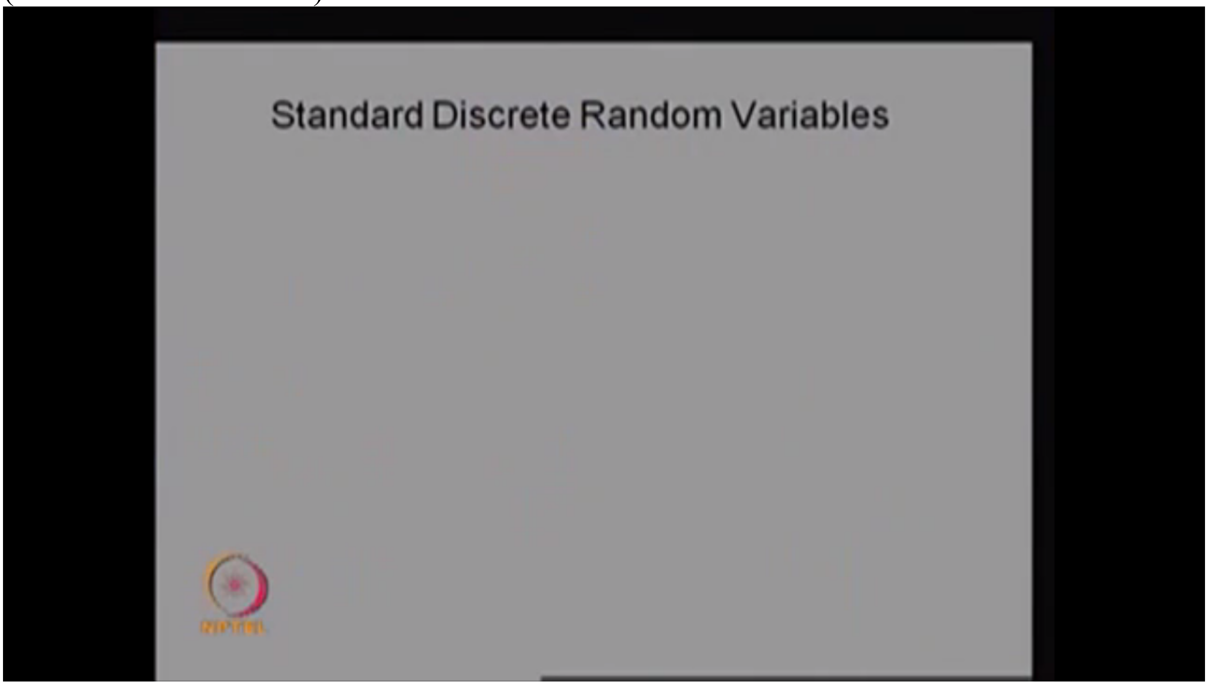
Discrete Uniform Distribution

Dr. S Dharmaraja
Department of Mathematics, IIT Delhi

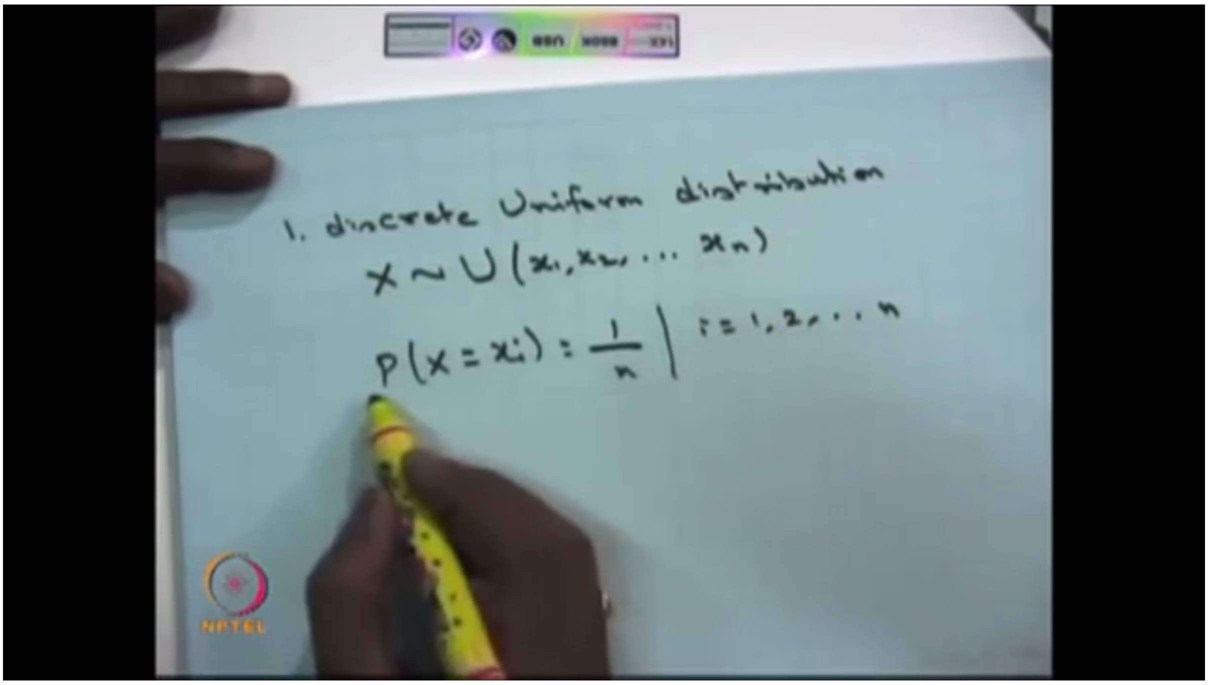
(Refer Slide Time 00:02)



So here I am going to list out a few standard discrete and continuous random variable. So these are all the standard one we are going to use it in our course. So the first one is discrete uniform distribution or the random variable is a discrete uniform distributed random variable.

