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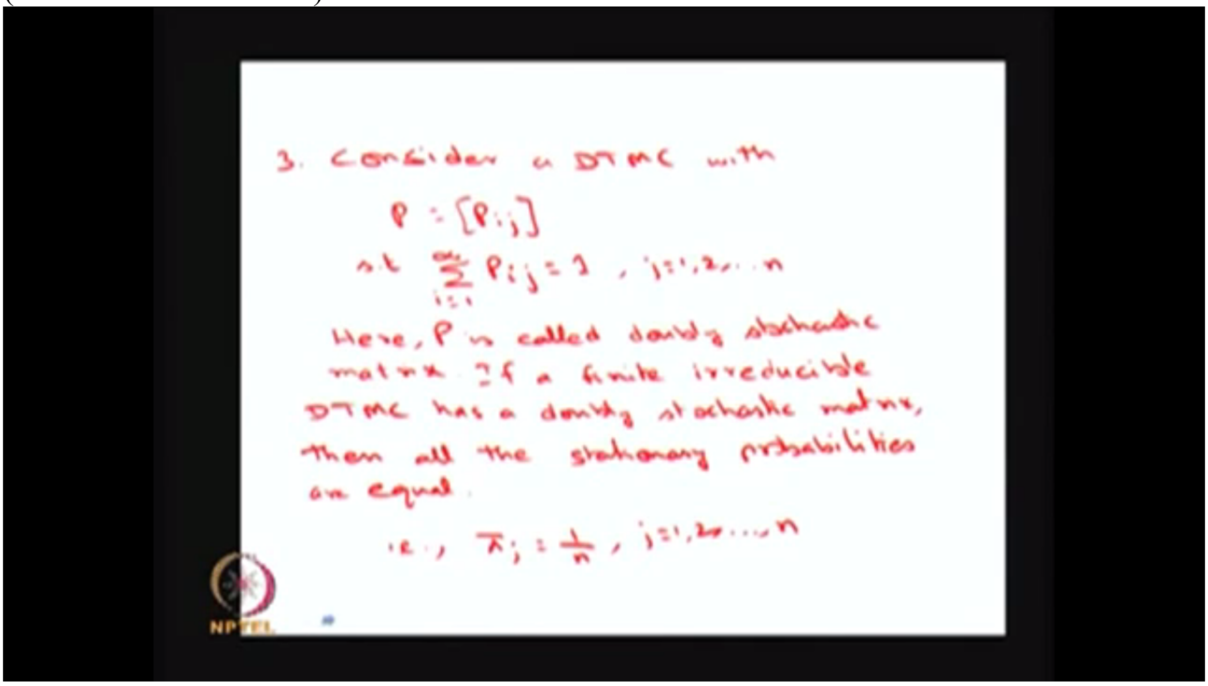


The third example I'm considering a discrete-time Markov chain. Obviously, it is a time-homogeneous discrete-time Markov chain with the one-step transition probability matrix satisfies the additional condition that is the column sum that is also going to be 1. Obviously, the stochastic matrix means the row sums are going to be 1 and here I am making an additional condition along with the row sum, the column sum is also going to be 1 for a finite Markov chain

In this model, in this situation, this stochastic matrix is going to be call it as a doubly stochastic matrix. That means it's a stochastic matrix that means each entities are lies between 0 to 1 and row sum is going to be 1. Along with the row sum, the column sum is also going to be 1. Then that matrix is going to be call it as a doubly stochastic matrix.

If you have a discrete-time Markov chain with the finite and doubly stochastic matrix and also it is irreducible, I am making additional condition, if it is a finite, irreducible with the probability, one-step transition probability matrix, it is a doubly stochastic matrix, then the stationary probabilities exist as well as that stationary probabilities are going to be a uniformly distributed. That is that values are 1 divided by n where n is the number of states of the discrete-time Markov chain.

To get this result, you can use all the previous results also. It is a irreducible Markov chain. Therefore, and also it is a finite. So for a finite irreducible Markov chain, all the states are going to be a positive recurrent. You can use the previous result. Only the aperiodcity is missing, but since it is a doubly stochastic matrix, that aperiodicity is taken care. Therefore, the stationary probabilities exist.

