INDIAN INSTITUTE OF TECHNOLOGY DELHI

NPTEL

National Programme on
Technology Enhanced Learning

Video Course on
Stochastic Processes-1

by

Dr. S Dharmaraja
Department of Mathematics, IIT Delhi

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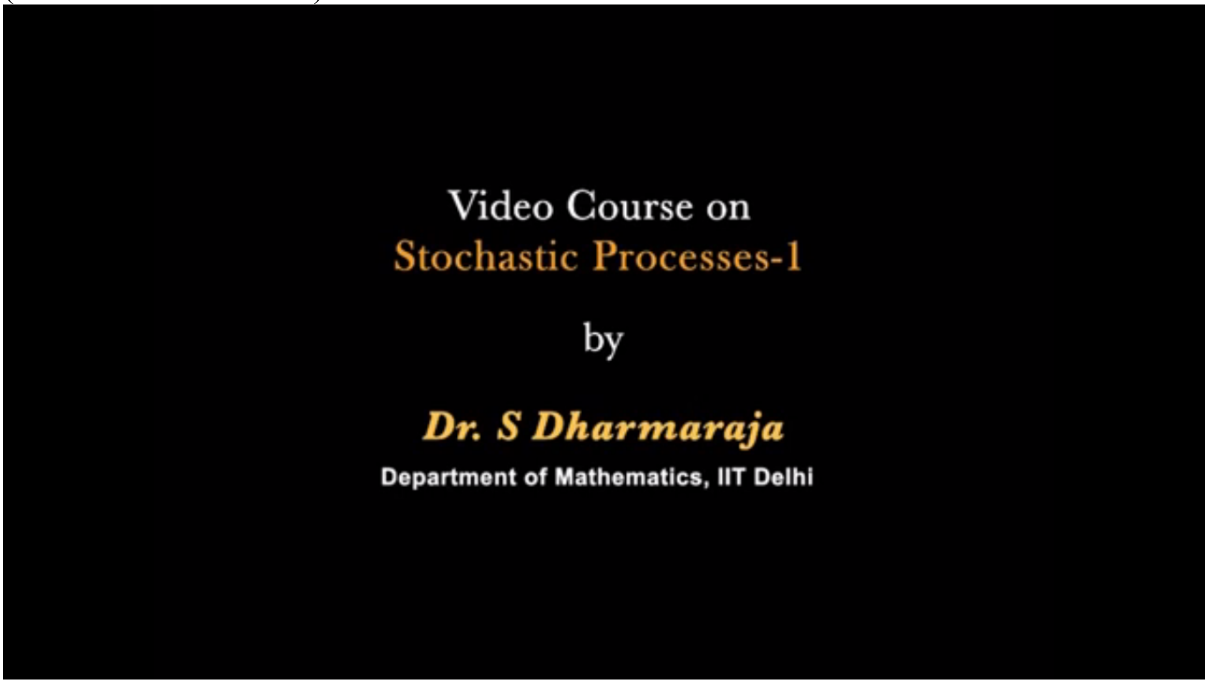

Module 1: Probability Theory Refresher

Lecture # 1
Introduction to Stochastic Processes(contd.)

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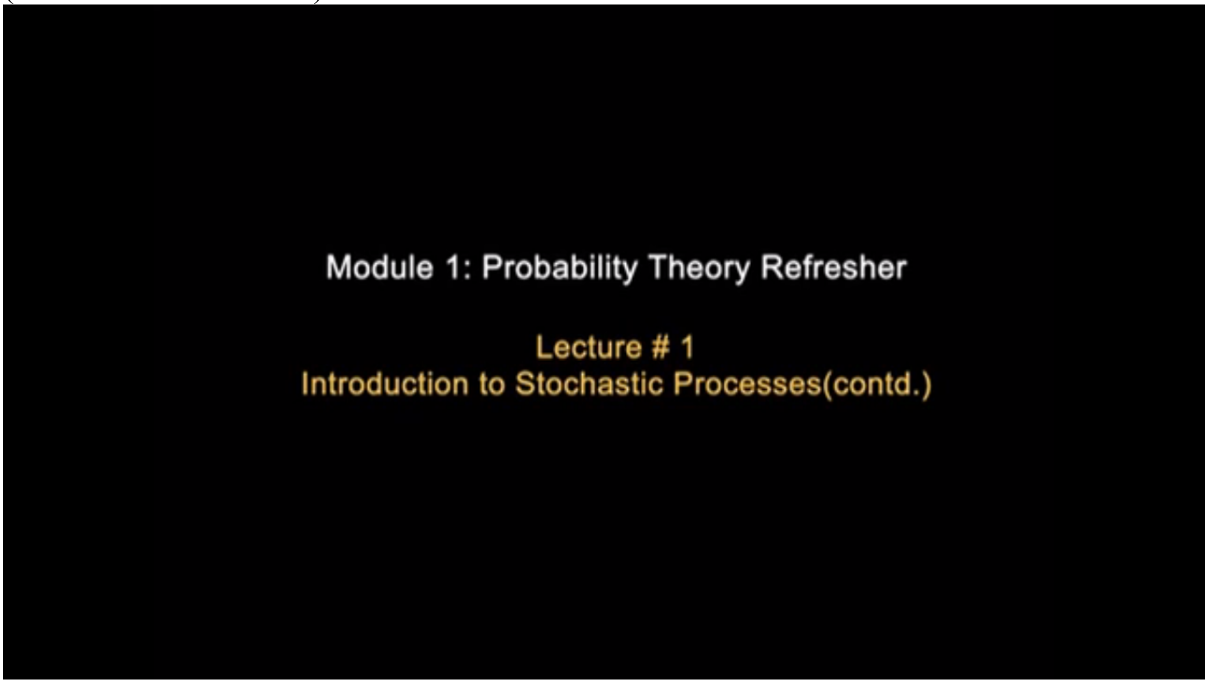

