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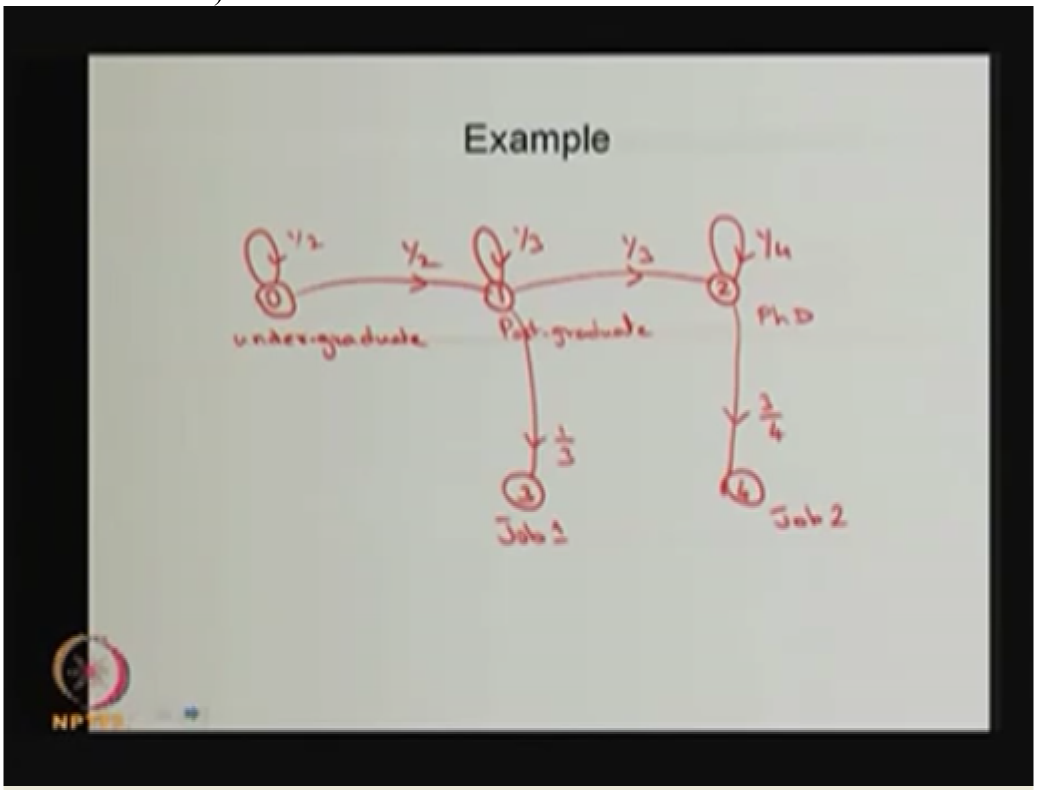


I'm going to give one simple example for this type of reducible Markov chain with the transient states and one or more absorption state. I'm making the assumption that is a positive recurrent, so instead of positive recurrent I have a finite Markov chain, so the finite Markov chain at least one state is a positive recurrent therefore this is, both are going to be absorbing state therefore we don't want those conditions also.

So here in this model the states 0, 1, 2 are the transient states, 3 and 4 are absorbing states, this is a easy example in which you can visualize someone is doing the undergraduate with the probability of, he's not able to complete the undergraduate in the next step with the probability of, he's moving into to the post graduate in the next step, so I'm making a DTMC with the assumption them a medalist properties so satisfied and so on, from the post graduate either someone gets the job 1 with the probability one third are not able to complete the post graduate, that probability is 1/3rd or he'll completes and go to the PhD program 1/3rd. From the PhD 1/4th is not able to complete the PhD in the next step or with the probability 3/4th is getting the job 2, now you can visualize the questions, what is the probability that I observed into the state job 1 or job 2, that's the probability of absorption.

The next question how much time on average I'll be spending in the transient states in the study before I get the job? So this is the way you can visualize the reducible Markov chain with this type, so these two questions are going to be answered by finding the probability of absorption and mean time up to absorption.

