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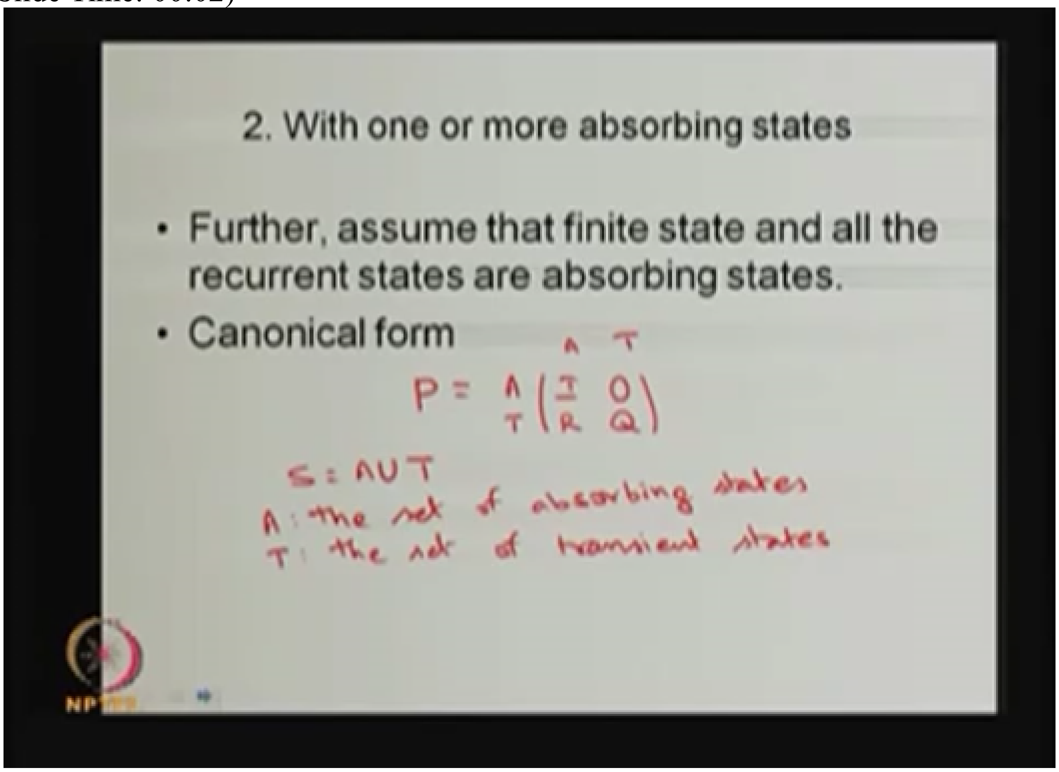

Now we are moving into the second type. The second type this is a reducible Markov chain, but here each close communicating class consists of only one element that is nothing but the absorbing states, but more than one close communicating class are possible, therefore this type is called with one or more absorbing states.

Here also my interest is to find out the stationary distribution. The stationary distribution here the interest are of the different way one is the probability of absorption, the other one is what is the need, time before absorption, so for that I'm making a further assumption, the state space is going to be finite, okay.

So with that I'm making a canonical form, the canonical form consist of all the absorbing states that I label it as capital A, and all the transient state as the capital T, therefore the state space S is A union capital T, therefore the canonical form I collect all the absorbing states in the first few rows and then remaining will be the all the transient states.

Since the absorbing states PII = 1, therefore you'll have a identity matrix for the sub matrix of the matrix P corresponding to A to A, whereas A to T absorbing states to the transient states that elements are going to be 0, so that's a sub matrix with entities 0. Whereas T to A will be some matrix with capital R, and T to T will be sub matrix Q.