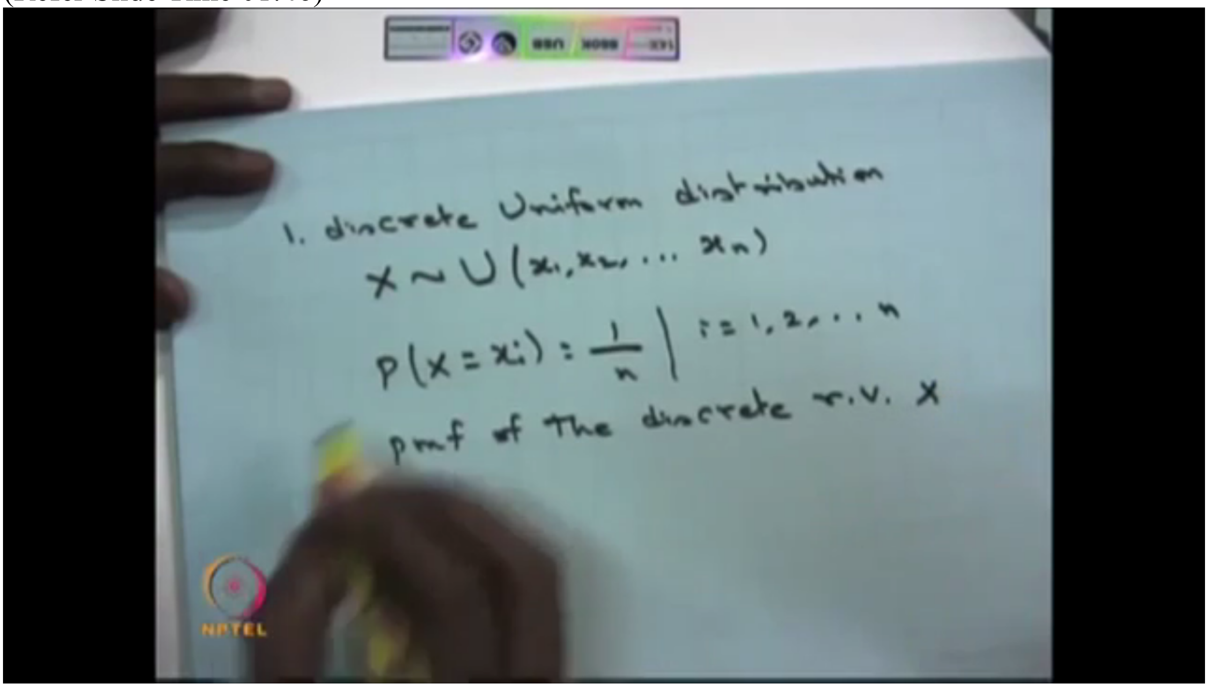
Suppose I make the random variable X is a uniformly -- discrete uniformly distributed with the -- with the discrete points x_n. That means that the random variable takes the possible values x_1 to x_n and it has the masses at the x_i's of equal mass for i vary from 1 to n and all otherwise it is going to be 0. Then in that case we say the random variable is going to be call it as a discrete uniform distribution.

(Refer Slide Time 01:10)



That means it is going to satisfies the property the summation of all the x_i's as going to be 1 and the probability of X equal to x_i is going to be greater than or equal to 0. That means for this x_i it is going to be greater than 0 and all other points it is going to be 0. Therefore, it is satisfying the probability mass function of the discrete random variable. Therefore, this is the probability mass function of the random variable of the discrete random variable X.

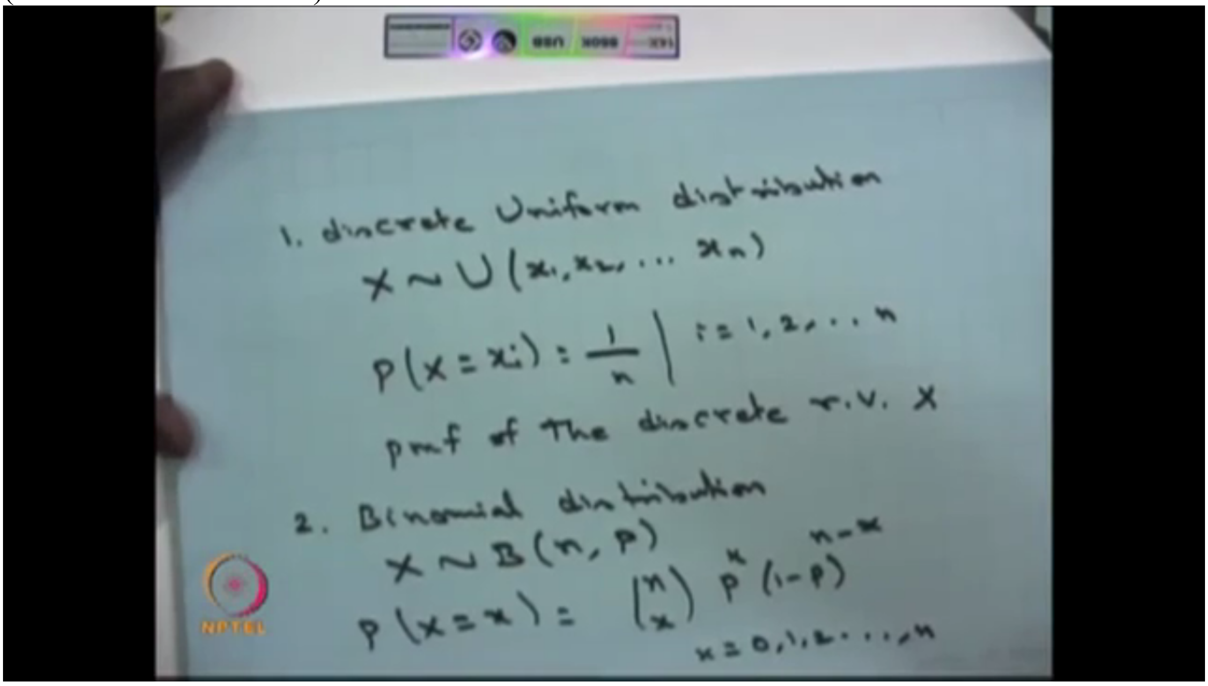
(Refer Slide Time 01:46)



So the P, probability of X equal to x_i is going to be the probability mass function of the discrete random variable x_i.

The second one, the discrete case that is a binomial distribution. When we say the random variable X is going to be call it as a binomially distributed with the parameters n and p, then the probability mass function for the random variable is going to be n C x P power x (1 minus P) power n minus x where x takes the value from 0, 1, 2 and so on.