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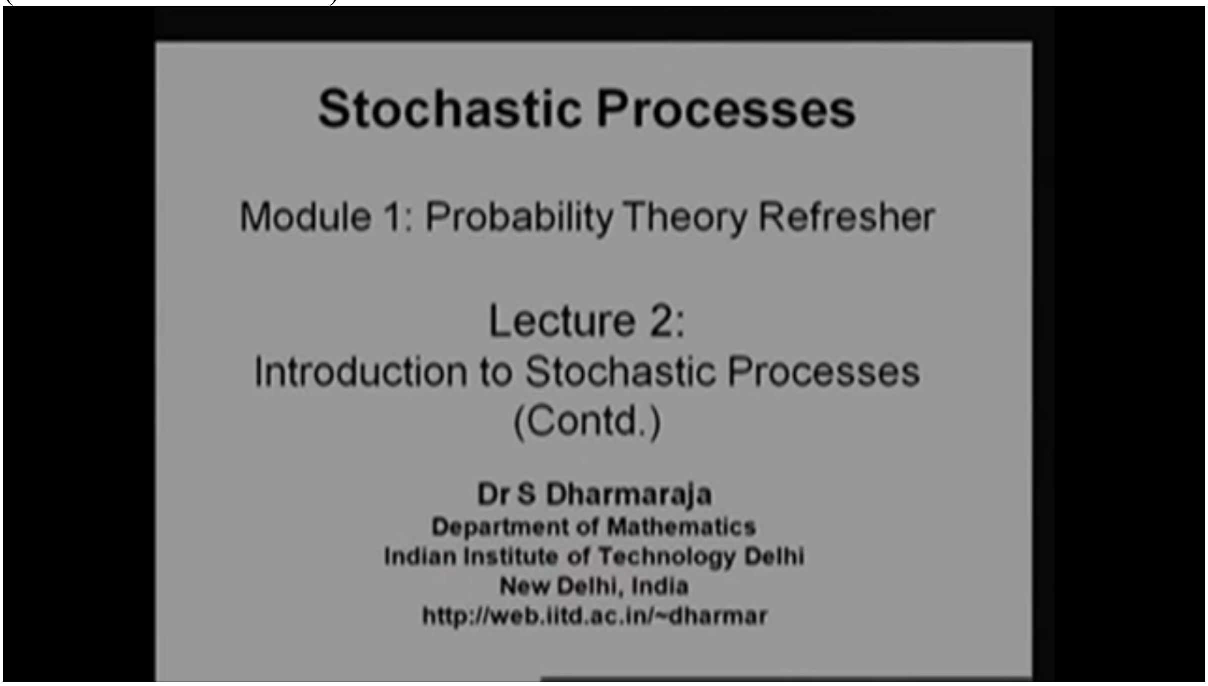

This is the continuation of Model 1: Probability Theory Refresher and this is the Lecture 2.

So the Lecture 1 we have covered what is the motivation behind the stochastic process. Then we have given a few examples and followed by that we have explained what are all the minimum things is necessary to study the stochastic process. We started with the random experiment, then events, then the probability space and to create the probability space we need a sigma-algebra. Then after creating the probability space, then we have defined the conditional probability. Then we discussed independent of events.

Then we have listed out a few standard discrete random variable as well as a standard continuous random variable. Even though we have discussed only three or four discrete and continuous random variable, there are more, but whenever the problem comes, we will discuss those standard distributions when we come across those distribution.

And the Lecture 2 we are going to continue whatever we have discussed in the Lecture 1, basically, the Probability Theory Refresher.