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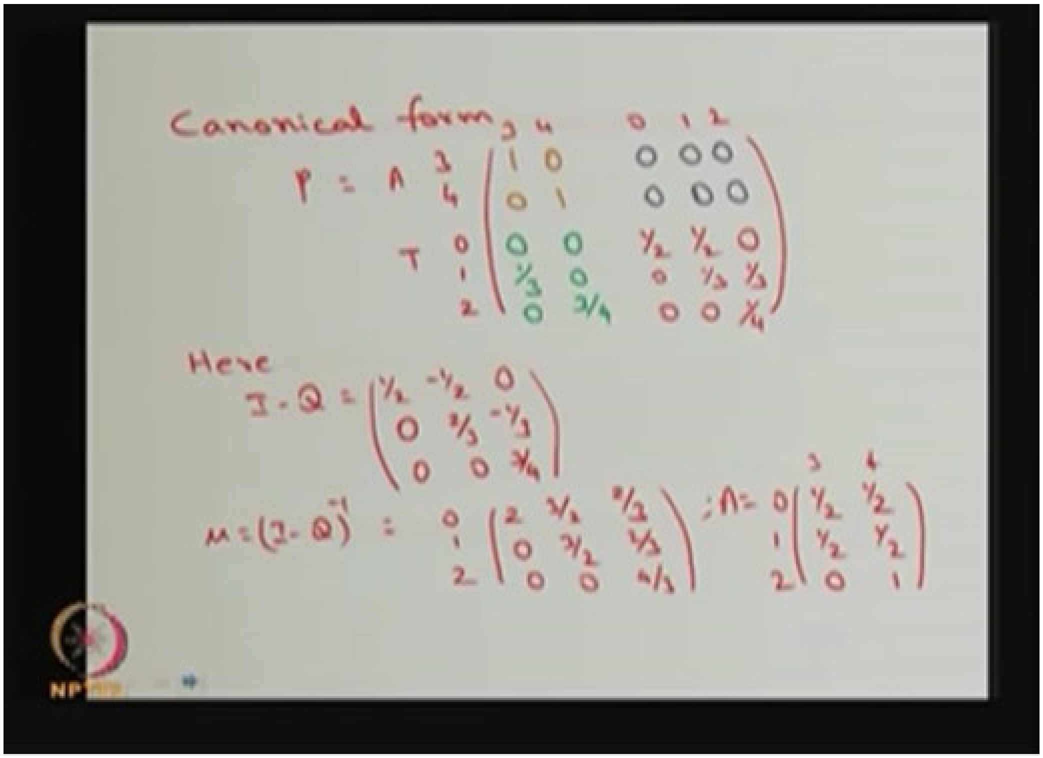


First let me write the P matrix in the canonical form and all the sub matrix I made it in the different colors, so 3 and 4 are going to form a, each one is going to be absorbing states, so therefore A to A that's identity matrix, A to capital T that is a 0 matrix, sub matrix, then T to A that is again a matrix that is R, then T to T that is a Q matrix, so what do we need the Q matrix and the R matrix? Both are sub matrix of capital T, that's a one-step transition probability matrix, so you find out what is I – Q, I is identity matrix of the same order, I3 here – Q matrix so you know the Q matrix is this, so I-Q matrix to find out the inverse, that inverse is this much, so from these if you multiply it the vector E that is 1, 1 you will get the mean time to absorption.

And also you can find out the probability of absorption after find the I-Q inverse that's a fundamental matrix multiplied by the R this matrix you'll get the probability of absorption. I'm not giving here the numerical calculation, see the result,