Now if you compute the stationary probabilities for a doubly stochastic matrix situation, then the $\pi$ is equal to $\pi P$ if you solve with the summation of $\pi_i$ is equal to 1, since the matrix is going to be a doubly stochastic, that means its column sums are going to be 1, therefore, it is going to be boils down or the -- the simplification is boils down to the state probabilities are going to be 1 divided by n. I am not going to give the derivation for that. That can be worked out.

That is Example 4. You consider a two-state model. The system is going from the state 0 to 1 in one step. That probability is 1 and the system is going from the state 1 to 0. That probability is also 1. Therefore, the P matrix, one-step transition probability matrix, 0 to 0 is 0. 0 to 1 that is 1. 1 to 0 that probability is 1 and 1 to 1 that is 0.

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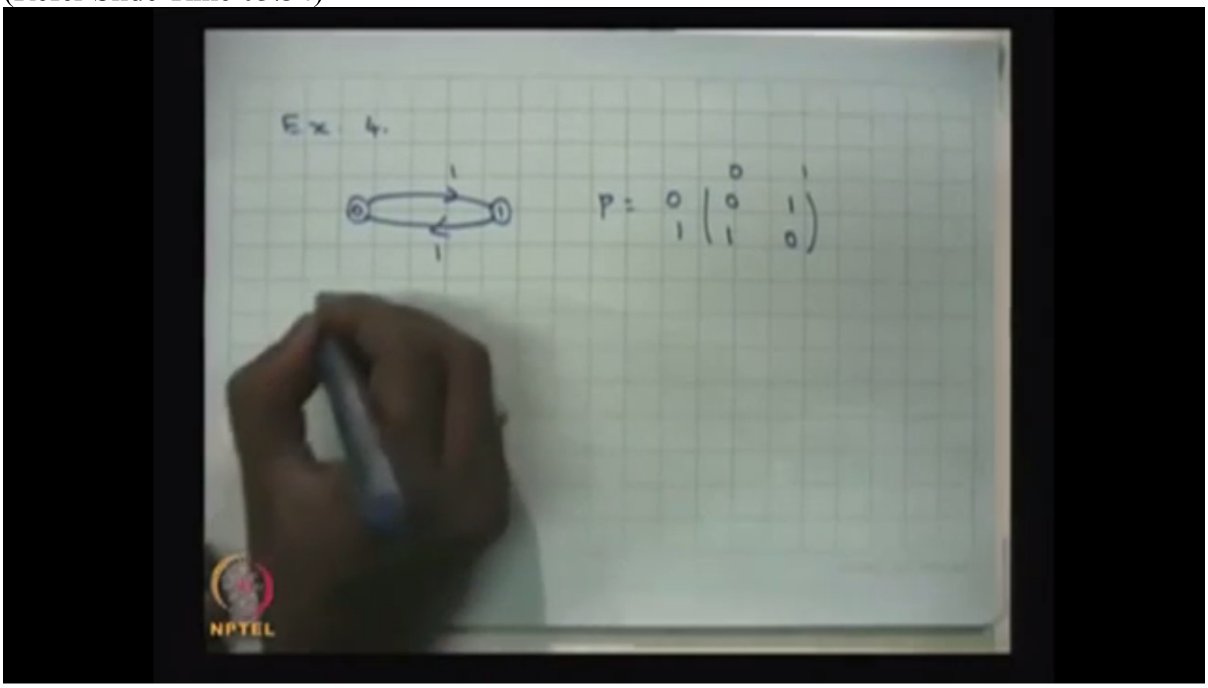


So this is the one-step transition probability matrix and if you see that this is a finite state model, irreducible. It's not aperiodic because there is no self-loop. So if you find out the periodicity for the state 0, the greatest common divisor of system starting from the state 0 to coming back to 0 in how many steps, you find out the greatest common divisor of that and since it can come back in two steps or four steps and so on, therefore, the greatest common divisor is 2.