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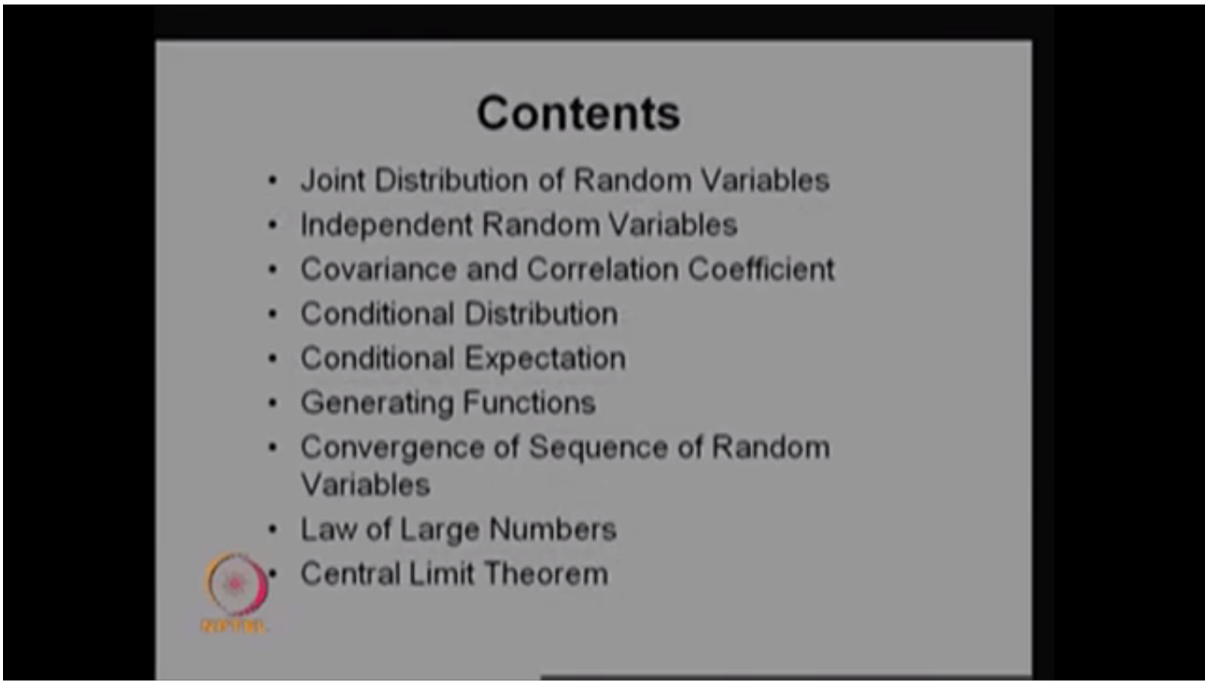

In this we are going to give a brief about what is joint distribution and if the random variables are independent, what is the behaviour of joint distribution and so on. Then we are going to discuss the covariance and correlation coefficients. After that we are going to discuss the conditional distribution, then followed conditional expectation also. And we are going to list out a few generating functions, a probability generating function or moment generating function and also the characteristic function.

Then at the end of the probability theory part, we are going to discuss how the sequence of random variable converges to some random variable and for that we are going to discuss the Law of Large Numbers also and at the end of the Lecture 2, we are going to complete with the Central Limit Theorem.

So let -- let me start with the Joint Distribution of Random Variable.

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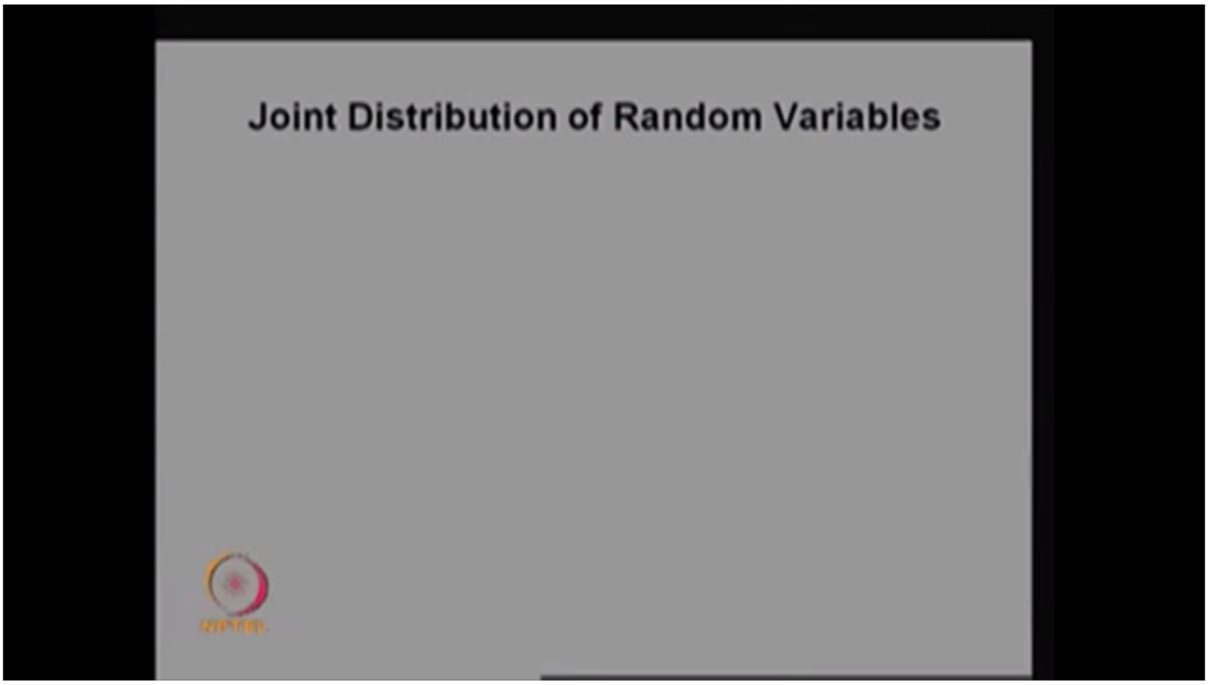