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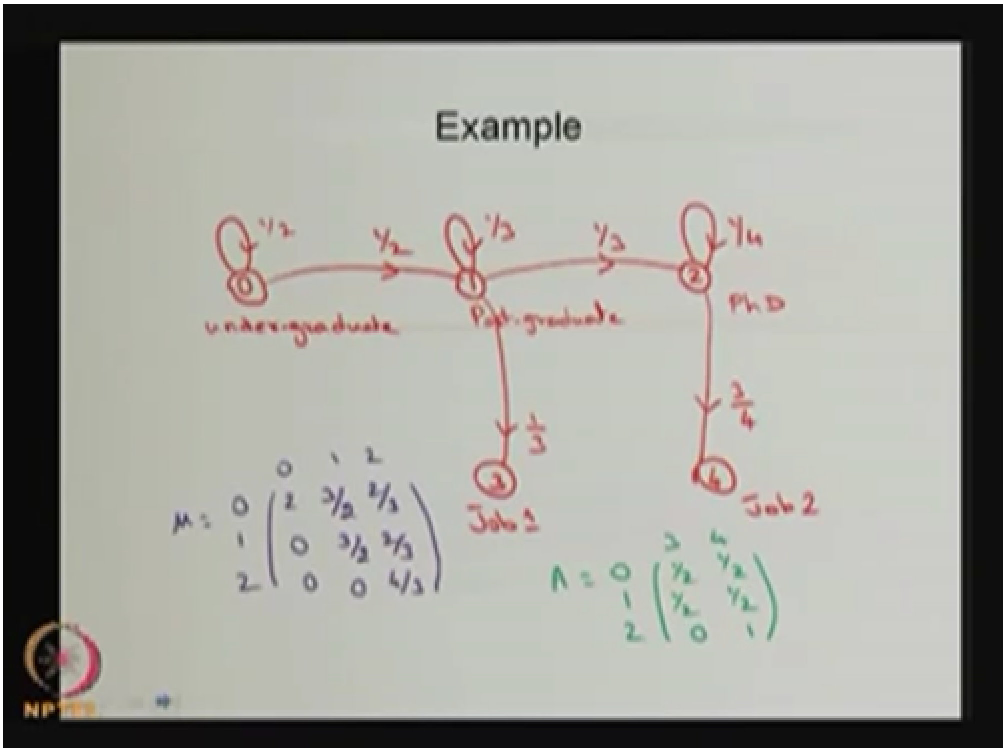


so this is the mean time up to the absorption and this is the probability of absorption, first let us discuss the probability of absorption, if the system start from the state 0, state 3 is nothing but the job 1, so with the probability of you would have been absorbed into the job 1 with the probability 1/2 if the systems start from the state 0 from the undergraduate.

Similarly with the job 2 that probability is 1/2, it's a probability mass function either you will be in the job 1 or job 2 that's the probability of absorption, if you would have started, starting with the post graduate then with the probability 1/2 and 1/2 you maybe in the job 1 and 2.

Whereas if you beginning with the PhD program, not this two programs that is not possible but still this is just example, so if you start with PhD program then definitely you land up with the job 2 with the probability 1 because there is no arc from 2 to 1 and land up the job 1, therefore the probability of absorption into the job 1 that probability 0 is for illustration, therefore you can make out how the calculation goes, so here the probability of absorption starting from the state 2 that probability 0 to the job 1, whereas job 2 that probability is 1, and so this is the probability distribution of probability of absorption starting from this transient states.

Similarly you can visualize the mean time up to the absorption. These 0s can be discussed first, so if the system start from the state 2, what is the average a number of steps, the system goes from the state 2 to 0, then it goes to the absorption state, that is not possible, the system is going from 2 to 0, therefore the mean time is going to be 0, because the minimum time is 1R, minimum number of steps system spending in the transient trades are 1 and so on, therefore mean is 0 here.

Similarly the system is starting from the state 2 and land up 1 and from there it goes to the absorption state that is also not possible therefore that mean is also 0, whereas all other values greater than 0 that gives what is the average number of steps the system is starting from these transient states and reaching this transient states before absorbed into any one of the absorption states accordingly it'll have these values.

