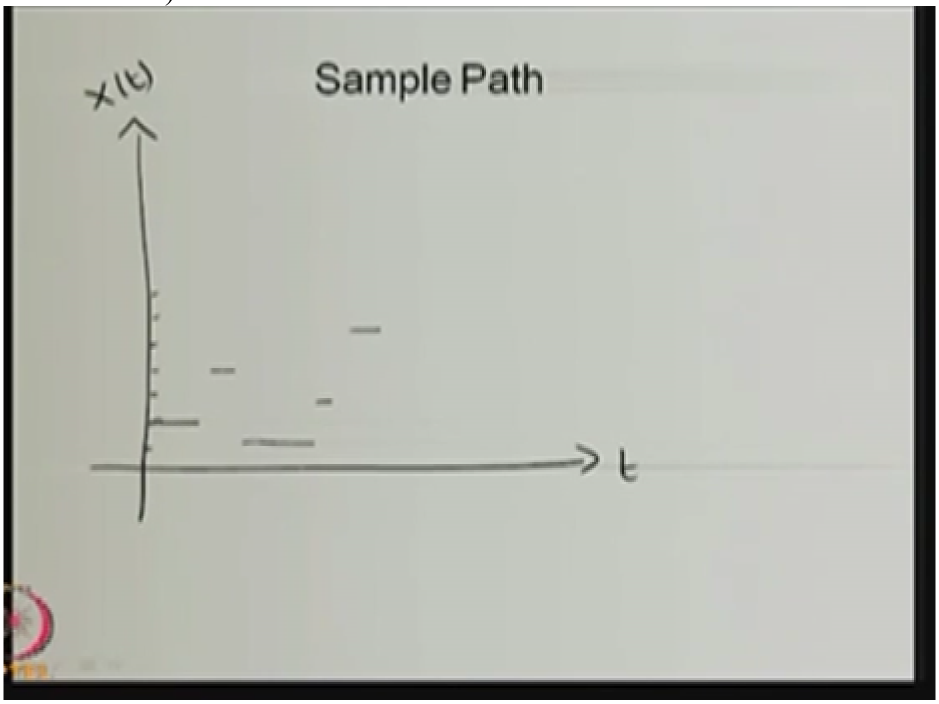

I can say that the time the system spending in this particular state that is exponentially distributed with some parameter, then it moved into some other state, the time spending in this state that is also exponentially distributor with some other, it could be some other parameter, it depends on that particular state, then it moved into the some other state and the time spending in this state that is also exponentially distributed.

And later we can conclude all these the time spending in each state because of it is a Markov property satisfied the time spend in this state, the time spend in this state all are exponentially distributed which is independent of the other, so all are going to be mutually independent random variables, then only the Markov properties going to be satisfied, that means whenever the system is moving from one state to another state you'll have a exponentially distributed time spending in each state and they are from a mutually independent. And since the exponential distribution has the memory less property, the system spending in this state, if you just observe at any time T and what is the probability that the system will be for some more time in the same state given that it was spending already this much time in the state, than that is also exponential distribution because of memory less property of exponential distribution, and which is independent of how much time spend in the same state already, therefore the Markov properties going to be satisfied throughout the time, whether the system spending in this state or the other state and so on, so the Markov properties going to be satisfied for all the time points and the time spending in each state is exponentially distributed and all the random variables spending in each state all are going to be mutually independent random variables.

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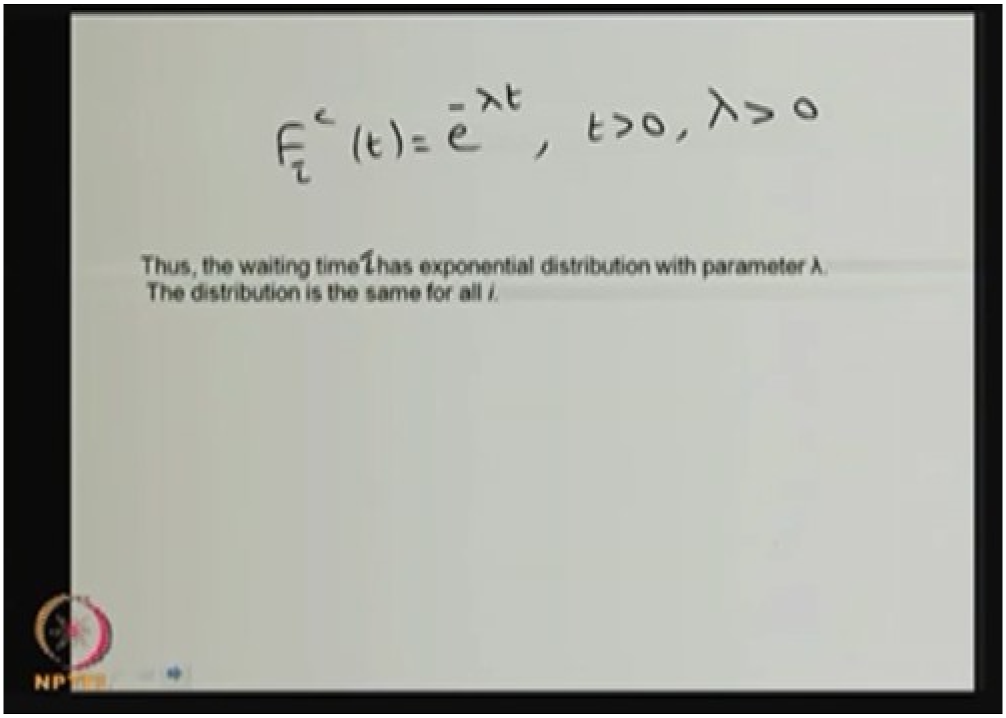

Now we found out what is the time spending in each state and that is exponentially distribution with some parameter lambda I and the distribution is same for all I, whereas the value of the parameter lambda maybe depends on the I.