So suppose you have a random variables x_1, x_2, x_n, we say this random variable is a n dimensional random vector means each random variable x_1, x_2, x_n are the random variables. Either it could be a discrete random variable or continuous random variable and you are going to make it as a together in the vector form and each one is going to be a random variable, then it is called a n dimensional random vector.

Once you have together as a vector form, then we can go for giving the joint distribution. So the joint distribution we can discuss in two ways. Either it is a joint probability mass function or we can define as a joint probability density function.

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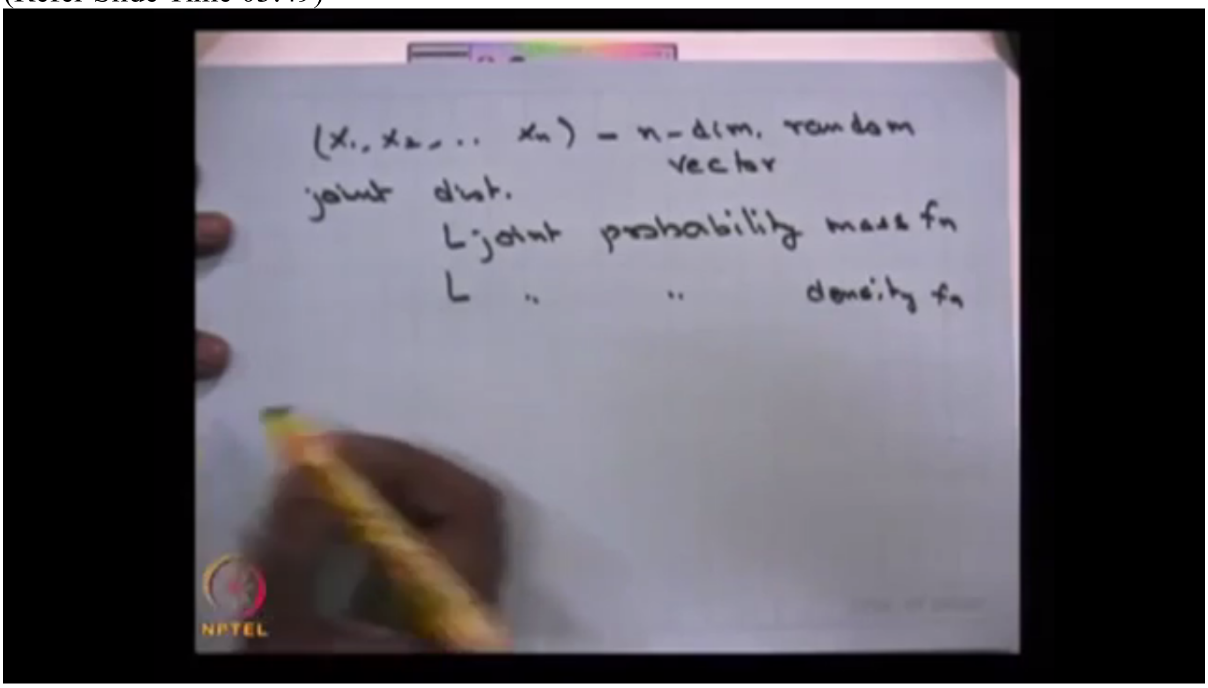

Suppose you take an example of two dimensional random variable (x, y), if both the random variables are discrete, random variable x is discrete as well as the random variable y is discrete, then you can define what is a joint probability mass function of this two dimensional discrete random variable as probability of (x, y). Here the small x, small y are the variables and this (x, y) denotes the two dimensional random variable.

This is nothing but what is the probability that X takes the value random variable X takes a value small x and the random variable Y takes the value small y, and based on the possible values of x and y, you have the probability of this.

