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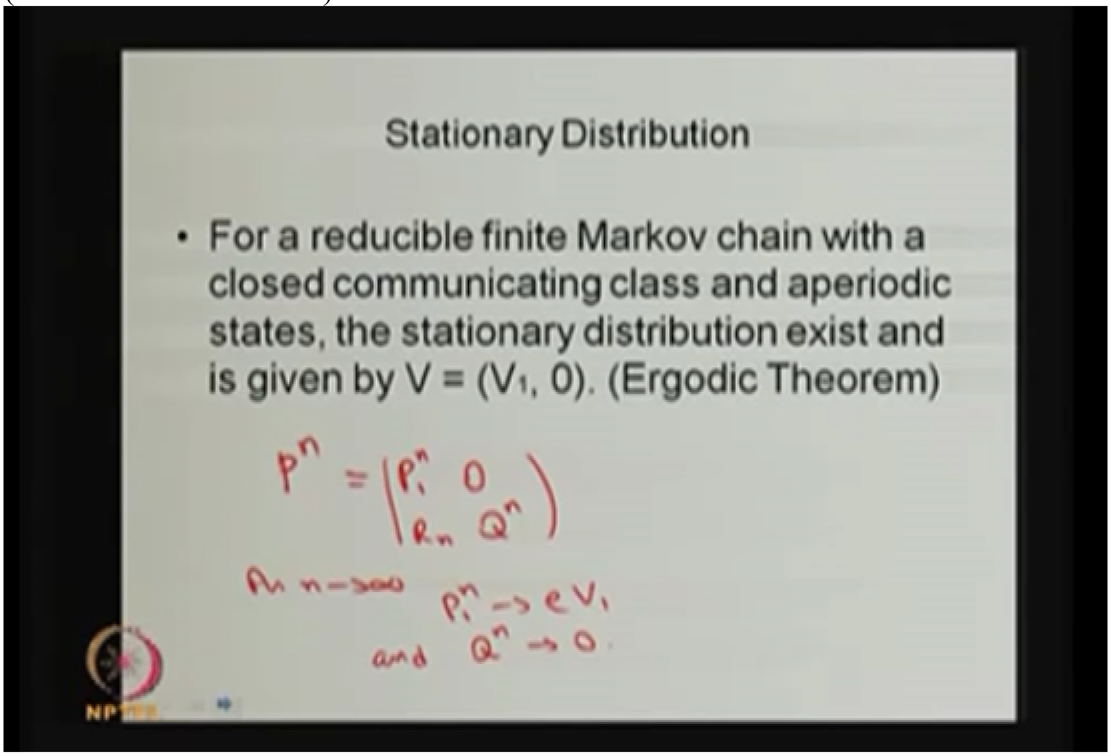

How to study the stationary distribution for a reducible Markov chain along with the assumptions, one is a periodic and the finite state.