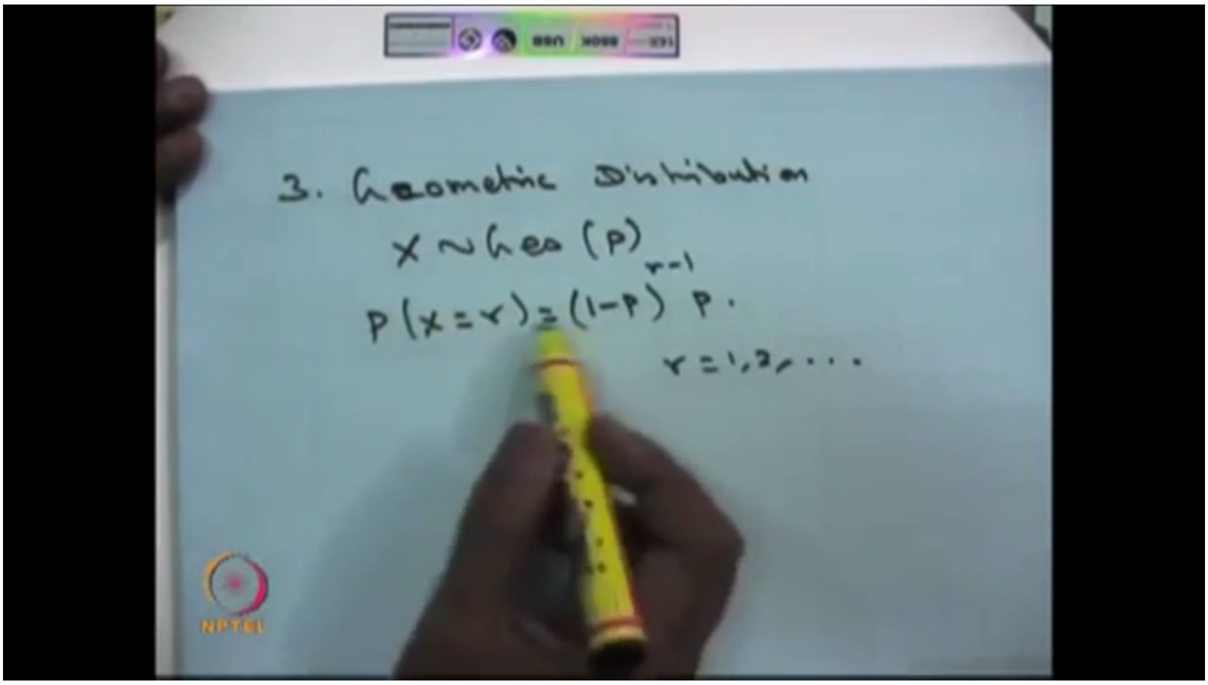
(Refer Slide Time 02:34)



That means this is the probability mass function of a binomial distribution. It takes a value 0 to n. That means it has the jump points n plus 1 points. It has the jump points n plus 1 jump points and this we call it as a binomial distribution. If you put n is equal to 1, then that is going to be the Bernoulli distribution random variable and here the P is nothing but the probability of success in each trial. And you can create the binomial trials by having a n independent Bernoulli trials and each trial, the probability of success is going to be P.

The third discrete random variable, which we are going to use, that is a geometric distribution. When we say the random variable X is geometrically distributed with the parameter P, then the probability mass function of this random variable is going to be 1 minus P power R minus 1 into P where r can take the value from 1, 2 and so on.

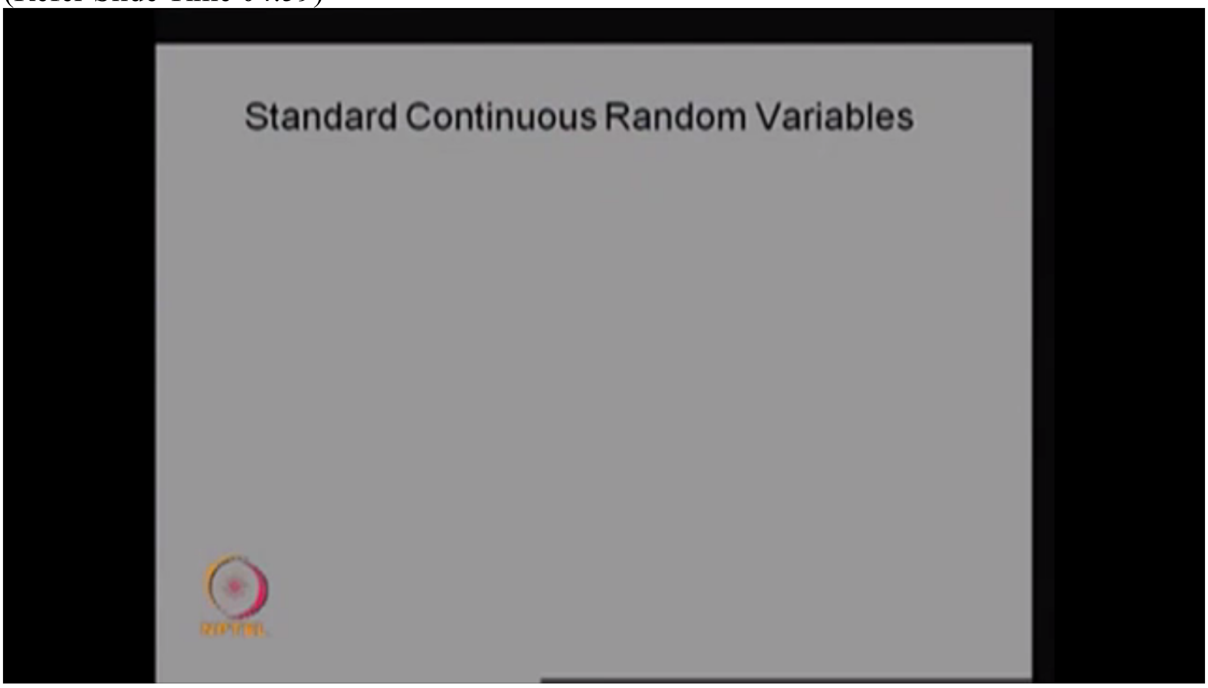
(Refer Slide Time 03:57)



That means if you have any discrete random variable and that random variable probability mass function is going to be of this form, then we say that random variable is geometrically distributed with the parameter P and here the P can be treated as the probability of success in each trial and you can say, what is the probability that the r^{th} trial getting the first success? That is same as all the trials are independent. Therefore, you have a r minus 1 trials. You have the success subsequently, failures subsequently and you get the success of first time in the r^{th} trial. Therefore, you land up 1 minus P power r minus 1 for all such failure or all such non-success r minus 1 trials and first success in the r^{th} trial.