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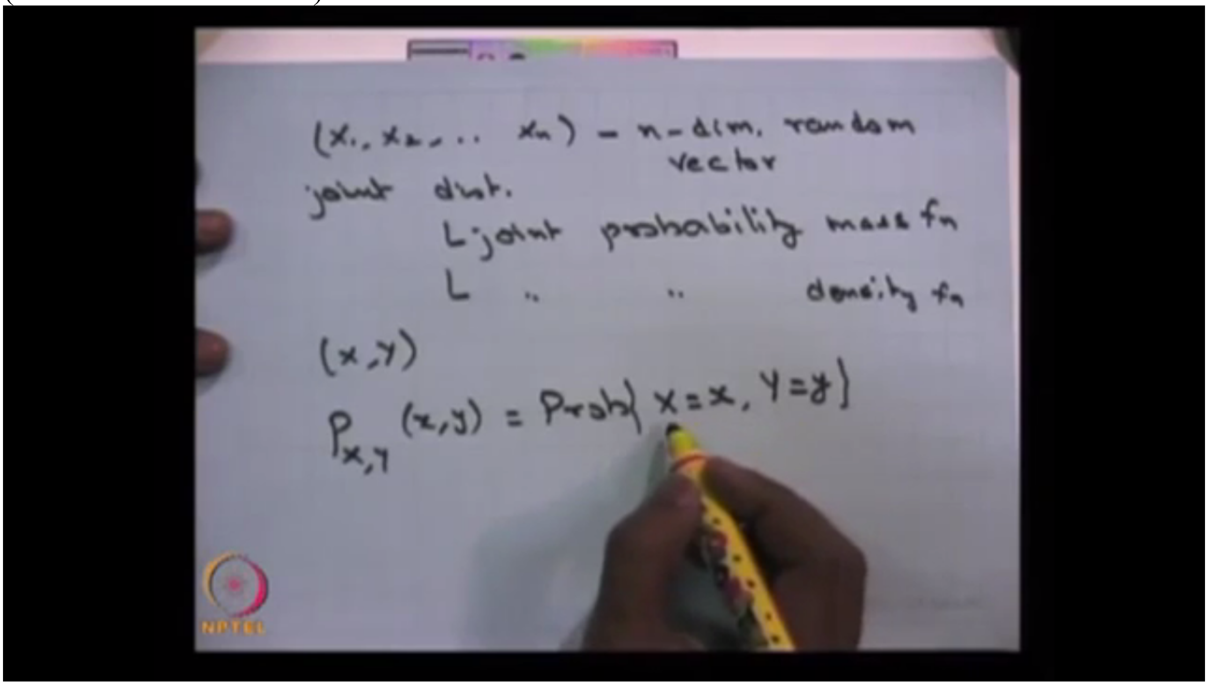

That means you land up creating what is the event which corresponding to X equal to x and Y equal to small y. That means if you not getting any possible outcomes, that gives some possible values of (x, y), then it may be the empty set. Otherwise, you land up with a different possible -- possible -- you can collect the -- that means X is equal to x and Y is equal to y for all possible values of (x, y) may relate with what is the event in which X of w gives the value x as well as Y of w gives the value y where w is belonging to omega.

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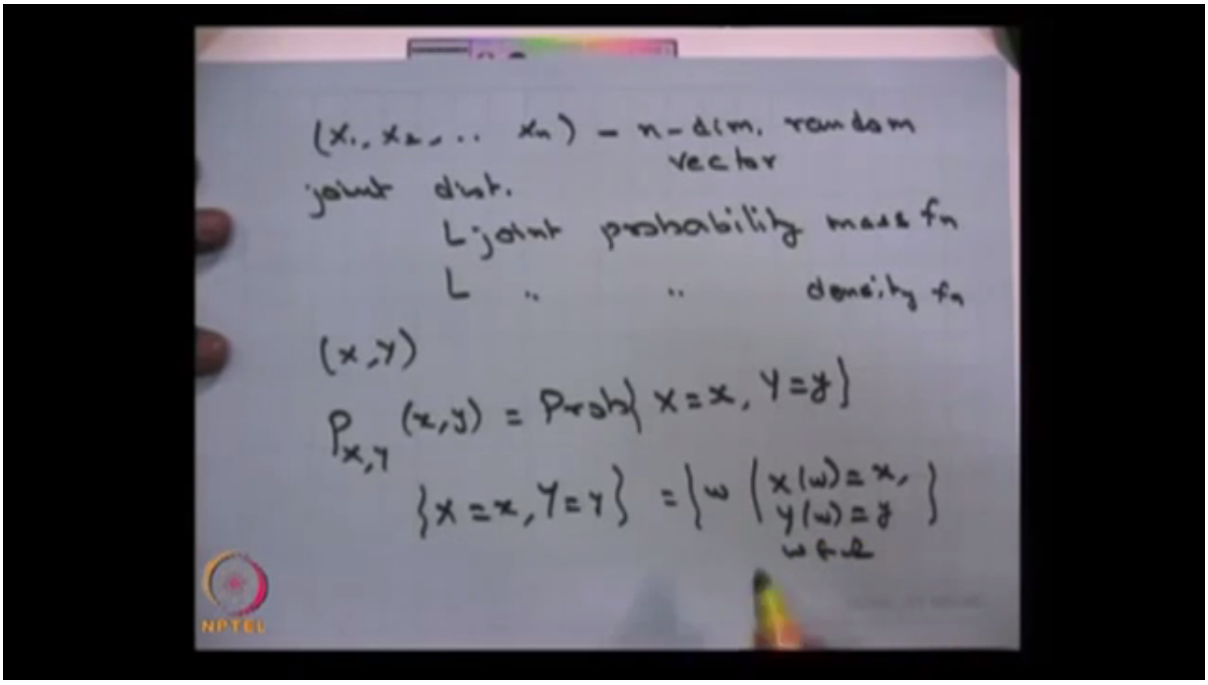

That means you collected a possible outcomes w such that it satisfies both the conditions where w is belonging to omega. That means this is going to be the event. Therefore, this is the probability of event and you know by using the axiomatic definition, the probability of event is always greater than or equal to 0 and the probability of omega is equal to 1, and if you take mutually exclusive events, then the probability of union of events is going to be the summation of probability.