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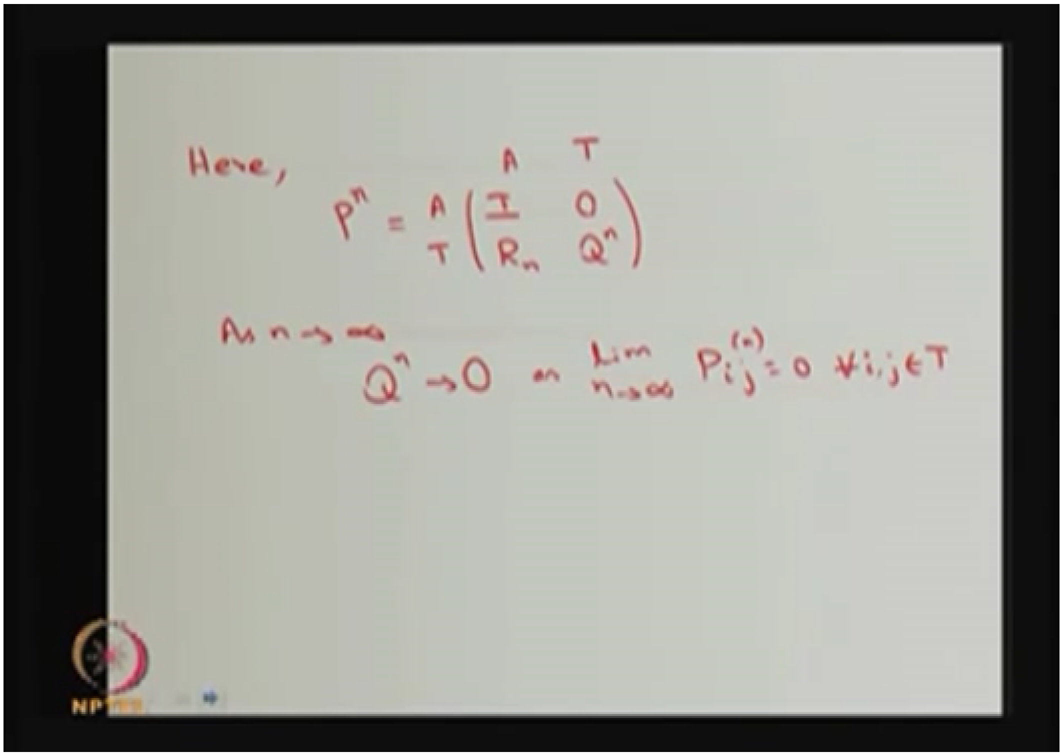

So if you go for, what is the N step transition probability, since it is identity matrix again also you will have identity matrix, whereas T to A that is going to be a function of N, whereas T to T will be a power N, that is Q raise to power N.

As N tends to infinity the system won't be in the transient state therefore QN will tends to 0 sub matrix, as N tends to infinity and this probabilities are going to be 0 for all I, J belonging to E, T is nothing but the set of transient states.