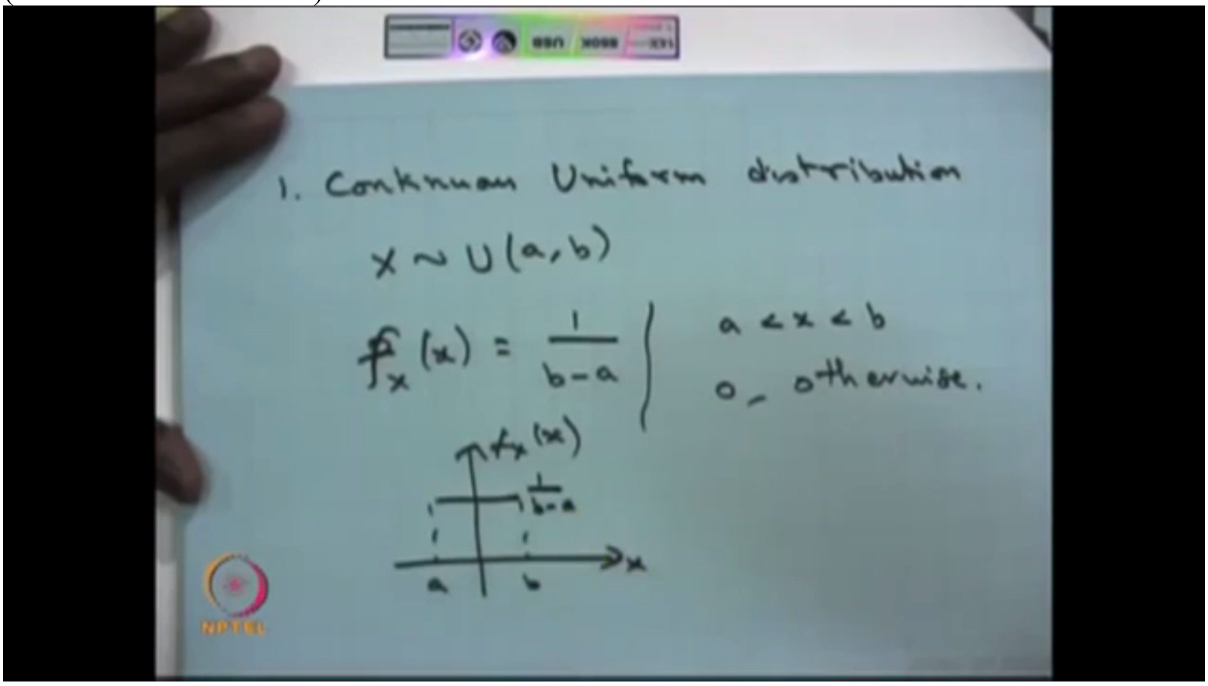
Next we are moving into the discrete continuous random variables.

(Refer Slide Time 04:59)



The first one is a continuous uniform distribution. When we say the random variable X is continuous uniform distribution between the interval a to b, then the probability density function for the random variable x is going to be of the form 1 divided by b minus a between the interval a to b and all other it is going to be 0. That means the probability density function for this random variable is have the height a, and if you treated this as b and this height is 1 divided by b minus a.

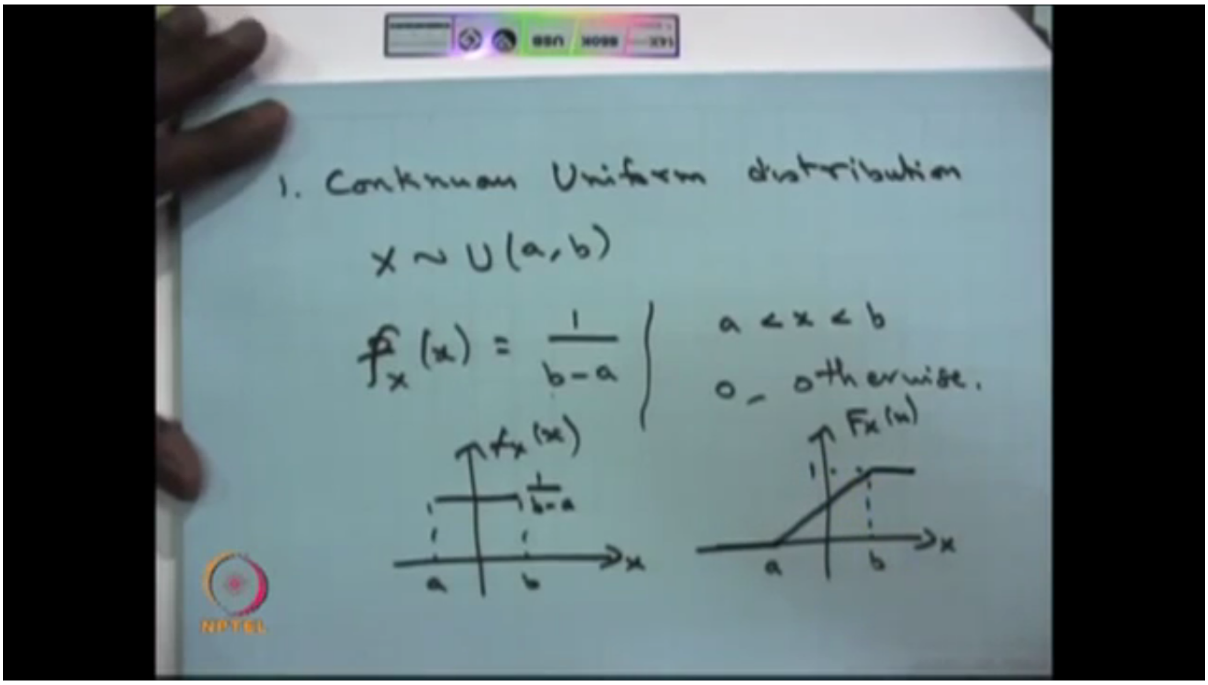
(Refer Slide Time 06:01)



That means if you find out the integration between the range a to b of height 1 divided by b minus a, then that is going to be 1 and this is going to be greater than or equal to 0 always. Therefore, this is going to be the probability density function of the continuous random variable and for any continuous random variable, the probability density function is going to be 1 divided by length of the interval in which it takes the value 1 divided by this much and all other it is 0. Then that random variable is going to be call it as a continuous uniform distribution between the interval a to b.

And if you see the CDF of this random variable, till a it is going to be 0 and after a, it is going to be increasing and at the point b, it reaches 1.