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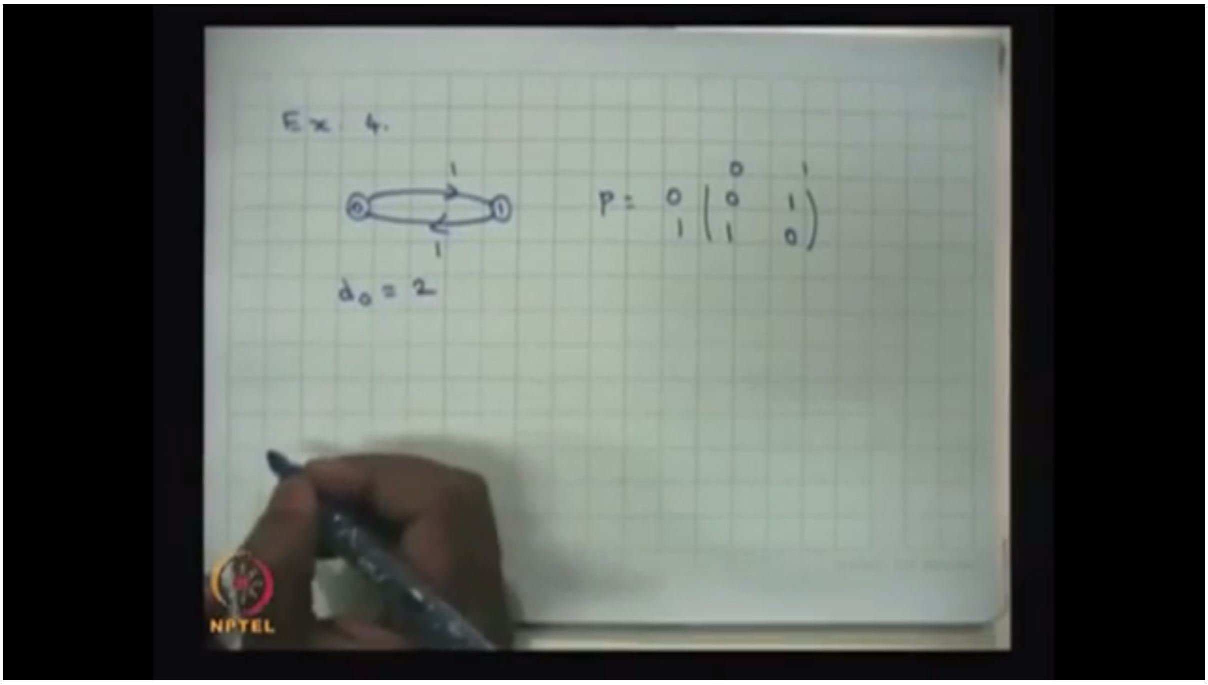


Similarly, since it is a finite state model, if one model is -- one state is of periodicity, then all other states are also going to be a same periodcity as long as it is irreducible. Therefore, the periodicity for the state 1 that is also going to be 2 or you can compute separately coming back to the state 1 starting from the state 1, that is going to be either two state or two steps or four steps or six steps and so on. Therefore, the greatest common divisor is 2. Since it is a irreducible model, all the states are going to be of the same type. Since it is a finite, 1 is going to be a positive recurrent. Therefore, both the states are going to be a positive recurrent and aperiodic, sorry, periodicity 2 and irreducible.