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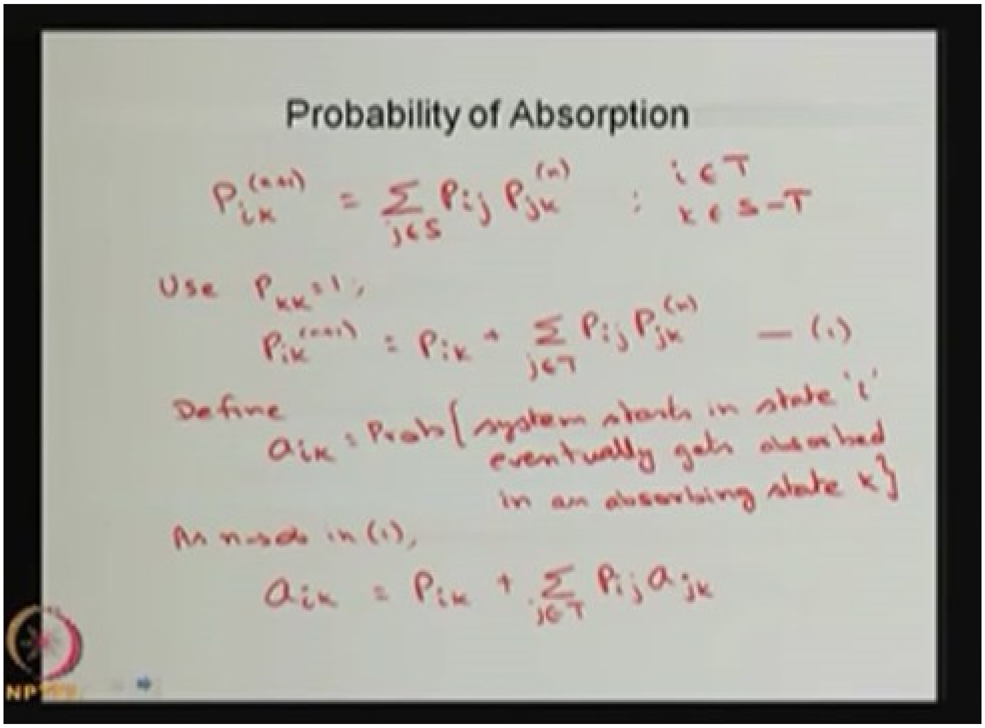

Our interest is here what is the probability of absorption? Because we have a few one or more absorbing states, so if the system starts from some transient state what is the probability of that the system will be absorbed into this absorption state. So for that I'm going to start with the Chapman Kolmogorov equation, that's Chapman Kolmogorov equation for the N+1 at the step, the system going from the state I to K that probability is same as, what are all the possible? The system can go make a one step from I to J, and then J to K union steps all the possibilities J belonging to S, where S is the state space.

I know either I have a 1, sorry, either I have a transient states or all other states are absorbing states therefore if K is going to be the absorbing state then PKK = 1, that means 1 step transition probability of system moving from state K to K that is 1, therefore I to K in N+1 steps that probability I can split, I can make I to K in 1 step then for our IB in the state K + I would have moved to the state I to J, where J is another transient state, it could be same also, it could be same also, then J to K union steps.

Now I'm defining what is the meaning of probability of absorption, that I am denoting with the letter A suffix I, K that is nothing but the probability that the system starts in state I, it starts with state I eventually get absorbed in absorbing state K, so the first letter is the starting state and the second K is the absorbing state, so this is the probability of absorption starting from the state I to the absorption state K.

Now I'm taking the equation one as I make N tends to infinity in both side, the left hand side will be A of I, K because as N tends to infinity so this will be AIK, similarly PJK(N) that is also AJK, therefore I'll have a AIK this side and AJ, so this is sort of recursive equation, so this is in the element form I can go for in the matrix form,

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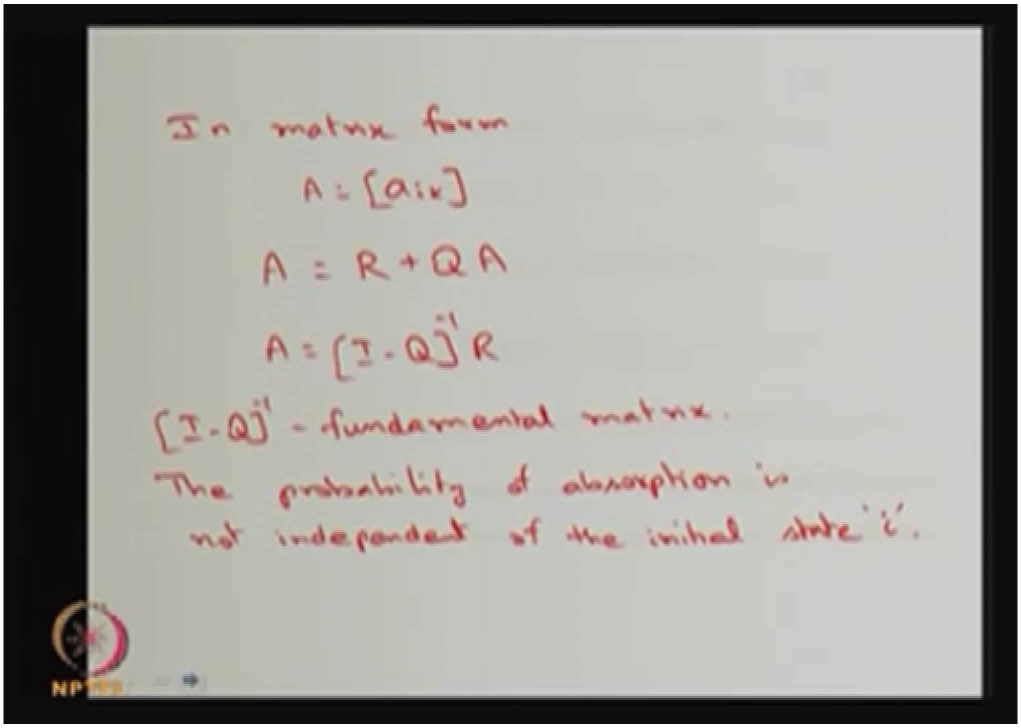