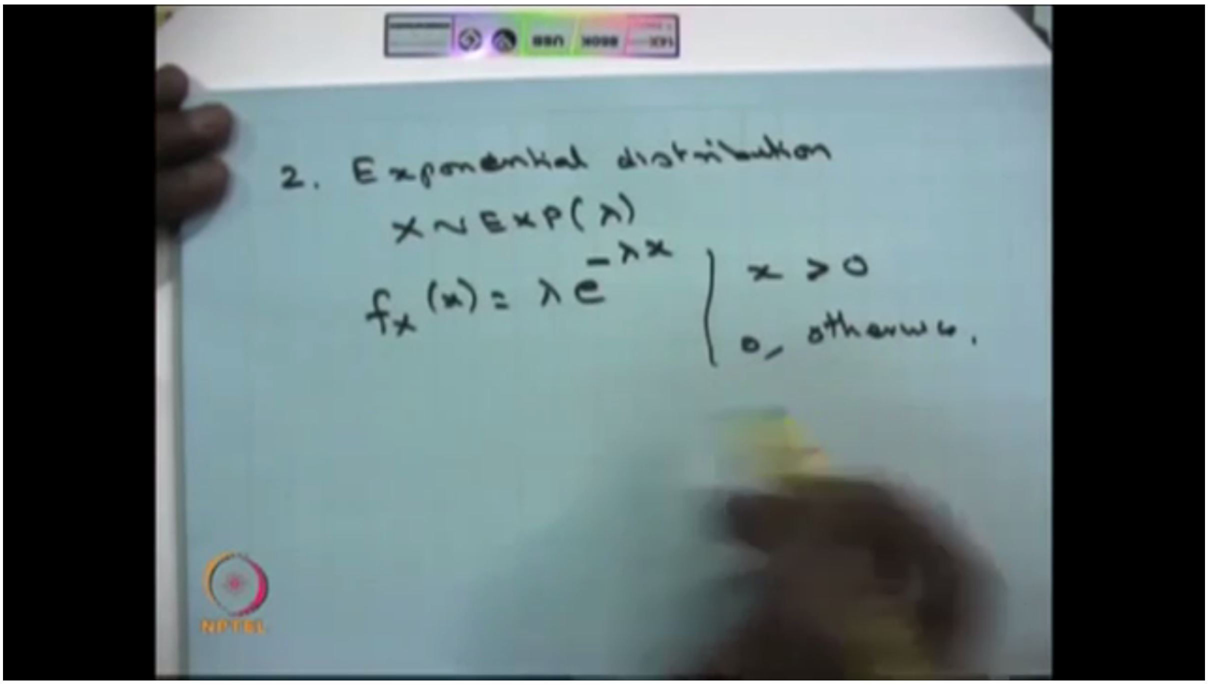
(Refer Slide Time 006:59)



That means you can come to the conclusion if any random variable CDF is going to be between 0 to 1 in the interval a to b with this standing line, then you can come out what is a point in which a and b and you can come to the -- find out what is the random variable in which it is going to be a continuous -- it is going to be a uniform distribution between the interval a to b.

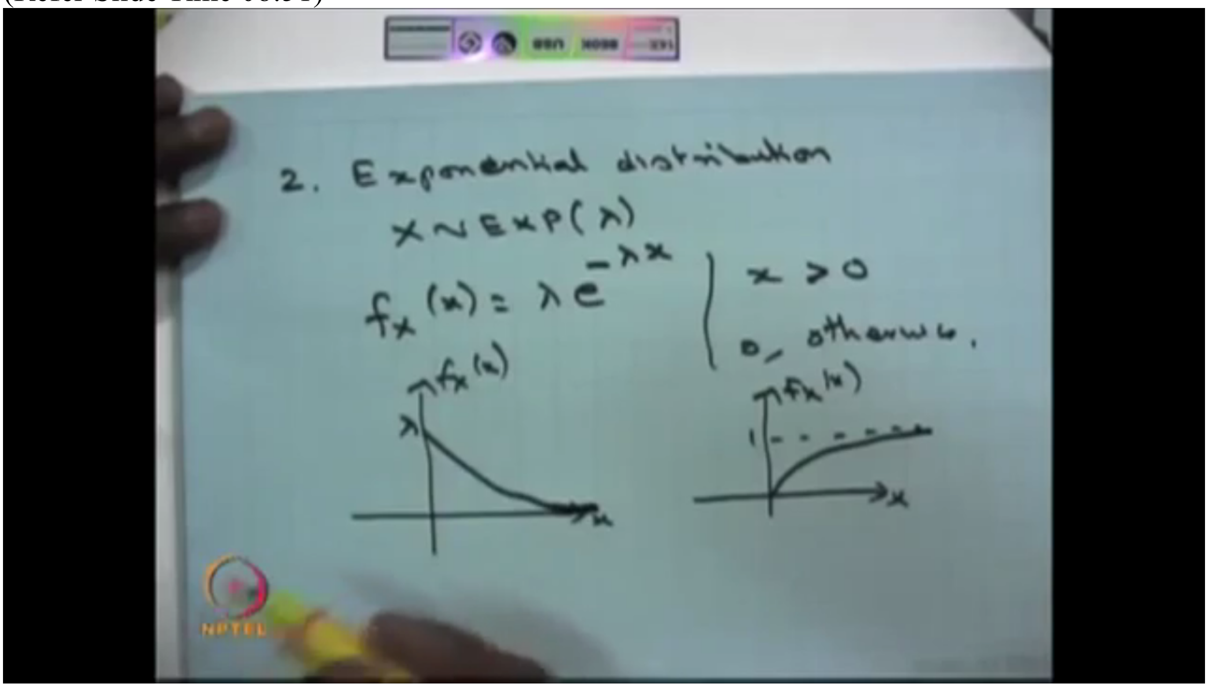
The second one is exponential distribution. When we say the continuous random variable X is going to be exponentially distributed with the parameter lambda, if the probability density function for that random variable is going to be lambda times e power minus lambda x and between the x is going to be greater than 0 or if it is going -- it is going to be 0 otherwise. That means within the range of 0 to infinity, the f of x is going to be lambda times e power minus lambda x. Otherwise, it is going to be zero.

(Refer Slide Time 08:16)



So if you see the probability density function of that continuous random variable, it is going to start from lambda and asymptotically it touches 0. So this is the probability density function of the exponential distribution and if you see the CDF of this, it reaches 1 at infinity.

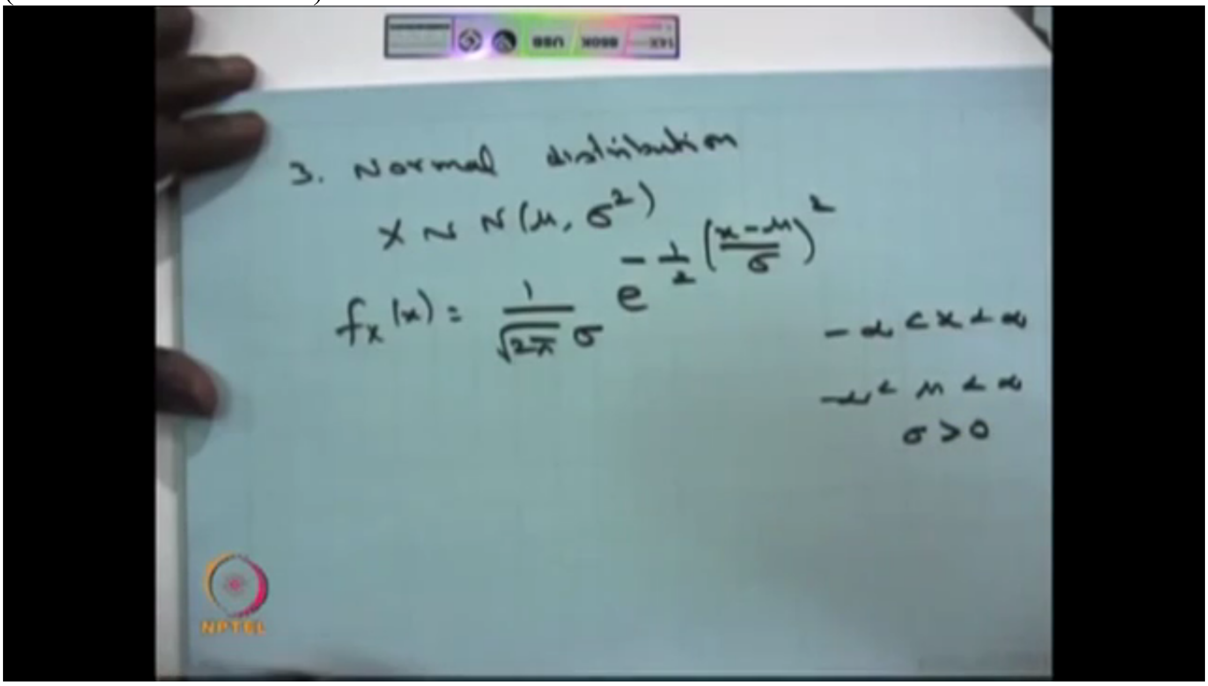
(Refer Slide Time 08:51)