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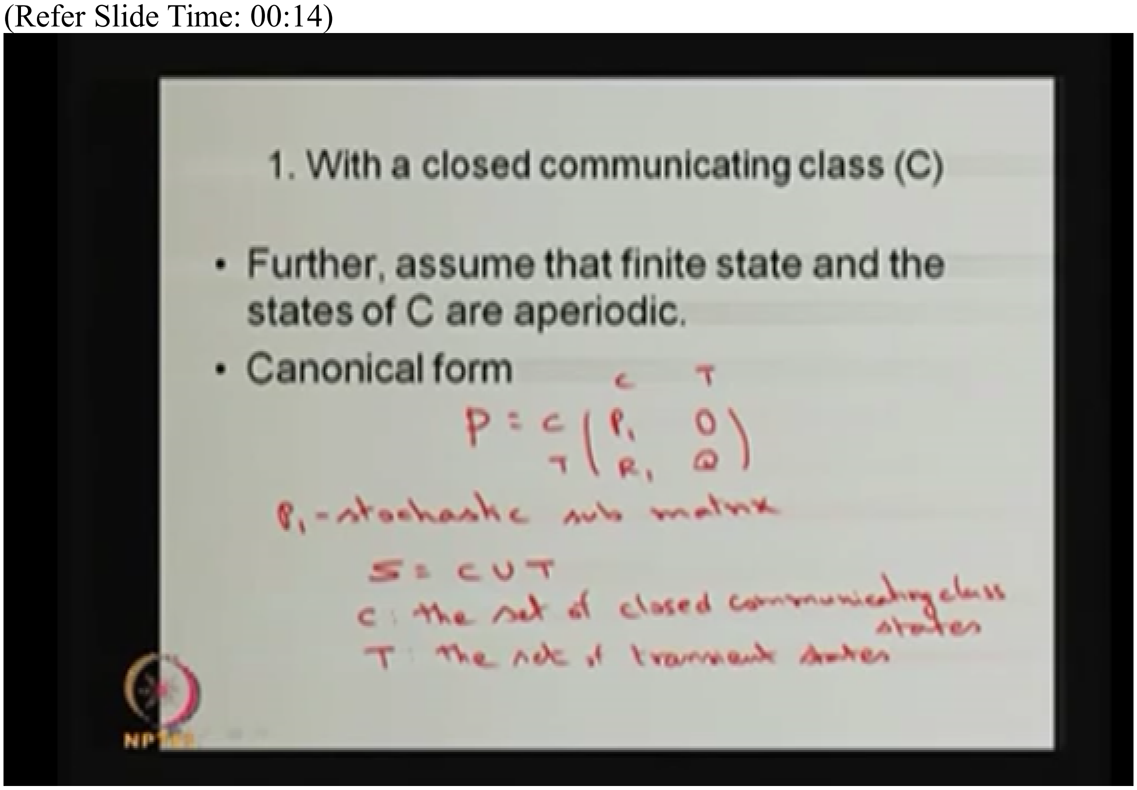

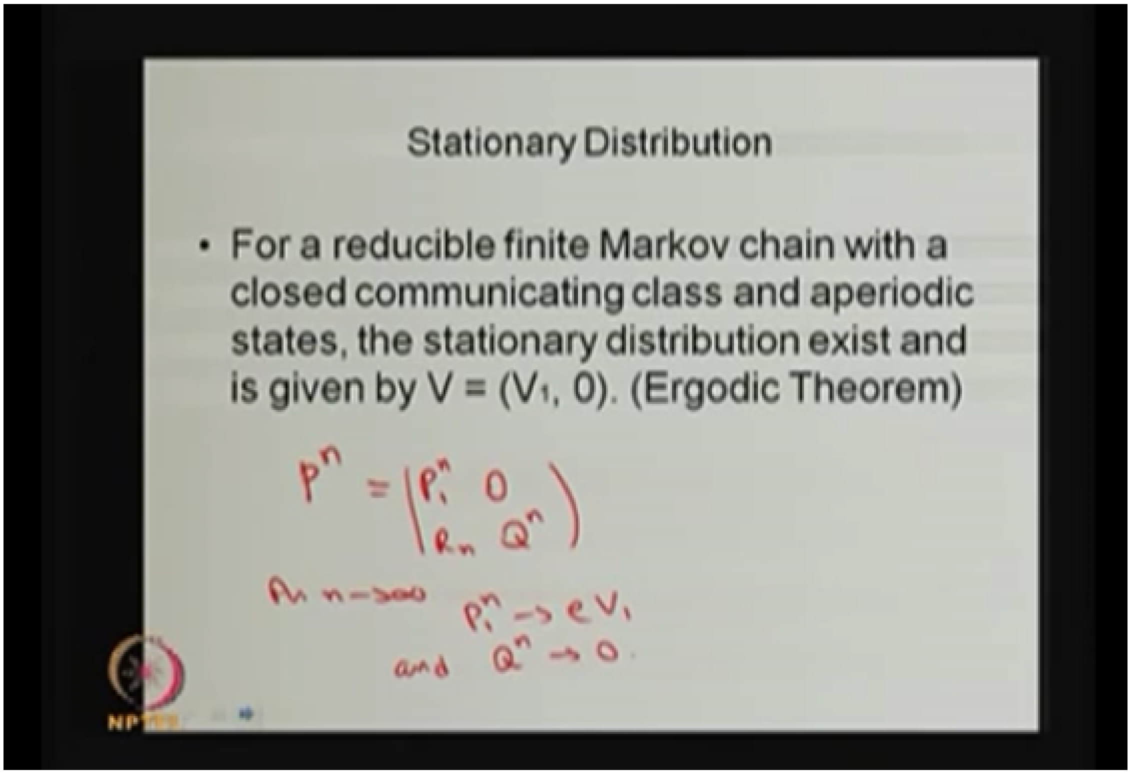

Here I'm making one more, here I'm giving the stationary distribution, so I'm giving the result for a reducible finite Markov chain, Markov chain is in the finite state space and it is reducible one.