so I can write AIK as a matrix, capital A therefore in the matrix form the previous this equation for all values of I this equation as in the matrix form, capital A = R matrix because this is P I to K, where I is the transient state and K is the absorbing state, so transient state to the absorption state, transient state to the absorption state, that's sub matrix exists capital R, therefore in the matrix form capital A = R matrix + Q matrix that is the one step transition of system is moving from transient to transient multiplied by A matrix, so I can do the simplification so I get A matrix = I – Q inverse R matrix. And here this I – Q inverse that is nothing but the fundamental matrix, so once you are able to find out the fundamental matrix multiplied by the R matrix that will give the probability of absorption starting from the transient state and reaching absorption state, and this probability is not independent of initial state that is very important.

Whereas the previous type of reducible Markov chain that is independent of initial state. Whereas here the probability of absorption is not independent of the initial state I, so this we can visualize through one example that I'm going to present later.

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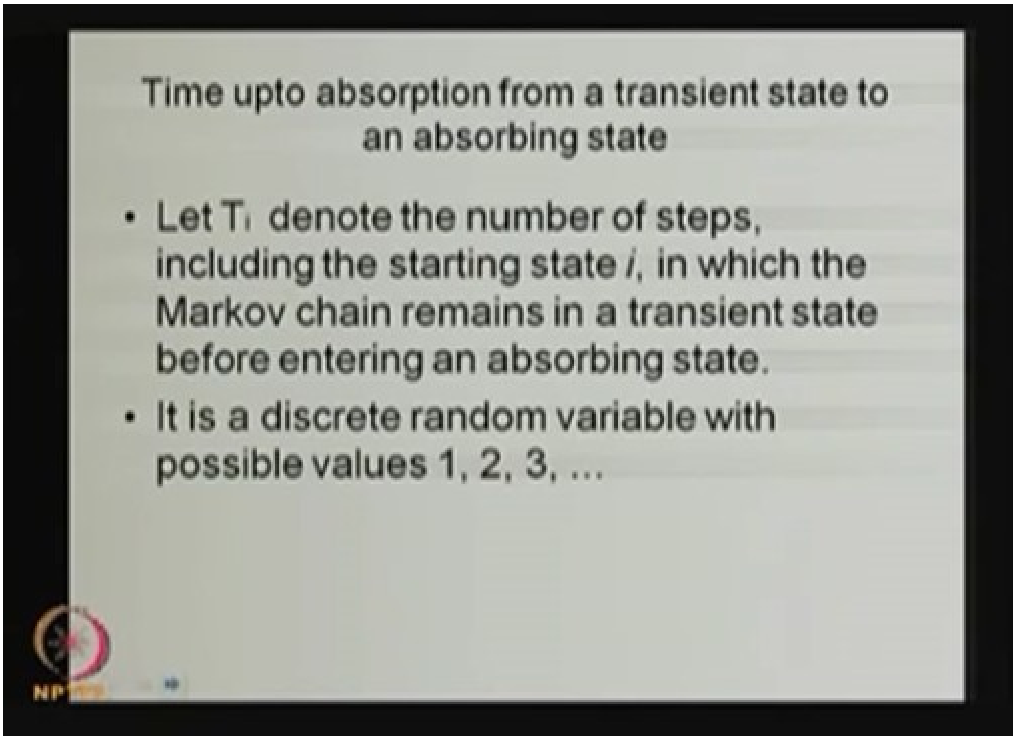