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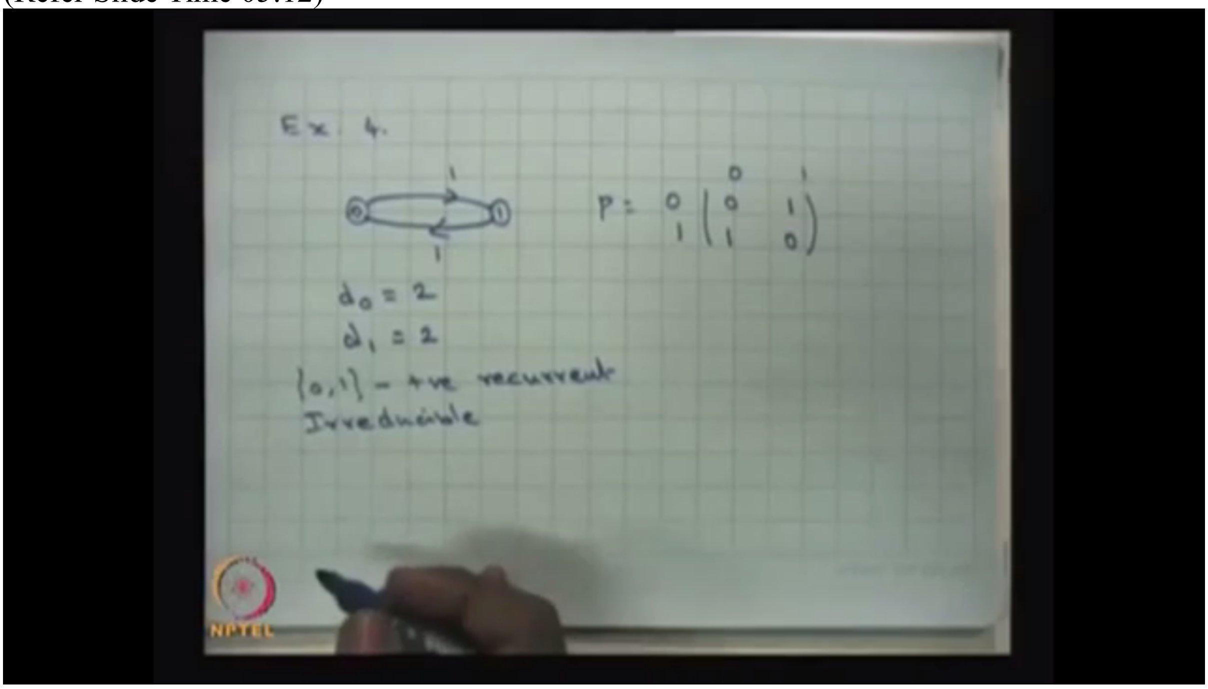


Note that the one -- the example which I have formulated, the column sum is also 1. Therefore, it is a doubly stochastic matrix. Therefore, you can use the previous result, the example which I have given, finite, irreducible, doubly stochastic. Therefore, the stationary distribution exists.

So if you solve $\pi$ is equal to $\pi P$ with the summation $\pi_i$'s is going to be 1 where $\pi$ is nothing but $\pi_0$ $\pi_1$ vector. So if you solve $\pi$ is equal to $\pi P$ with the summation of $\pi_i$ is equal to 1, you will get ($\pi_0$ $\pi_1$) that is same as (1/2 1/2).

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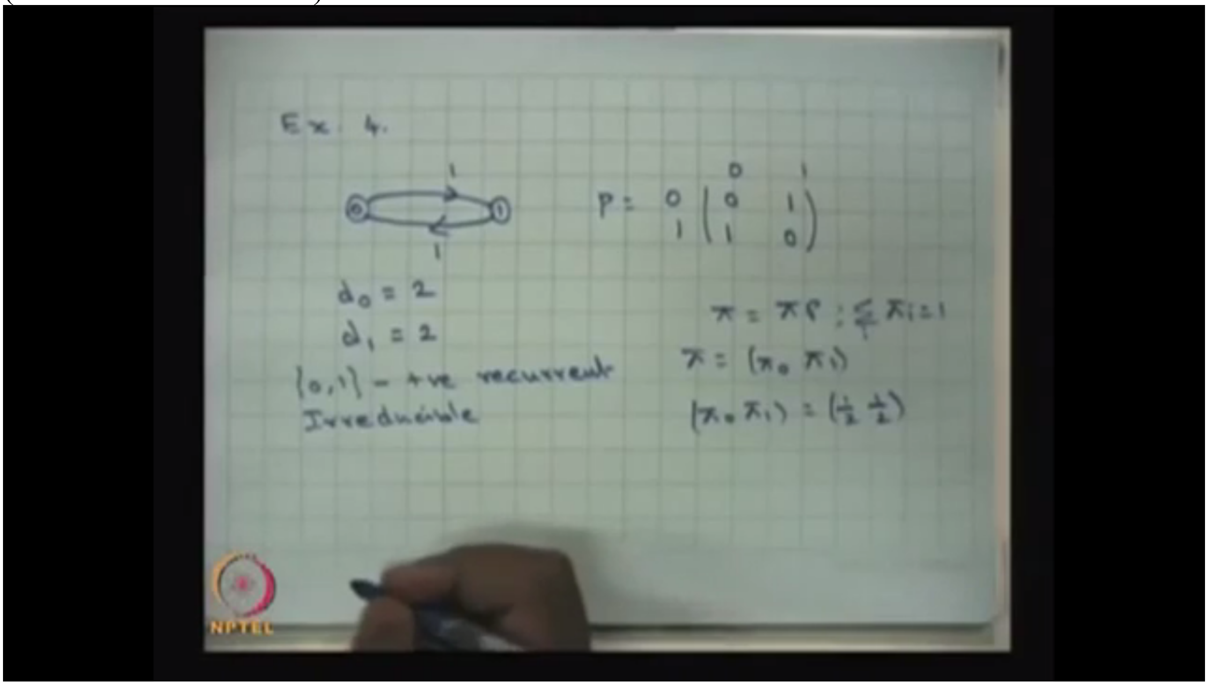