With the closed communicating class has aperiodic states, there is a mistake, a closed communicating class of states has aperiodic states, aperiodic, the closed communicating class of states has aperiodic, then the stationary distribution exist, if that is going to be unique also, and that is given by the vector V which consists of a two sub vectors V1, 0 vector that you can find out.

And this is nothing but the Ergodic theorem for the reducible Markov chain with the assumption finite state space and the states of a closed communicating class has aperiodic states, in that case we'll get the unique stationary distribution, and that unique stationary distribution has a two sub one that the vectors are V1 and vectors of 0 elements.

Before we get the stationary distribution we can find out what is the N step transition probability for the same reducible Markov chain model, so the PN is going to be, you have a sub matrix stochastic sub matrix P1, therefore that is going to be P1 power N whereas for every N this is going to be a function of N. R is the sub matrix which is the one step going from the transient state to the closed communicating class.

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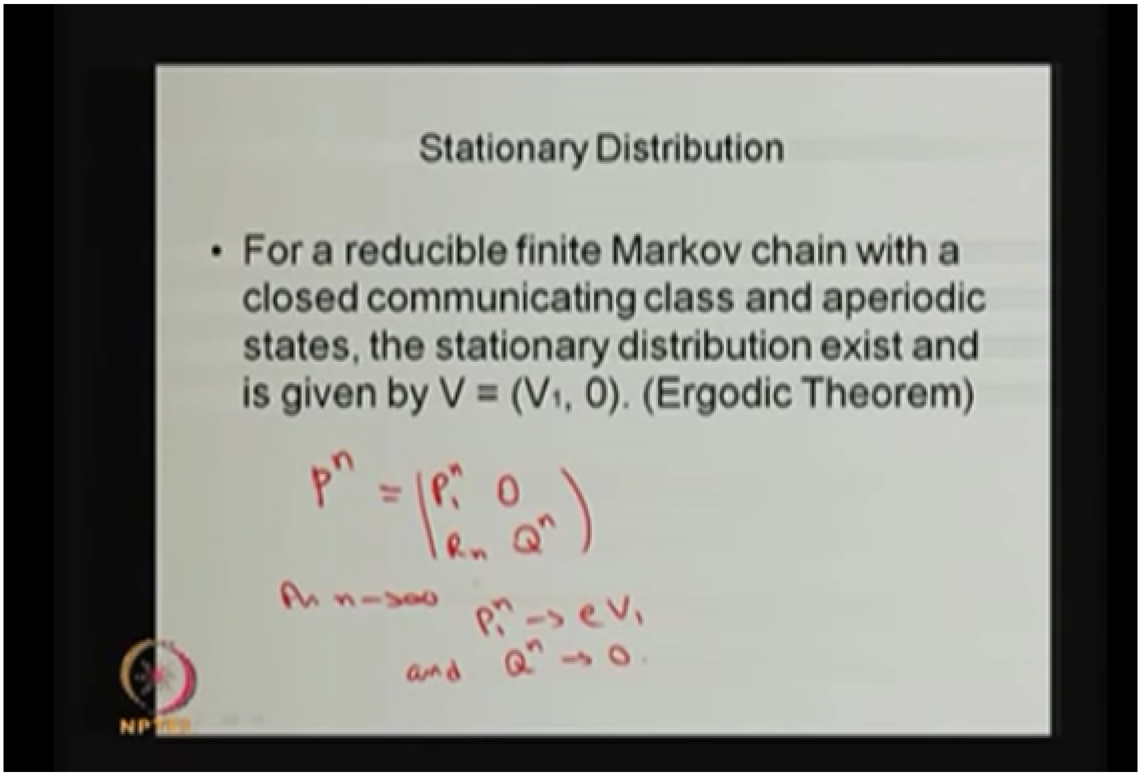

Now the RN is nothing but a function of N that sub matrix is a corresponding to the transient state to the closed communicating class, whereas the transient to transient that is going to be a power, that is Q matrix, Q matrix is the sub matrix for one step T to T, whereas Q power N is the element corresponding to the N step transition probability matrix, so as intense to infinity the Stochastic sub matrix that power N, that will tense to E's vector of V1, V1 is the sub few elements that is corresponding to the stationary state probabilities for the states corresponding to the closed communicating class of states, so E is the vector of entities 1 on 1 and so on, multiplied by the V1.