Therefore, this is the way you can define when the random variables x and y both are discrete type. Then you can give the joint probability mass functions. Therefore, here this is the joint probability mass function and this satisfies all the values are always greater than or equal to 0 for any x and y and if you make the summation over x as well as summation over y, then that is going to be 1. Therefore, it satisfies the property of always greater than or equal to 0 for any x and y and summation, double summation over x and y is equal to 1. Therefore, this is going to be the joint probability mass function corresponding to the discrete type random variable.

Suppose the random variable x and y are each random variable is a continuous type random variable. Therefore, you land up with the two dimensional continuous type random variable or random vector. In that case, you can define what is the joint probability density function is of the form f of x, y, small x and small y, that is going to be what is -- so you can have a joint probability density function and you can relate with this joint probability density function with the CDF by what is the CDF of the random variable that is nothing but what is the integration from minus infinity to x and what is integration from minus infinity to y what -- of the integrant is going to be (r, s) where r is with respect to x that is dr and this is ds.

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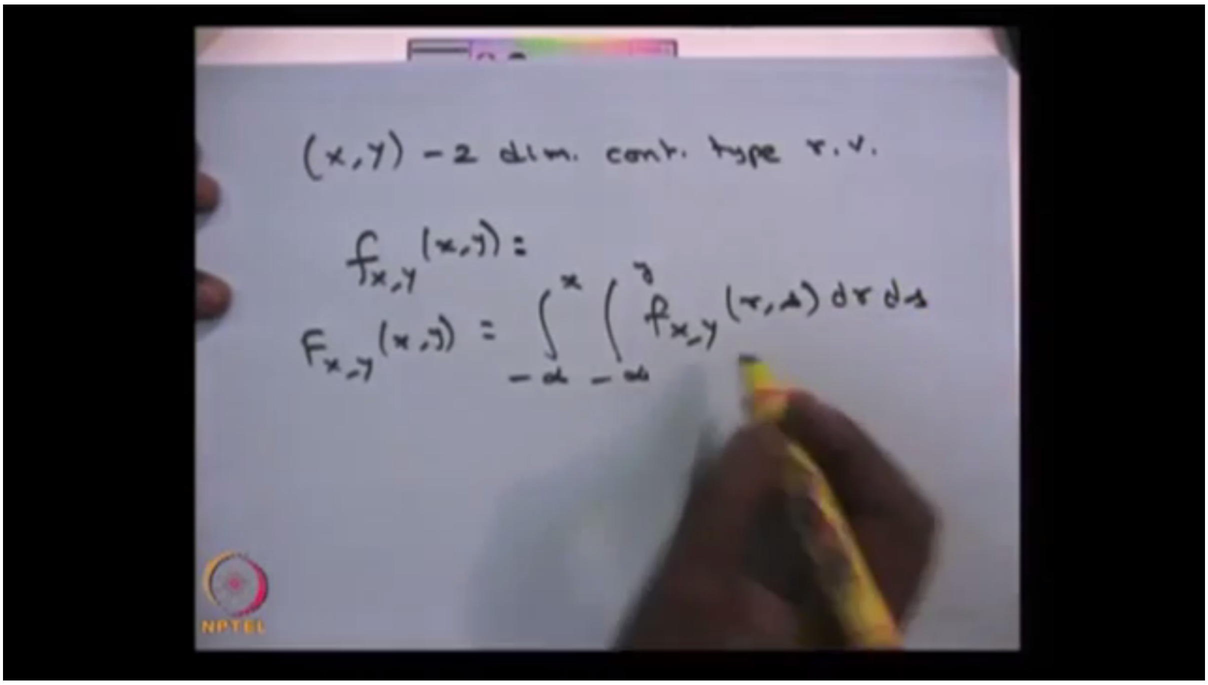

That means since you have a continuous type random variable, therefore you may land up with the continuous function with the two variables (x, y) by using the fundamental theorem of algebra, you can always land up a unique integrant and that is going to be the density function for this two dimensional continuous random variable and you can able to write the left-hand side continuous function in x and y can be written in the form of integration from minus infinity to x and minus infinity to y of this integrand and ds where this function is going to be call it as a joint probability density function for the random variable (x, y).