The next result interested in the reducible Markov chain with one or more absorbing states that is what is the time to absorption, basically our interest is to get the mean time to absorption starting from the transient state to absorbing state, that means how much time on average the system is pending in the transient states before absorption, that is very important because many application has a reducible Markov chain in which more than one absorption states are there with the transient states therefore what is the mean time up to absorption that means how much time spending in the transient states before the absorption, so for that I'm going to be define the random variable capital TI. The TI denotes the number of steps including the starting state I in which the Markov chain reminds in the transient state before entering a absorption state, absorbing state.

So there is a possibility, the system would have been spending at least one step before absorption or 2 steps or 3 steps and so on therefore that is going to be a random variable, it is a discrete random variable with the possible values are 1, 2, 3 and so on. Our interest is not only finding out the distribution of TI, our interest is to find out what is the mean time up to absorption from the transient state to a absorbing state.
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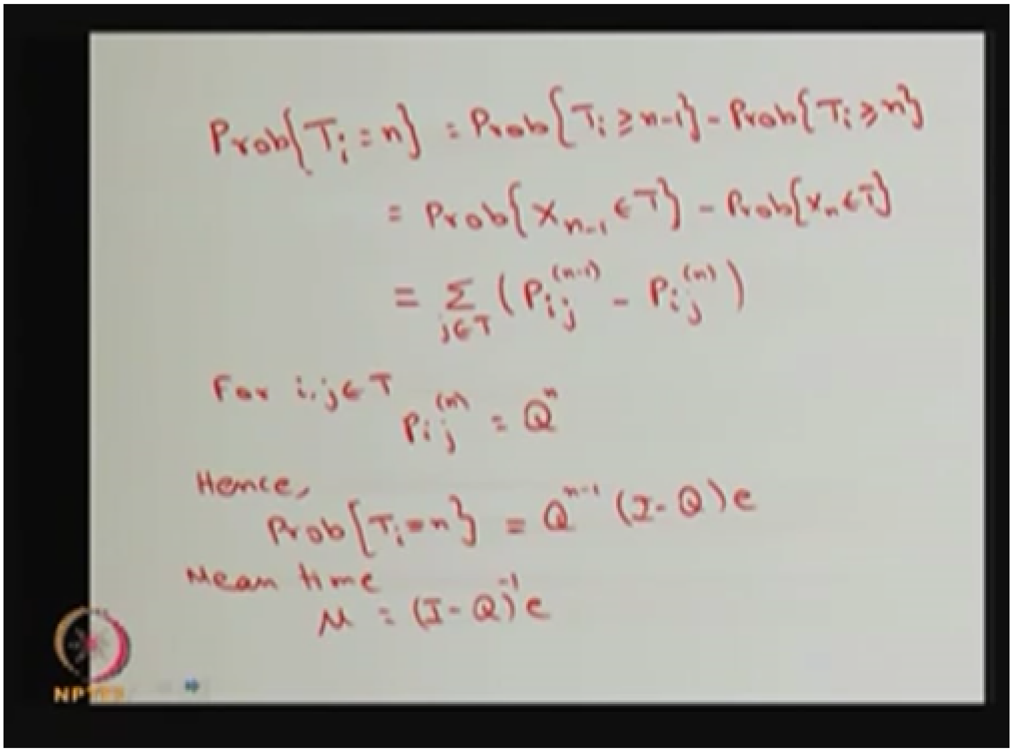

So this probability can be computed by find out what is the probability of a TN = N for some N, N can take the value 1, 2, 3 and so on, so that discrete random variable probability, probability mass function can be computed in this way, you find out what is the probability of TI is greater than or equal to N-1, minus what is the probability in that TI is greater than or equal to N if you find the difference that is same as the probability mass at N.