So this is the stationary distribution that exists and that values is state probability -- stationary state probabilities are going to be 1/2  1/2. That means in a longer run, the system will be in the state 0 or 1 with the probability 1/2 whereas if you try to find out the limiting state probabilities for limiting distribution, that means the limit n tends to infinity P of n, that means find out the n-step transition probability matrix, then you make a n tends to infinity. This does not exist for this model.

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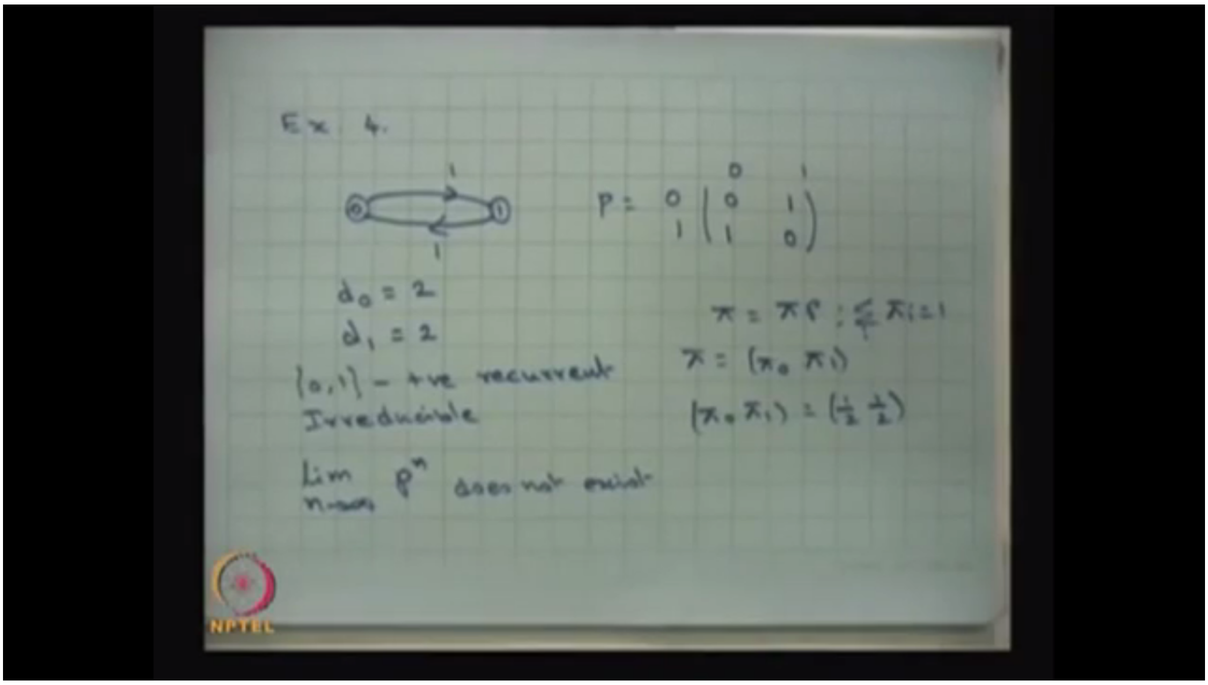


If you see the result which I have given the -- the limiting distribution, it's going to be exist and unique and so on, there I have made -- there I have not discussed the periodicity. There I have made it aperiodic. So here it is a periods -- period 2 model. So whenever you have a irreducible, positive, recurrent state, if the periodicity is not 1, that means it's not a aperiodic model, there is a possibility the limiting distribution won't exist, but still the stationary distribution exists.

