Example of Classification of states (Contd.)

Dr. S Dharmaraja
Department of Mathematics, IIT Delhi

I'm going to give you the next example that is Example 5, which has the finite states, states 0, 1, 2, 3, 4 and there is one-step transition probabilities that is one-third, two-third and for the state 1 with the probability 1, it moves to the state 2 and for the state 2 with the probability 1, it moves to the state 0. For the state 3 with the probability 1/2, it goes to the state 4. With the probability 1/2, it goes to the state 1. For the state 4, it is with the probability 1/2, it goes to the state 1/2, state 3 with the probability 1/2, it goes to the state 2.

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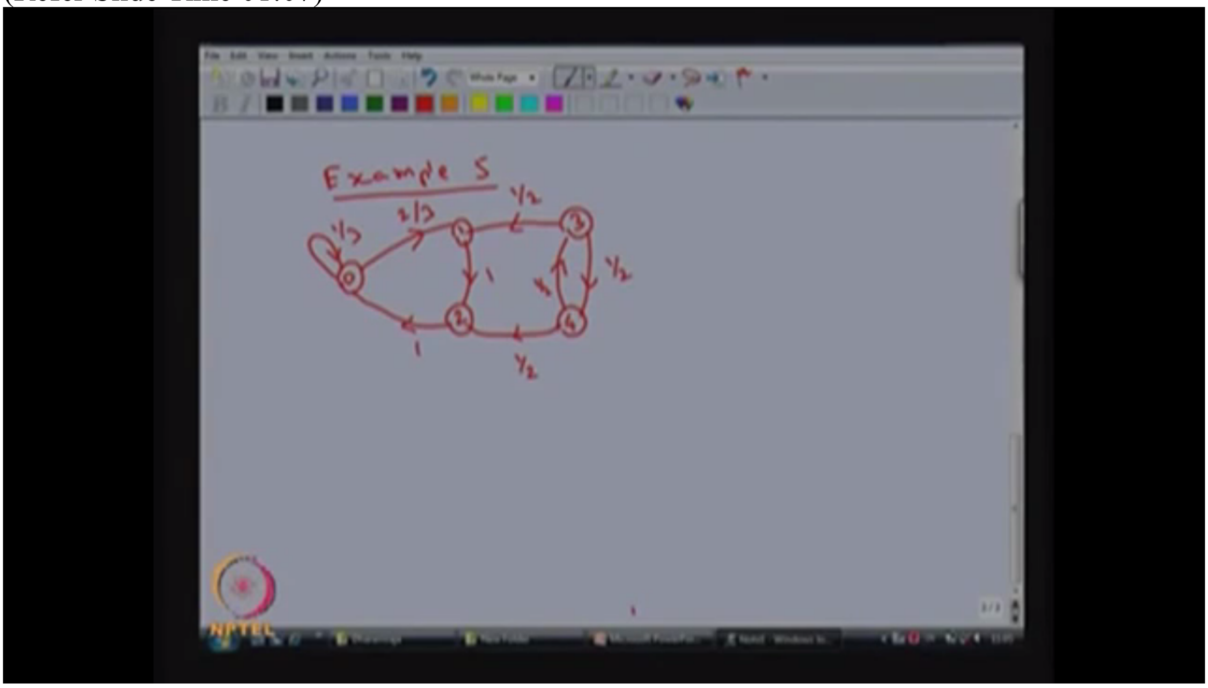

The way I have drawn the state transition diagram by taking care the row sum is going to be 1, so you can equivalently have a one-step transition probability matrix also. So here I have only the state transition diagram for this DTMC. From this diagram, either by calculating f_{ii} and the capital F_{ii}, you can conclude it is going to be a recurrent state or transient state. Then you can conclude whether it is going to be a positive recurrent or null recurrent, but whenever the Markov chain is going to be finite, without doing the calculation, from the diagram you can conclude these states are going to be a positive recurrent and these states are going to be transient states. So that I am going to do, but the same exercise you can do it and get the result also.

The way the arcs are here, if you see the state 3 and 4, states 3 and 4, it has the only outgoing arc to the states 1 and 2 whereas 0, 1 and 2 has a loop form and the state 0 has the self-loop with the probability 1/3. Sometimes if the outgoing arcs, the probabilities are not going to be 1, that summation. That means you can make out the self-loop as the probability 1 minus of all the outgoing arcs. But that's a default scenario, but always we should draw the correct state transition diagram. If it has some positive probability with the self-loop, we should always draw the self-loop with the positive probability. That's a correct way of drawing the state transition diagram.