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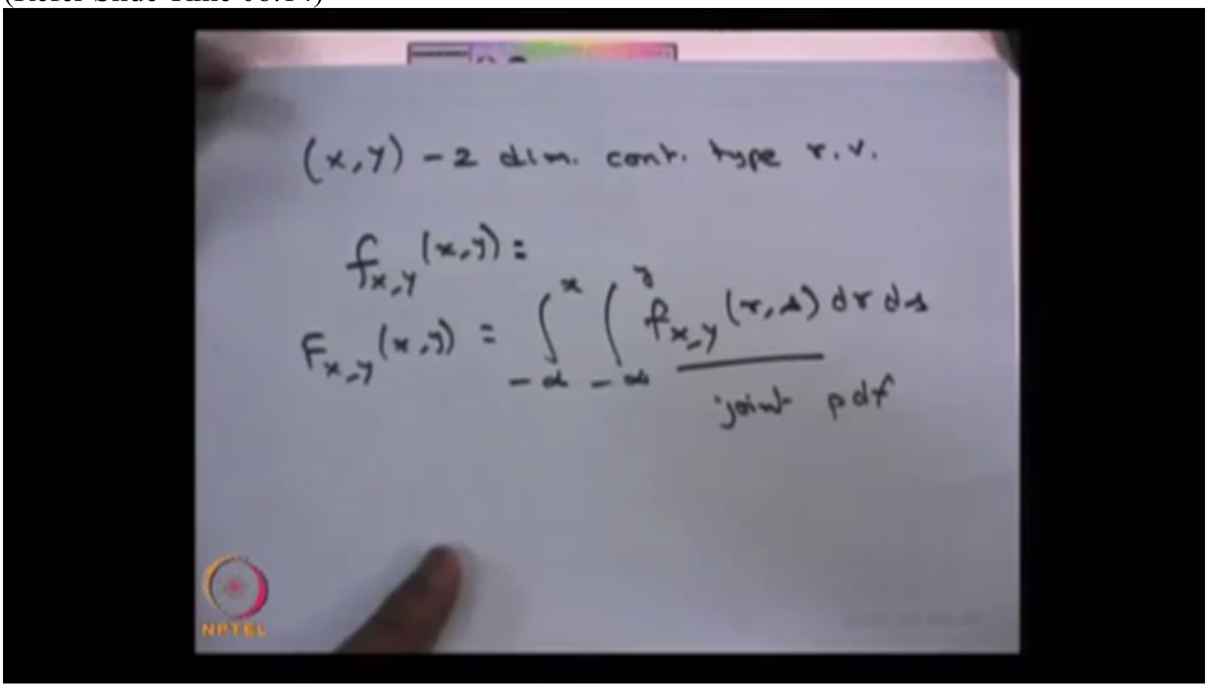

So there is a relation between a joint probability density function with the -- with the CDF. You can -- I can always able -- I can give a few examples and the example one, suppose you have a both the random variables as (x, y) is a discrete i, then I can give a one simplest example of a (i, j) that is going to be 1/2 power i plus j for i is belonging to 1, 2 and so on and j can take the value 1, 2 and so on and this is going to be the joint probability mass function for two dimensional continuous type random variable. If you made a summation over x and summation over i and summation over j, then that is going to be 1 and you can get by summing it over only j, you will get the probability mass function for the random variable x.

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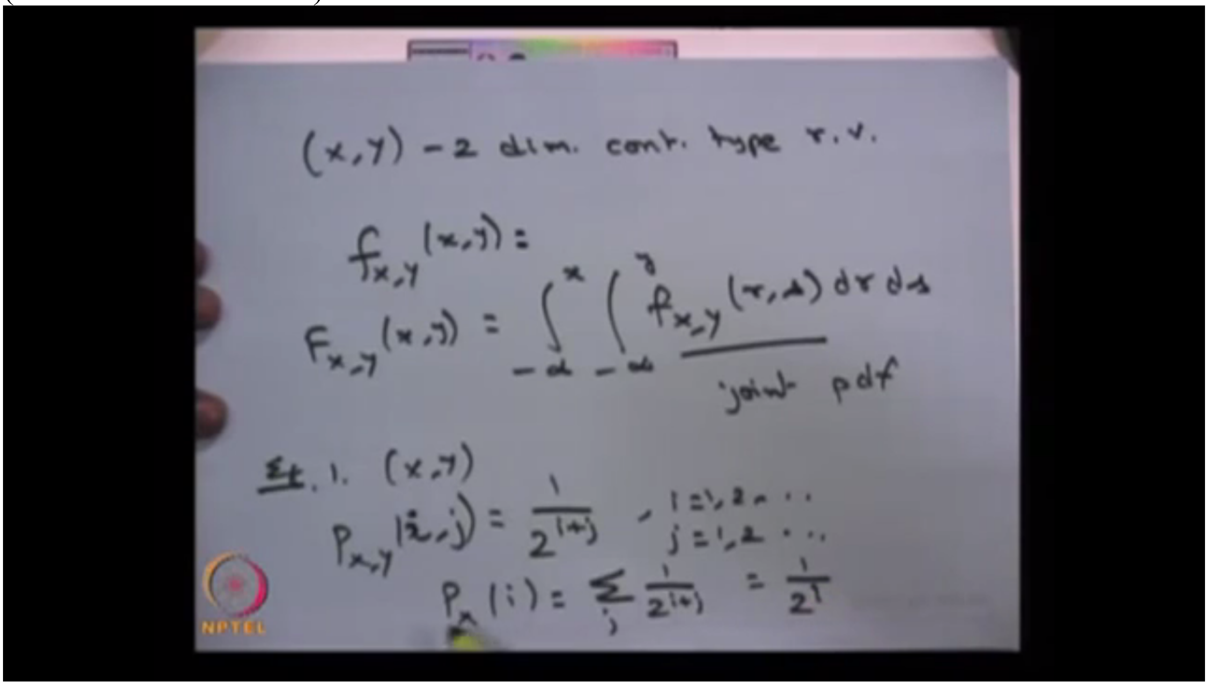