And the transient to transient N step transition probabilities Q power N has N tends to infinity, this will tends to 0, this is obvious because since the states are transient state, for a finite N you have a probability Q power whereas as N tends to infinity the system won't be in the transient state, therefore the long run proportion of the time the system being in the transient states that is 0 as N tends to infinity, therefore QN will tends to 0 whereas this will tends to the stationary state probabilities therefor this stationary distribution vector V consists of few elements of 0s that is corresponding to the transient states, transient state probabilities in a longer run.

And the V1 is the steady state probabilities in a longer run, it's not steady state, it's stationary distribution, stationary state probabilities in a longer run for the closed communicating class of states.

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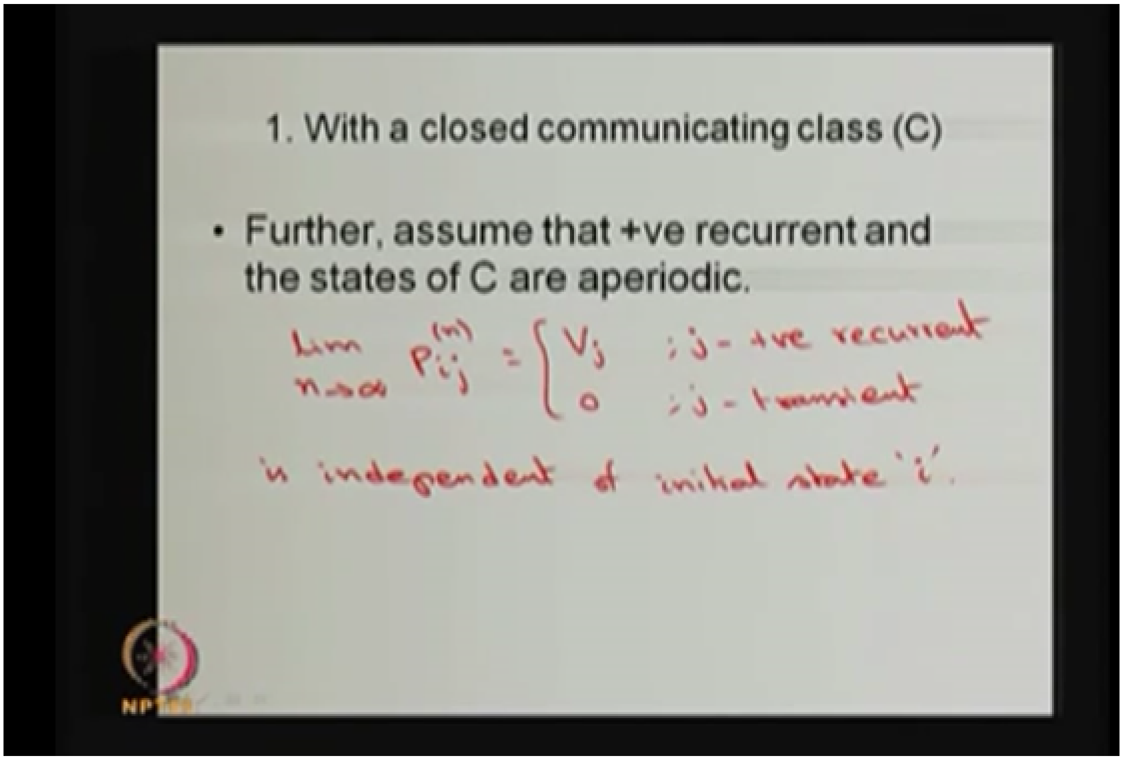

So this one can solve by using the equation by P = pi you can get this pi S, that pi S in the notation here it is VI is V1.