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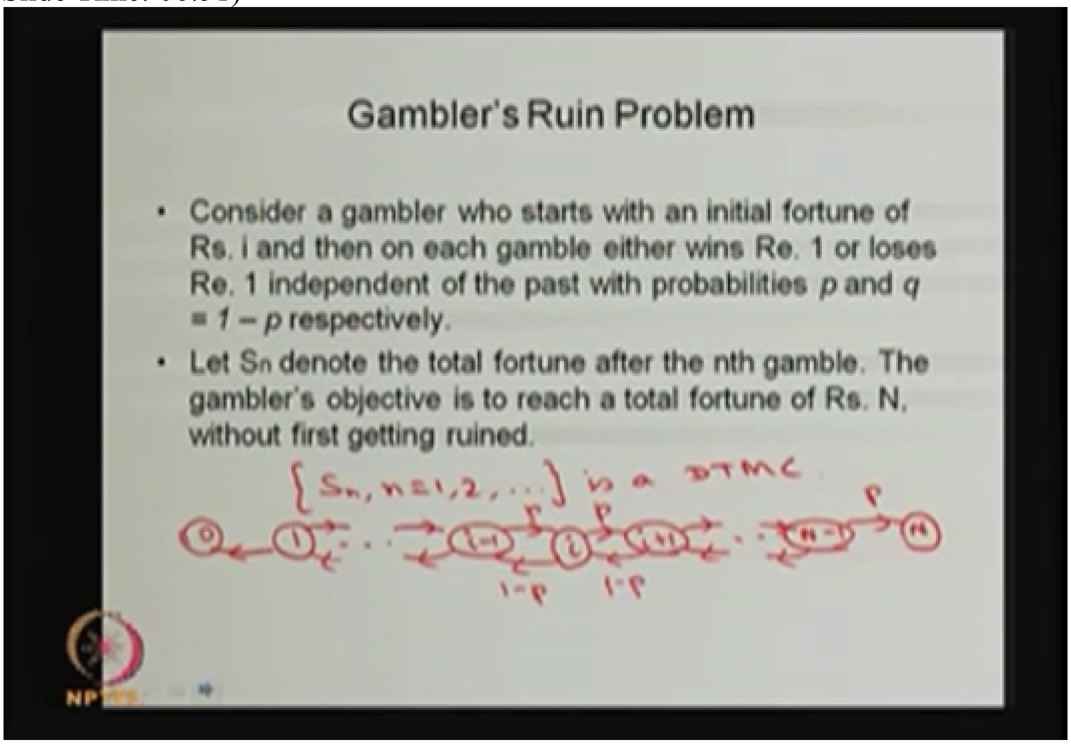


With this example I go to the next example that is reducible Markov chain and this is the special case of random walk also, let me discuss what is the example, this is called the gamblers ruin problem, let me define what is the gamblers ruin problem? Consider a gambler who starts with the initial fortune of rupees I, I amount he has at the time 0, and then on each gamble either wins rupees 1 or loses rupee 1 independent of the past with the probabilities P and 1 – P respectively, so in this game there is no draw, there is no tie, either he wins or he loses, wins with 1 rupee, loses 1 rupee and the corresponding probabilities are P and 1 – P, and he started with the initial amount small i.

And SN denote the total fortune after the nth gamble, that means S naught is small i, and S1 becomes if he wins, he is a total fortune after the first gamble that will be I + 1, if he loses then his money would have been I – 1 that is the way S1, S2, S3 sample parts goes. The gamblers objective is to reach the total fortune of rupees capital N, where N is some number, some positive integer, without first getting ruined. That means you can make a state transition diagram for this Markov chain, the SN is going to form a time homogenous discrete time Markov chain because of each games are independent and with the probability P and with the probability 1-P, he wins or he loses therefore the Markov properties going to be satisfied, therefore this stochastic process will form a discrete time Markov chain.

If you notice, if he is land up 0 amount at the nth game then he's ruin, if he's getting first time n rupees then the game is over, that's objective, therefore this is a special case of random walk on dimensional random walk in which the states 0 and N are going to form a absorbing barrier, once the system goes to the state 0 the system is absorbed in the state 0, once the system reach the state capital N then the system is absorbed in the state N, therefore the states 0 and N are absorbing states, and all other states are, states from 1 to N-1 are going to be the transient states, therefore this DTMC is a reducible, DTMC with transient states and two absorbing states, so this will fall under second type the one we have discussed.

Our interest in this model is what is the probability of absorption, what is the probability that he loses all the money at the end of some game, or what is the probability he reaches capital N that is his objective so that's the probability of absorption.