Similarly, if you make the summation over i with the joint probability mass function, then you may get the probability mass function for the random variable y. That means from the joint probability density function by the joint probability mass function by summing it over the other random variable, you can get the distribution of the single random variable.

Suppose if you have a n-dimensional random variable, suppose for n is equal to 5 you have a 5 dimensional random variable. Suppose all the random variables are of the discrete type, then you may have a what is -- you may have the joint probability mass function of this 5-dimensional discrete random variable and by summing over each random variable, and if you want to find out what is a probability mass function for the one random variable, you can always get -- make the summation over other random variables with respect to the other random variables. You can get what is the probability mass function for -- by summing it over the other random variables, you will get the marginal distribution of the random variable x_1. Similarly, you can get the marginal distribution of x_2 or any other random variable.

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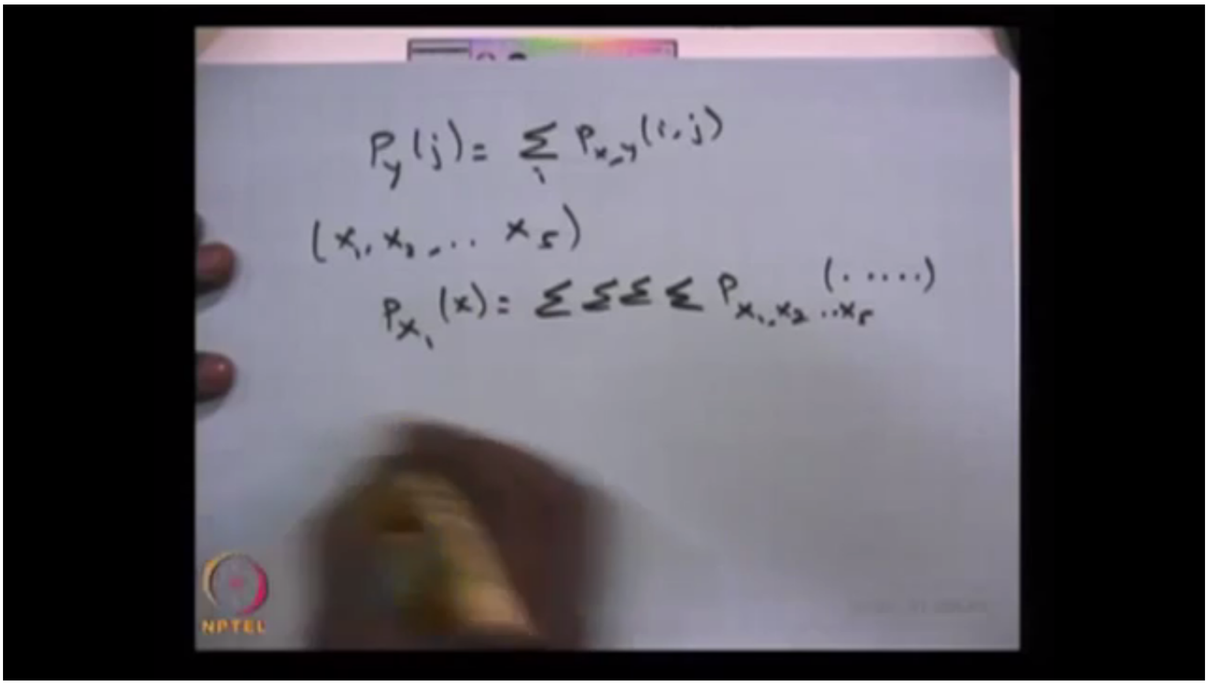

Similarly, suppose you have a joint probability density function, from that you can get -- you can get the marginal distribution of one random variable by integrating with the other random variable, by integrating with the other random variable.

So this is a joint probability density function. From the joint probability density function, by integrating with the other random variable, you will get the probability -- probability density function for this random variable y. Similarly, you can get the probability density function for y is -- so you can by integrating with respect to r, so that means you are just finding out the marginal distribution of the random variable y and here you are finding the marginal distribution of x.

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