But this is same as the TI greater than or equal to N-1 that is same as the N-1th step, the system is in the transition state, if TI is going to be greater than or equal to N-1 that means the system spends at least N-1 steps in the transient states.

Once it goes to the absorption state then it cannot come back to the transient states, therefore the meaning of TI greater than or equal to N-1 that is same as the N-1th step the system in the transient states, so both the evens are equivalent therefore the probabilities are equal.

Similarly you can argue if TA greater than or equal to N means at least N steps the system in the transient states before absorption that is same as in the nth step the system is in the transient state. The probability of N-1th step the system is in the transient state that is same as what is the, what are all the possibilities the system would have moved from the state I to J in N-1 steps, you add all the possibilities J belonging to T, you add all the possibilities of the transient states that summation will give this probability. Similarly for the XN belonging to capital T, this is in for fixed 5 where IE is belonging to the transient state.

Now I'll go for, I know that for I, J belonging to T the nth step transition probability is nothing but the sub matrix that is Q power N, if your recall the way we made a canonical form of P matrix that T to T that is a Q matrix,
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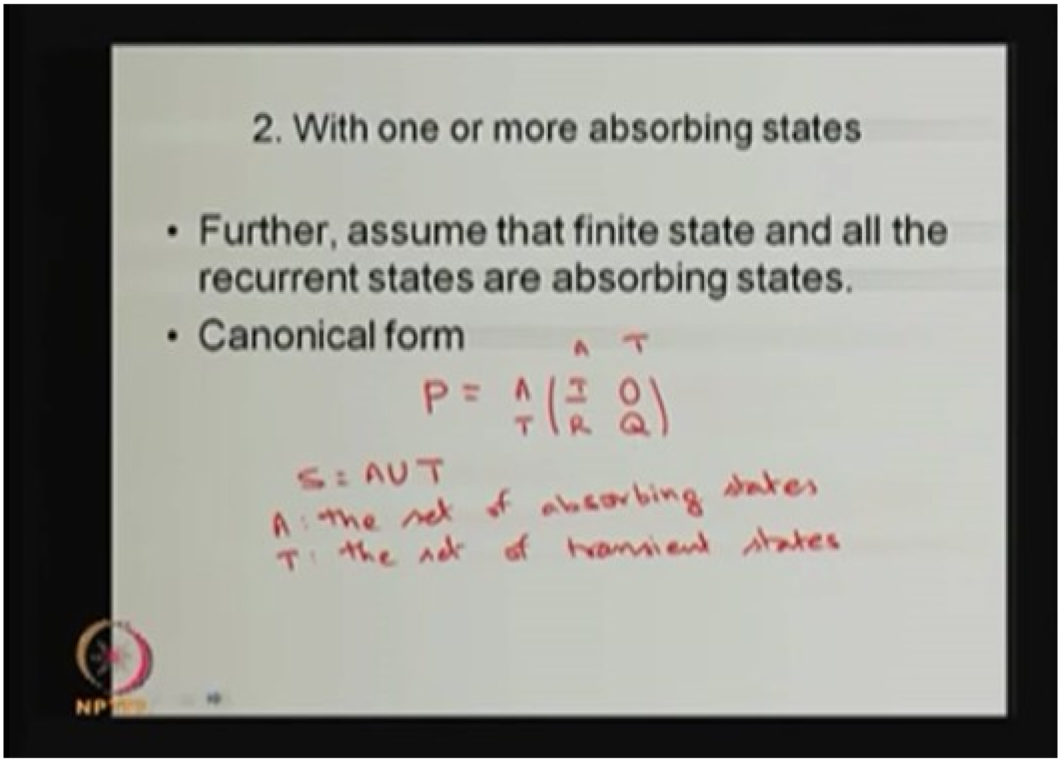