So now I am making a further assumption, the states are going to be a positive recurrent, so already I made a periodic states now I'm making the one more assumption it is a positive recurrent, once it is a positive recurrent then the limiting probabilities limit N tends to infinity, that probability is going to be VJs for the positive recurrent states and for all the transient states the probabilities are going to be 0, and since we have a reducible Markov chain with a one closed communicating class and all other states are transient states, this stationary distribution or stationary state probabilities, these probabilities are independent of the initial state I, that means either the system can start a time zero in one of the states in the closed communicating class or transient states.

In the longer run ultimately the system will be in one of the states in the closed communicating class whether it's started initially from the closed communicating class or transient states, therefore this stationary distribution is independent of initial state I. And for all the transient state you can conclude immediately these probabilities are zeros and for a positive recurrent states you can make it VJ's and you can compute these VJ's

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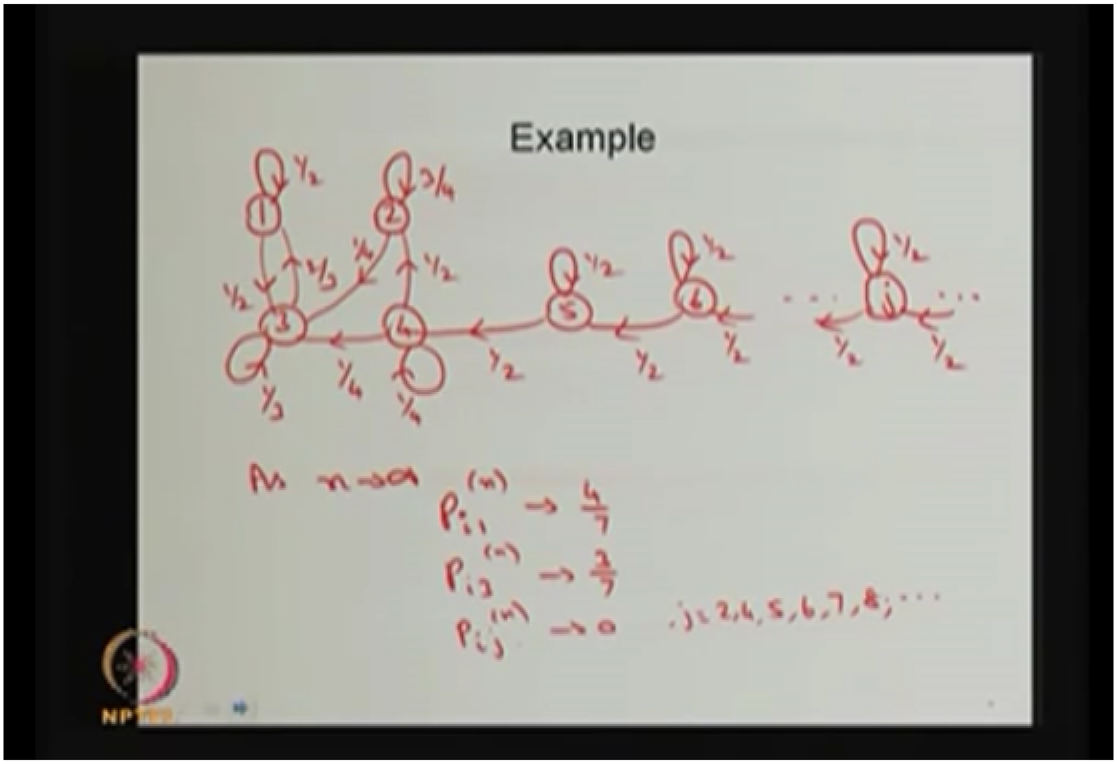

Now I am going to give one simple example in which we have a infinite state, this is going to be a reducible Markov chain because the states T5, not T5 including 4 and 2, the system come to the state 3, there is no arc from 3 to 4 or 3 to 2, therefore the states 2, 4, 5, 6 and so on all those states are transient states, whereas the states 1 and 3 are going to form a one closed communicating class, therefore this is the reducible Markov chain with one closed communicating class 1 and 3 and all other states are going to be transient states, therefore as N tends to infinity this probability are going to be 0 for the states 2, 4, 5 and so on, and this probabilities are independent of the initial state I, so wherever there the I, whether the I is belonging to the one of the elements, one of the states in the closed communicating class of states or transient states, M material of that, this is stationary distributions are 0s for the transient state for the, for the closed communicating class of states you can find out this probability by separately making the Markov chain, the states sorry, 1 and 3 you can make it separately and there is an arc from 1 to 3 with the probability of, there is a self-loop with the probability 1/2.