The other one is how much time he's in the transient states on average and what is the main time of absorption till he reaches the absorbing states either 0 or N.
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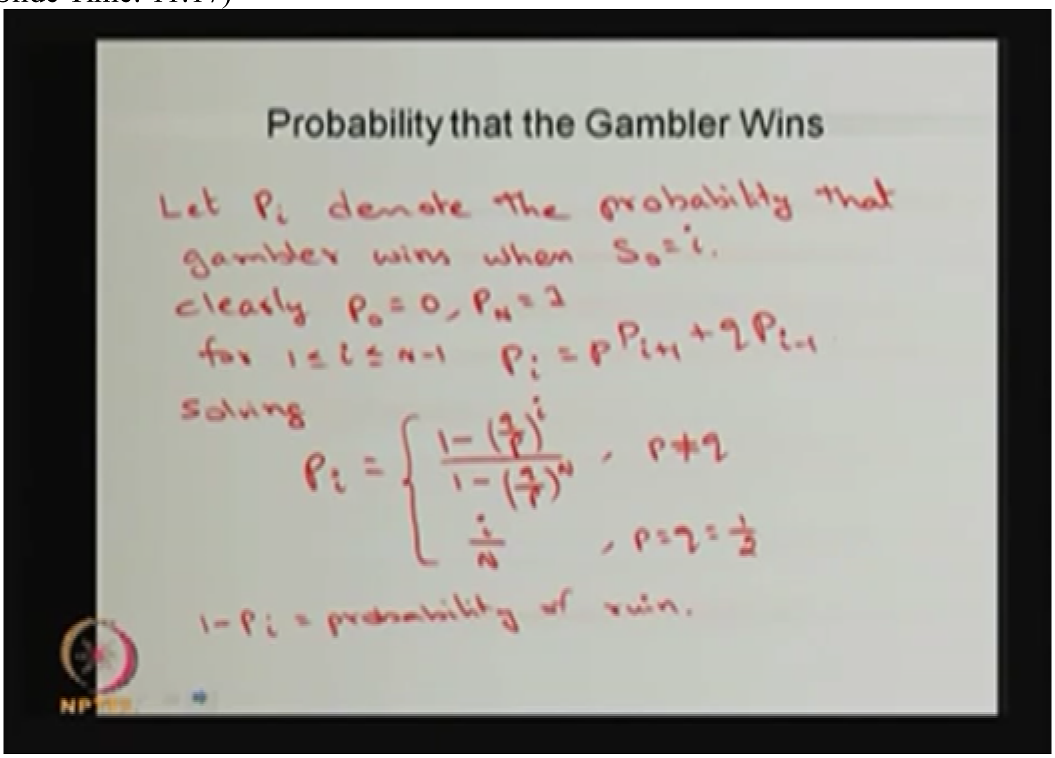


So for that I'm making a notation first piece of it's I that denotes the probability that the gambler wins then S naught = I, that is I, I means initially I amount he has, that is S naught, so what is the probability that the gambler wins? Clearly P naught = 0, similarly PN = 1, because no way if he's having initially 0 amount he cannot win, therefore that probability is 0, if he is having initially, the gambler has the N amount at that time 0 itself then you need not play at all, therefore that probability is going to be 1, therefore the probability that gambler wins that probability is going to be 1 if he's having N amount initially.

For all I in between 1 to N-1 you can make a recursive relation using the Chapman Kolmogorov equation that means the probability that the gambler win with the I amount initially that is same as either he has initially N + 1, sorry I +1 amount initially and with the probability P he wins, or with the probability I-1 the gambler wins multiplied by the probability Q, Q is the, he loses, so this two combinations will give the probability of gamblers win.

You can do the simple calculation the way we have PI's in terms of PI+1 and PI-1 you can write PI-1 also then you find out the difference, then you'll get the recursive way and you will get in terms of P1, and everything will get it, so you can use P capital N = 1 using that you will get all the PI's, you can use this relation P naught = 0 and capital PN, capital P, capital N = 1 using these two values you find out the difference and you make a recursive relation you will get a PI's.

So whenever the P is less than Q and P is greater than Q you will get and the PI's is 1 – Q – P power I divided by 1 – Q power P power N, for P and Q = same that means it is 1/2 because Q is 1-P therefore you will get the probability of gamblers win that will be I divided by N that you can get. And here the interest is, what is the probability that he's going to ruin, that means this is the probability that he is going to win and the one minus of that is going to be the probability that he's going to win in this gamble.