So this is an example in which the limiting distribution does not exist whereas the stationary distributions exist, but if the model is irreducible, aperiodic, positive recurrent, then the limiting distributions exist as well as the, sorry, stationary distribution exists as well as the limiting distribution exist and both are going to be same.

Now I am going to give the conclusion.

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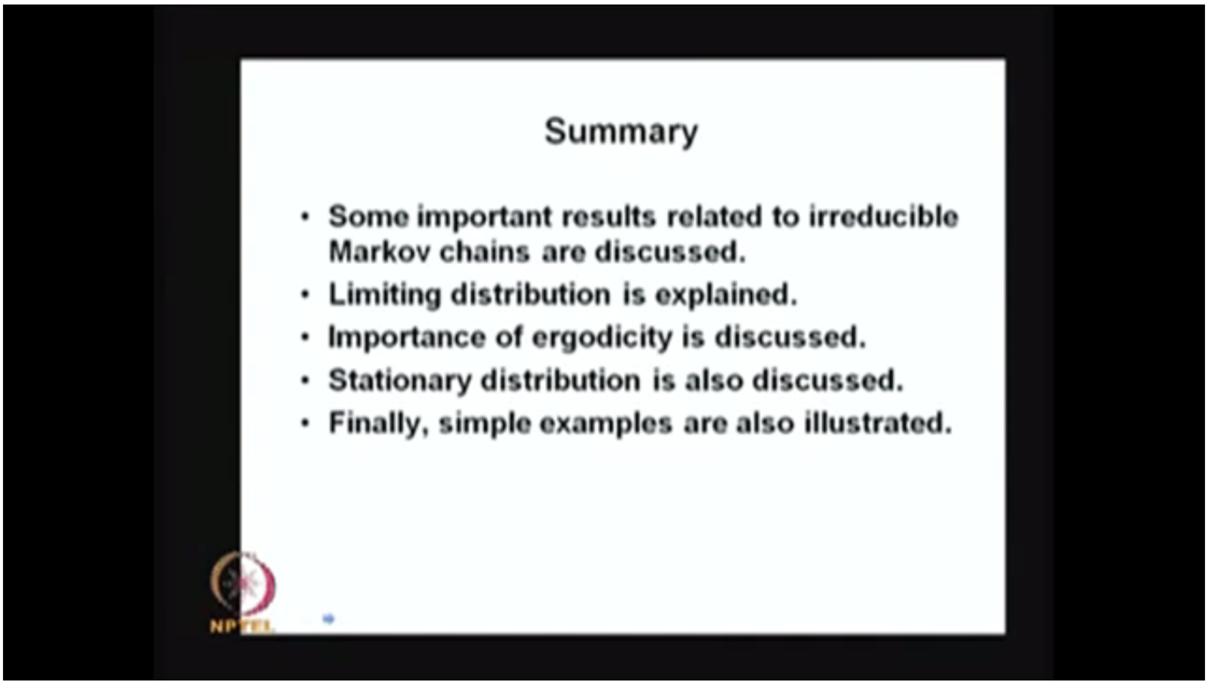


So in this talk, we have discussed some important results for the irreducible Markov chain. Then I have discussed what is the meaning of a limiting distribution and I have given one example of how to compute the limiting state probabilities. Then I discussed the ergodicity.

Then I have discussed the stationary distribution and how to compute the stationary distributions for a irreducible, aperiodic, a positive, recurrent, whether it is a finite state or infinite state Markov chain. I have given few examples and I have given an example in which the stationary distribution exists whereas the limiting distribution does not exist, and I have given some examples also. With this, I complete today's lecture. Thanks.

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