So this exponential distribution is going to be used in many of our problems later. Therefore, all the properties of the exponential distribution that I will discuss when we come -- when we discuss a stochastic process in detail.

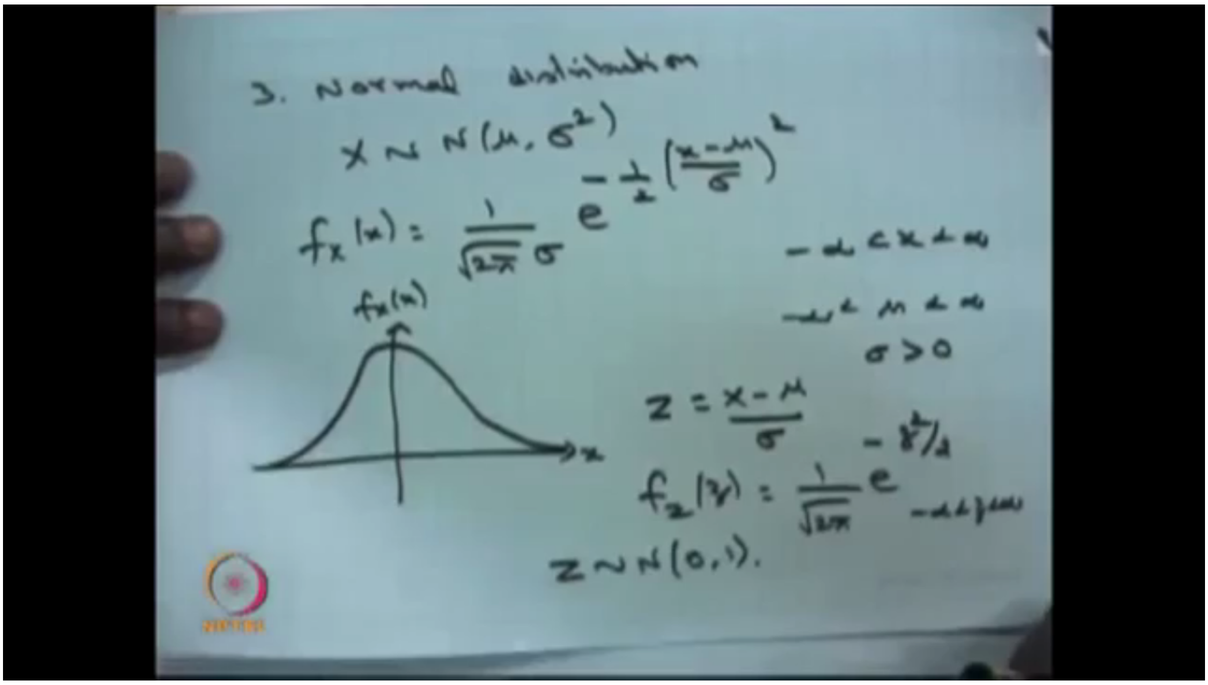
The third distribution is a normal distribution or Gaussian distribution. So when we say the random variable is normally distributed with the parameters mu and sigma square, the probability density function is going to be 1 divided by square root of 2 Pi sigma e power minus half times x minus mu by sigma whole square. Here the x can lie between minus infinity to infinity, and the mu also can lie between minus infinity to infinity and the sigma is a strictly positive quantity.

(Refer Slide Time 09:49)



And the mu is nothing but the mean of a normal distribution and the sigma square is the variance of normal distribution, and the sigma is the standard deviation, and the standard deviation is always strictly greater than zero. And if you see the probability density function of f of x, asymptotically, it start. So I made it with the mu is equal to 0 and this is the probability density. So it looks like a bell shape. So this is going to be a normal distribution and you can always convert the normal distribution into the standard normal by using this substitution Z is equal to x minus mu by sigma. So you land up with the standard normal one that is 1 divided by square root of 2 Pi e power minus z square by 2 where z lies between minus infinity to infinity.

(Refer Slide Time 10:55)