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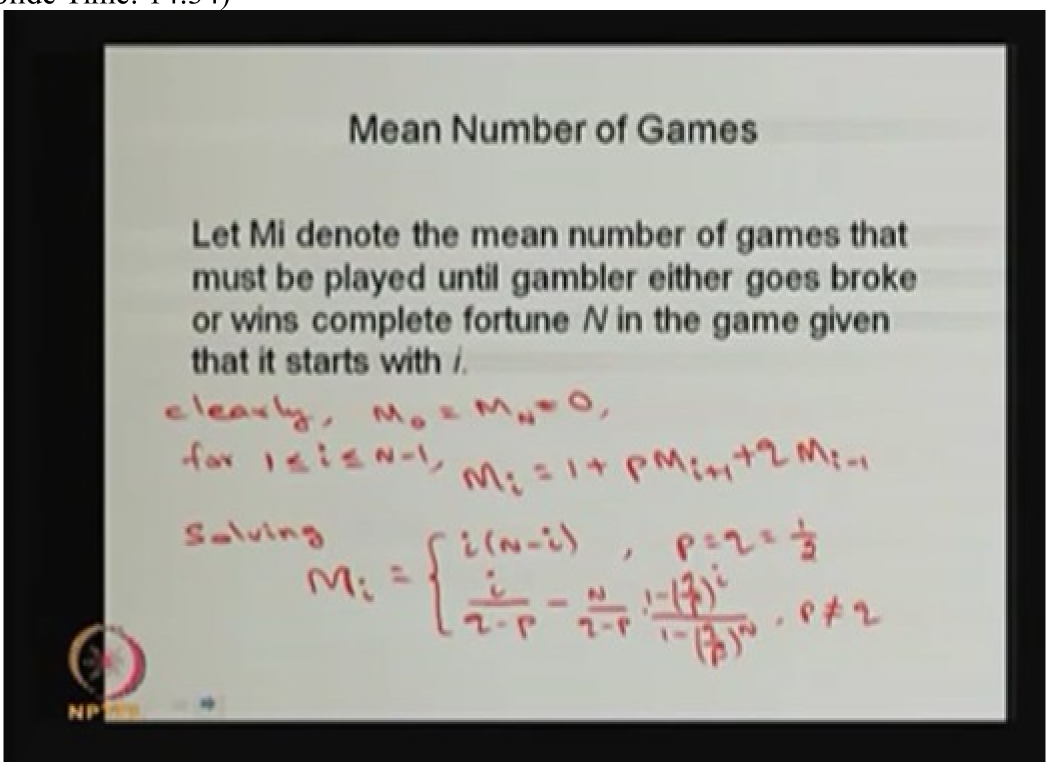


The next one is our interest is mean number of games, because the objective is he has to reach the capital N amount so the game is going to be over either he completely ruin or he's going to get the N amount, therefore I'm making here the random variable M suffix I, it's a suffix I, so M suffix I is denote the number of, sorry mean number of games I'm directly making a random variable for mean suffix I and I know the relation for this, and here also I'm making the similar relation by solving that till I get the MI's, so this is the mean number of games in the, mean number of games played by the gambler until he goes to broke or wins completely fortune N.