therefore as N tends to, for any nth step that is going to be Q power N, so this is what I'm using for I, J belonging to capital T, the sub matrix of a P power N that is Q power N, therefore for I, J belonging to T, the nth step transition of system moving from I to J that is Q power N, therefore I can substitute here the above equation, so the probability mass at N that is same as Q power N-1 into I-Q into E vector.

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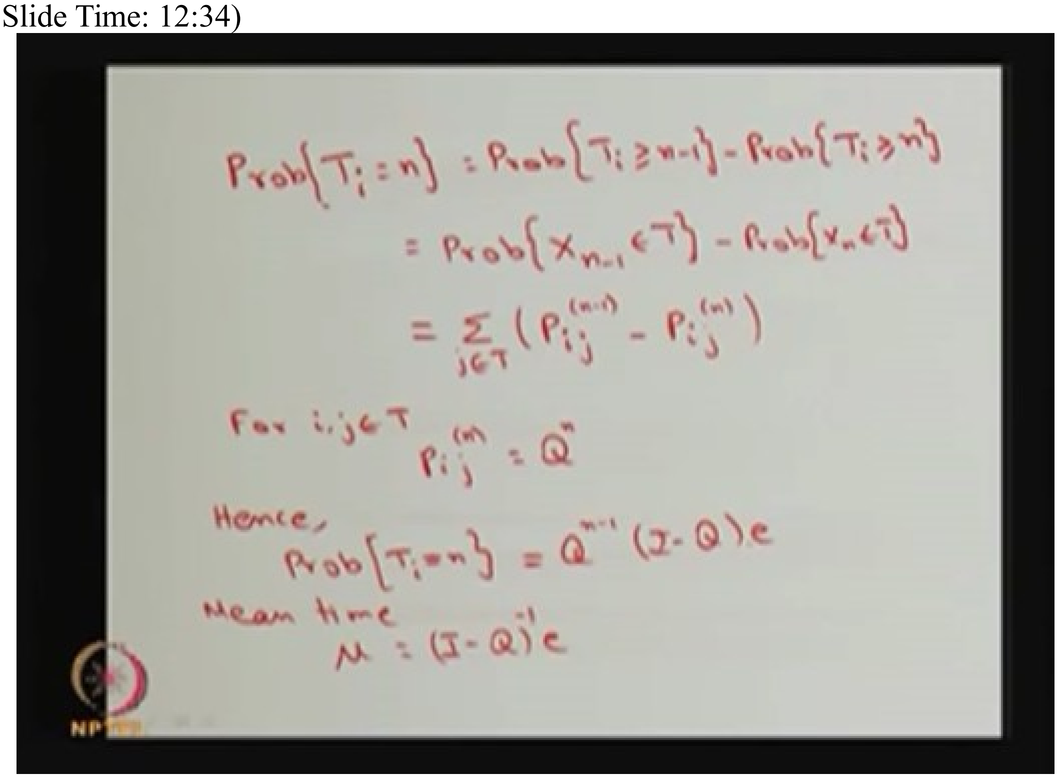

Once I know the probability mass function for the discrete random variable TI then I can find out the mean, mean is nothing but summation N times the probability mass at N, TI = N, if I add summation over N that is going to be the mean time up to absorption, that is going to be, I'll do the simple calculation you will get I-Q inverse into E vector, this I-Q inverse is nothing but the fundamental matrix that means if you find out the fundamental matrix multiplied by the R sub matrix you will get the probability of absorption, if you multiply by the E vector you will get the mean time up to the absorption.