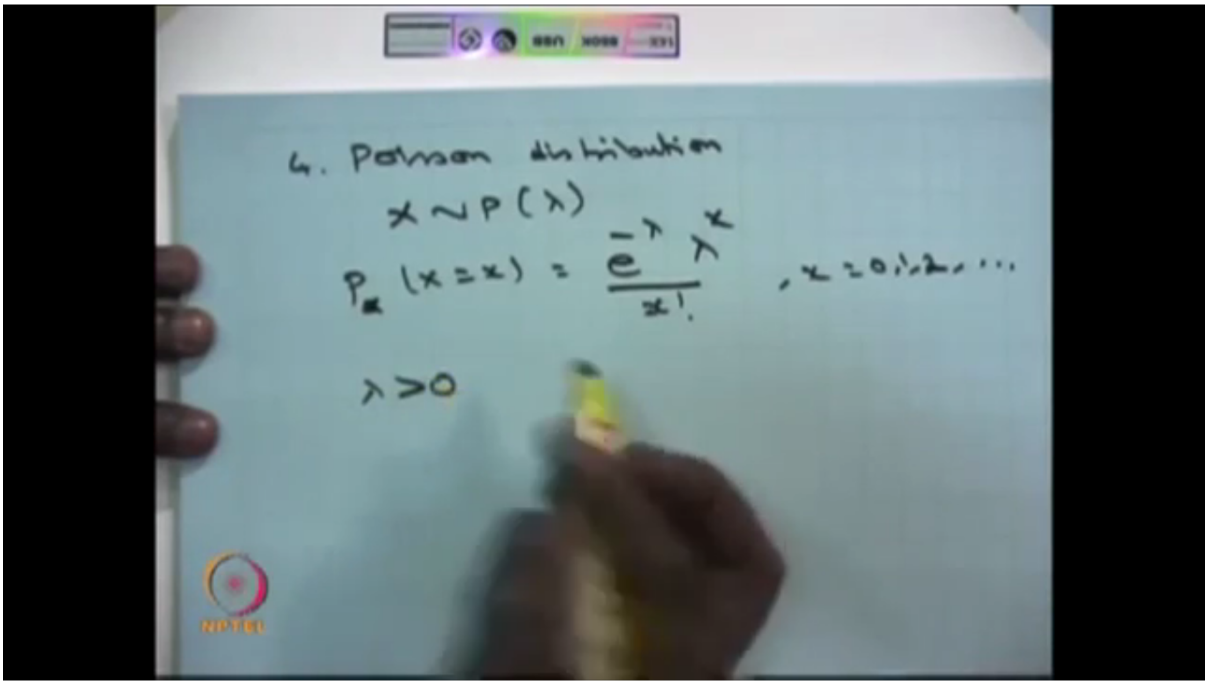


So this is going to be a standard normal distribution in which the mean is 0 and the variance is 1.

So other than the discrete standard distributions, we have discussed only the discrete uniform distribution. Then second we discussed the binomial distribution. Then we discussed a geometric distribution. The fourth one that is a very important that is a Poisson distribution.

When we say the discrete random variable X is going to be a Poisson distribution with the parameter lambda, if the probability mass function for the random variable x is going to be of the form e power minus lambda lambda power x divided by x factorial where x can take the value from 0, 1, 2 and so on, so that means this is the discrete type random variable in which it has the countably infinite masses and these are all the jump points and the masses are going to be e power minus lambda lambda power x by x factorial. Here the lambda is strictly greater than 0.

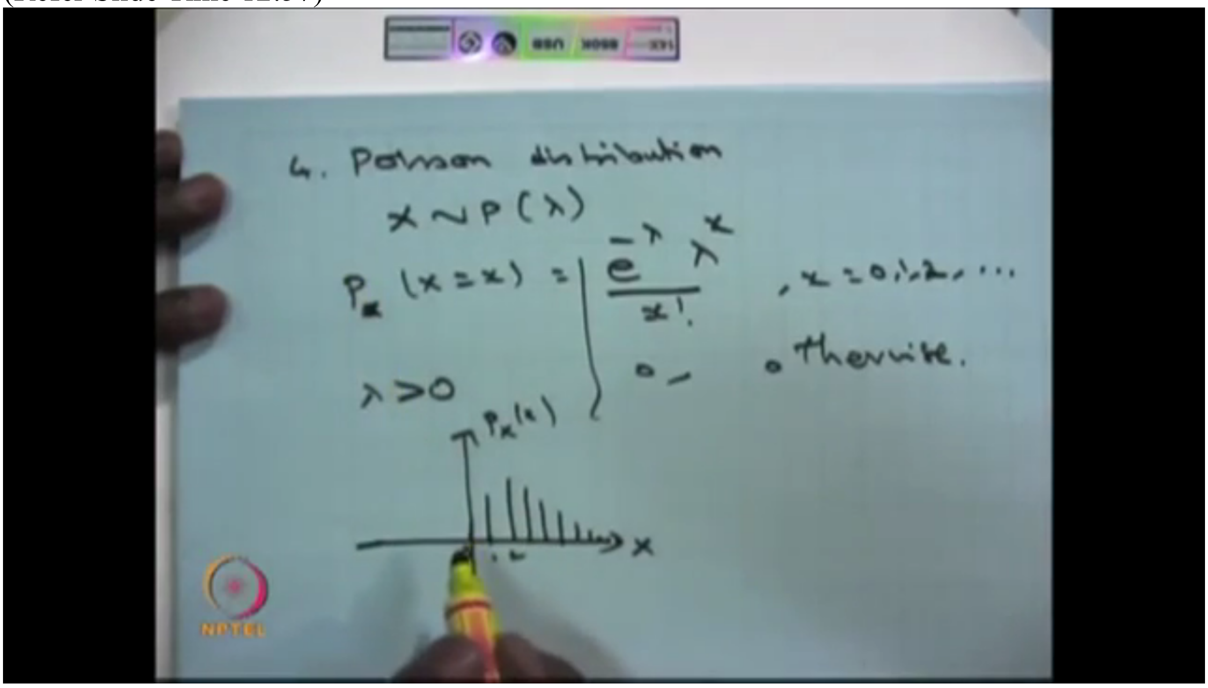
(Refer Slide Time 12:06)



That means if any discrete random variable has the probability mass function of this form, then we can say that that random variable is a Poisson distributed with the parameter lambda.

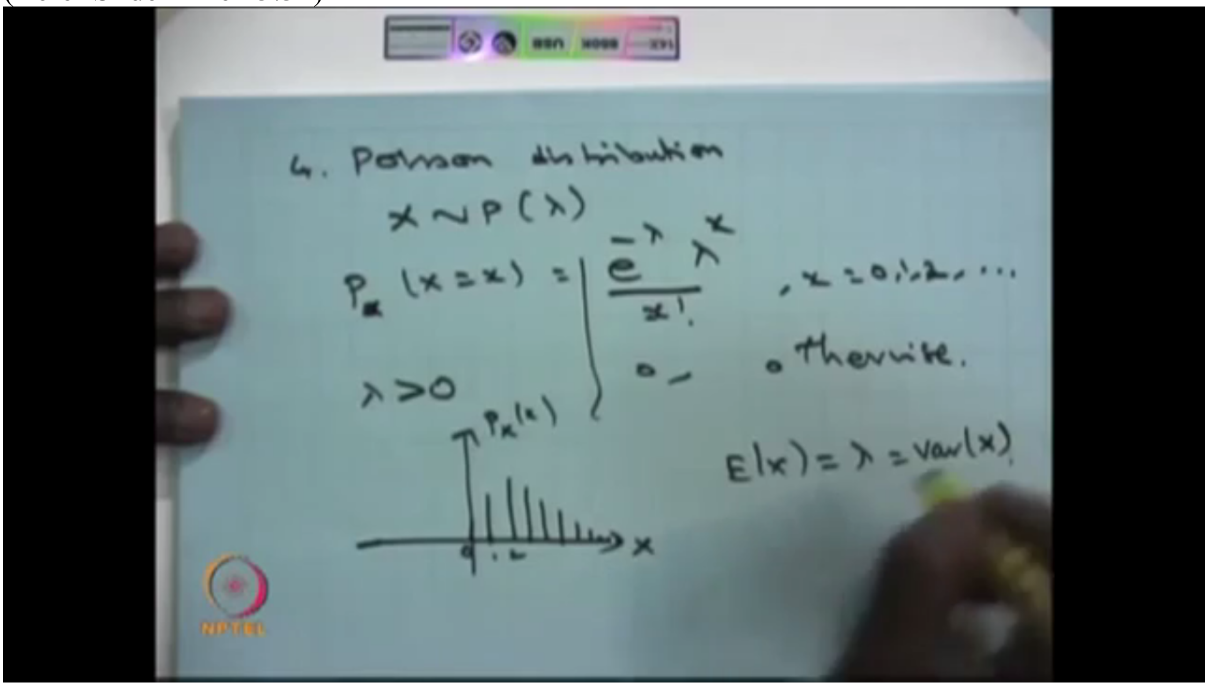
And if you see the probability mass function for the different values of x, so whatever the lambda you have chosen, so accordingly it is going to be at 0 it has some value and 1 it has some other value and 2 and so on. So that means for fixed lambda, you can just draw the probability mass function and this is going to be have a countably infinite mass. And if you add over the 0 to infinity, that is going to be 1, and the masses are going to be always greater than 0 and all other points it is going to be 0.