And there is a self-loop in the state 3 with the probability 1/3 and the arc from 3 to 1 is 2/3, so what do you want to find out this stationary distribution for this 2 states, therefore you make a stochastic sub matrix with the states 1 and 3, that is 1/2, 1/3, 2/3 and 1/3, this is also stochastic matrix you can verify.

Now if you want to find out the stationary distribution for these two states you solve P1 = pi, that means pi 1, pi 3 times p1 that is 1/2, 1/3 oh sorry, 1/2, this is 1/2, I made a mistake, 1/2, 1/2 and this is 2/3, 1/3, that is equal to pi 1, pi 3, you take the first equation that is pi 1, half times pi 1 + 2/3rd pi 3 that is equal to pi 1, so from here you will get pi 3 = 3/4 pi 1.

Now we use pi 1 + pi 3 = 1, so using these you will get pi 1 = 4/7, once you know the pi 1, the pi 3 is going to be 3/7.

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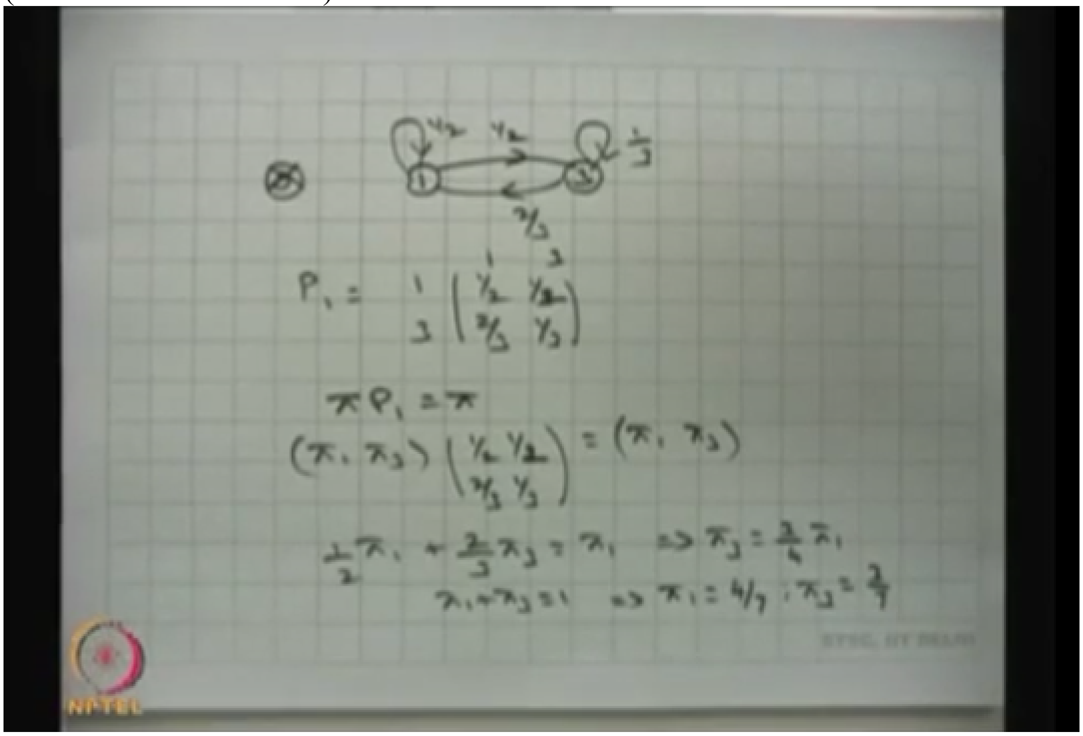

So you don’t want to find out the stationary distribution for the whole model, instead of that you can find out what is the closed communicating class and you can solve only the close communicating class that sub matrix by pi 1 = pi and you will get pi 1 and pi 3, and that is going to be in a longer run that is equal to 4/7 and 3/7 and all other states are going to be 0.