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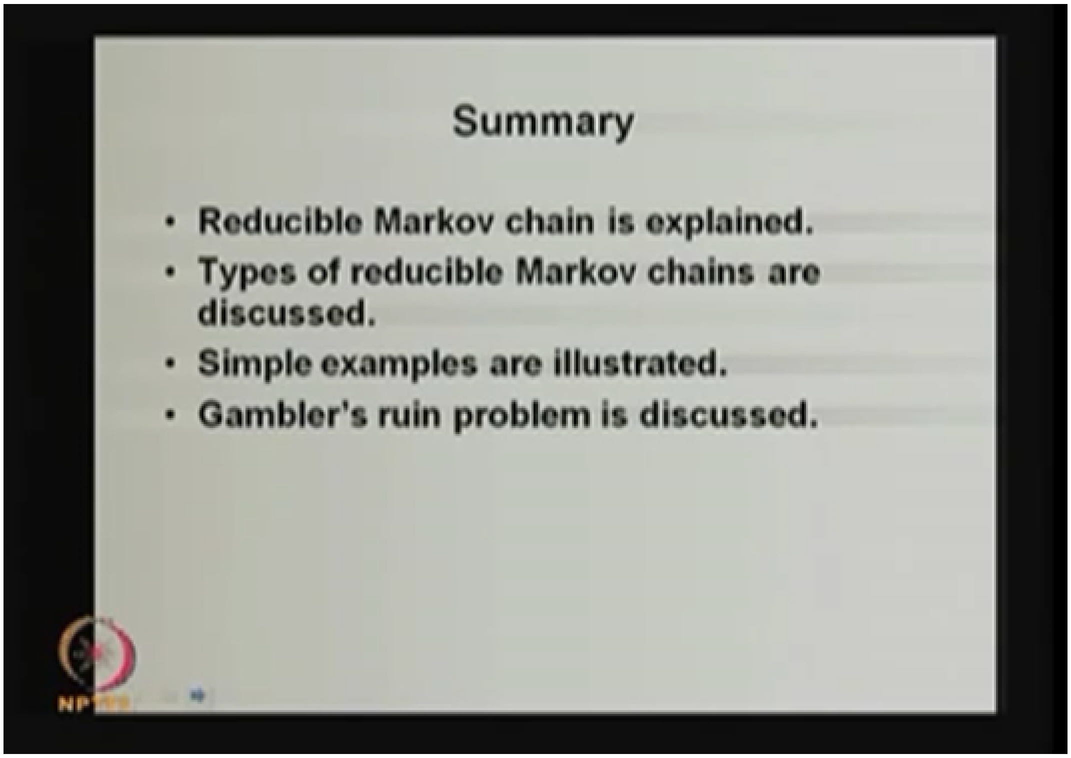


So in this lecture I have discussed a reducible Markov chain and the types of reducible Markov chain, and some examples also. And finally I have given gamblers ruin problem. References are this.

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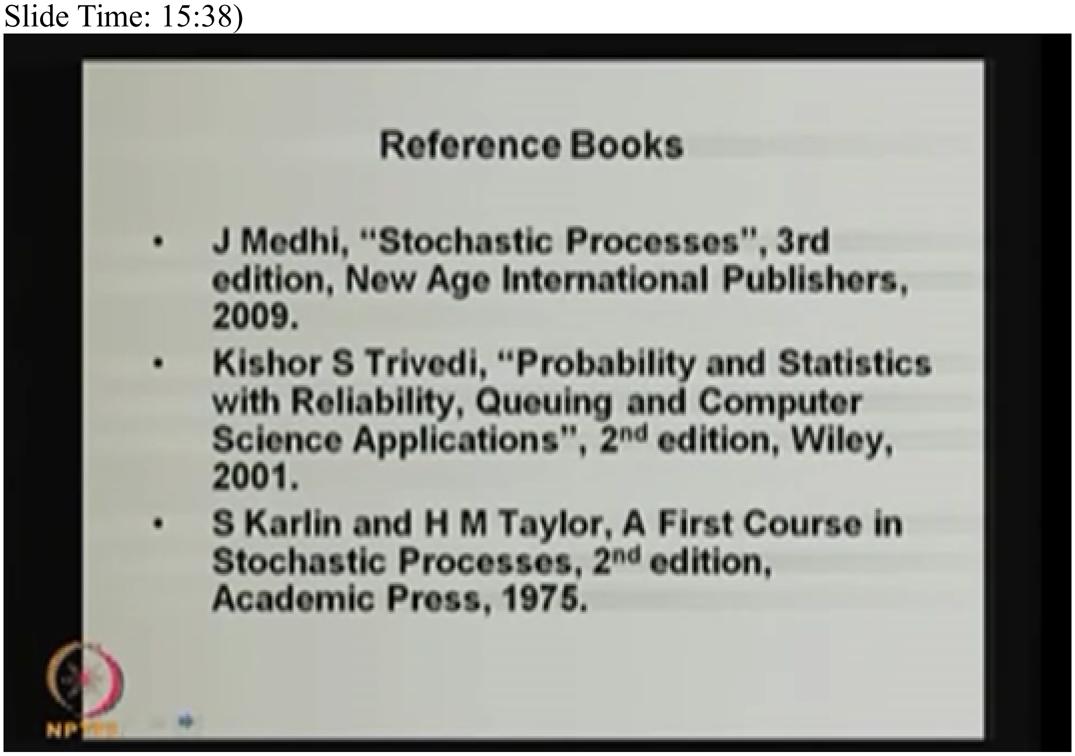


Thanks.

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