(Refer Slide Time 12:57)



And this is going to be the very important distribution because using this we are going to create a one stochastic process that is going to be call it as a Poisson process. That means in the Poisson process, the each random variable is going to be Poisson distributed. So for that we should know what is the probability mass function of the Poisson distribution and the properties, and here the lambda is same as if you find out the mean for this Poisson distribution, the mean is going to be lambda and the variance is also going to be lambda.

(Refer Slide Time 13:32)



So this is a one particular distribution in which the mean variance is going to be same as the parameter lambda.

So in today's lecture, what we have covered, introduction of a stochastic process by giving the motivation of the -- by giving four different examples to motivate the stochastic process. Then what we have covered is what is the probability theory knowledge is needed. In that I have covered only the probability space, and the random variable, and the discrete standard random variables as well as standard continuous random variables.

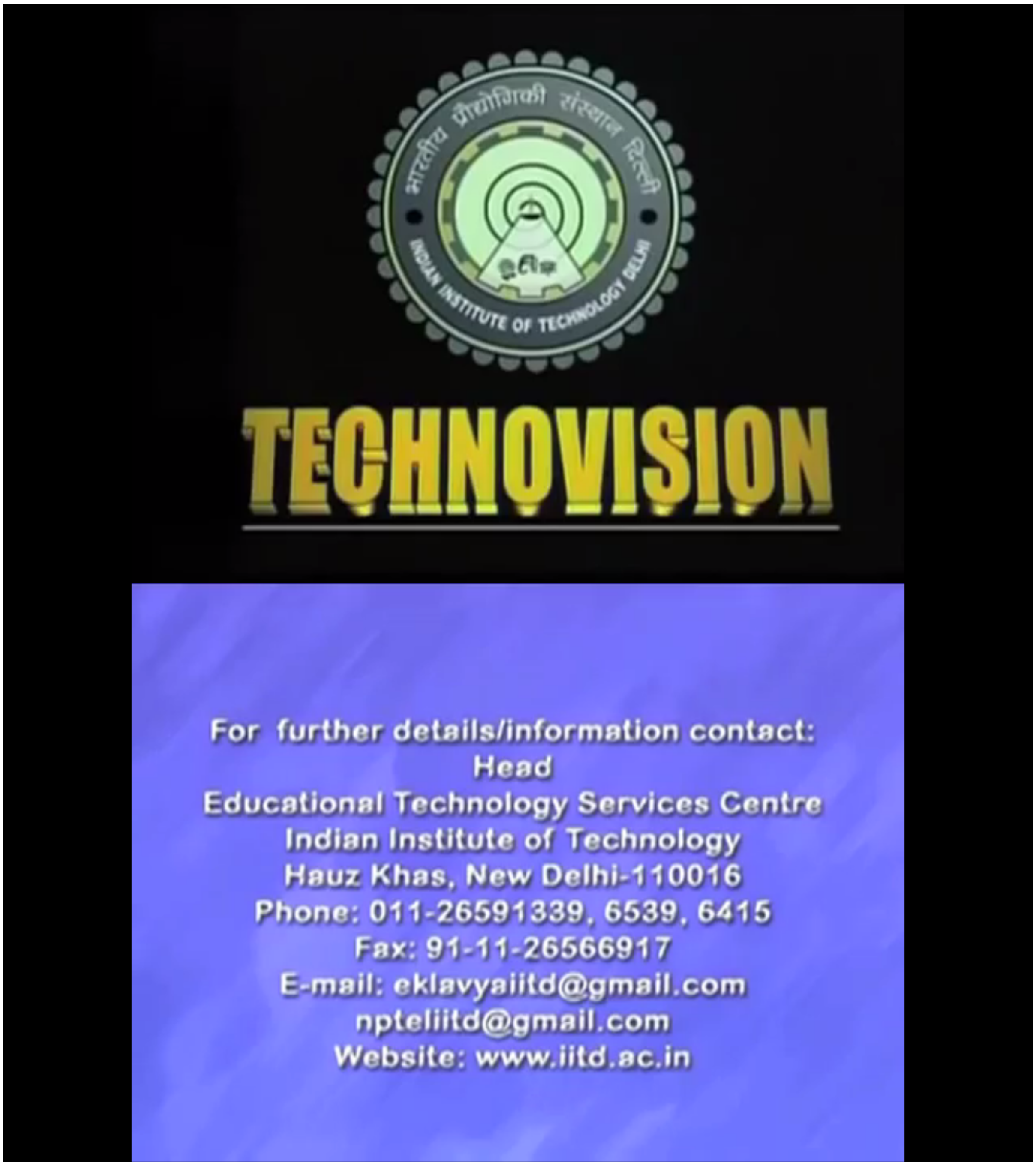
There are some more standard discrete random variables as well as there are some more discrete, there are some more standard continuous random variable that I have not covered here because it is a probability theory refresher and some of the distribution if it is needed, then we will be covered at the time of when we explain the stochastic process itself.

Therefore, with giving a few standard discrete random variable and few standard continuous random variable, I'll complete today's lecture and the next lecture I will cover some of the other probability theory concepts needed for the stochastic process that I'll cover it in the next lecture. Then the third lecture onwards, I will start the stochastic process.

Thank you.

INDIAN INSTITUTE OF TECHNOLOGY DELHI

TECHNOVISION




Produced by
Educational Technology
Services Centre
IIT Delhi