So now you can make out the state 0, 1 and 2 are forming some sort of loop. That means if the system start from the state 0 or 1 or 2, it will be only within these three states over the number of steps. Even for a longer run, the system will be any one of these three states only.

So these three states will be communicating each other, not communicating with the states 3 and 4 whereas there is accessible from the state 3 to 1, but there is no accessible from 1 to 3. Therefore, 1 and 3 are not communicating states. Similarly 2 and 4 are not communicating states because one side accessible is here, not the other side accessible.

Therefore, you can make a set 0, 1 and 2. You cannot include any more states to form a set and this set satisfying the property closed as well as the communicating, so this -- this set is called closed communicating class and all these three states are communicating each other.

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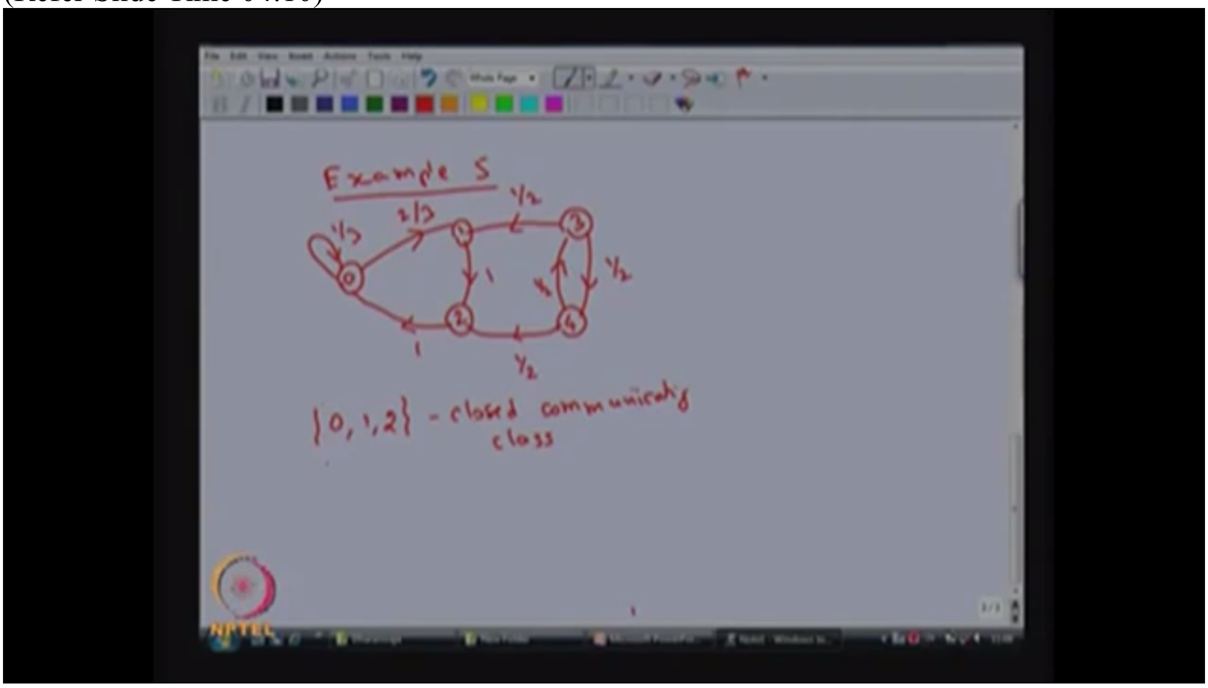

And if you find out F_{00}, F_{11}, F_{22} and so on, you come to the conclusion that value is going to be 1 and all these three states are going to be a positive recurrent states whereas the states 3 and 4, if the system starts from the state 3 or 4, it has the loop structure with the probability 1/2, but with the probability 1/2, it can go to the state 2 or it can go to the state 1 via state 3 or via state 4 accordingly, then land up. The system is not coming back to the state 3 or 4. Once it is going away from the state 3 and 4 starting from this -- these states, it is not coming back. Therefore, these states 3 and 4 will form a transient states.

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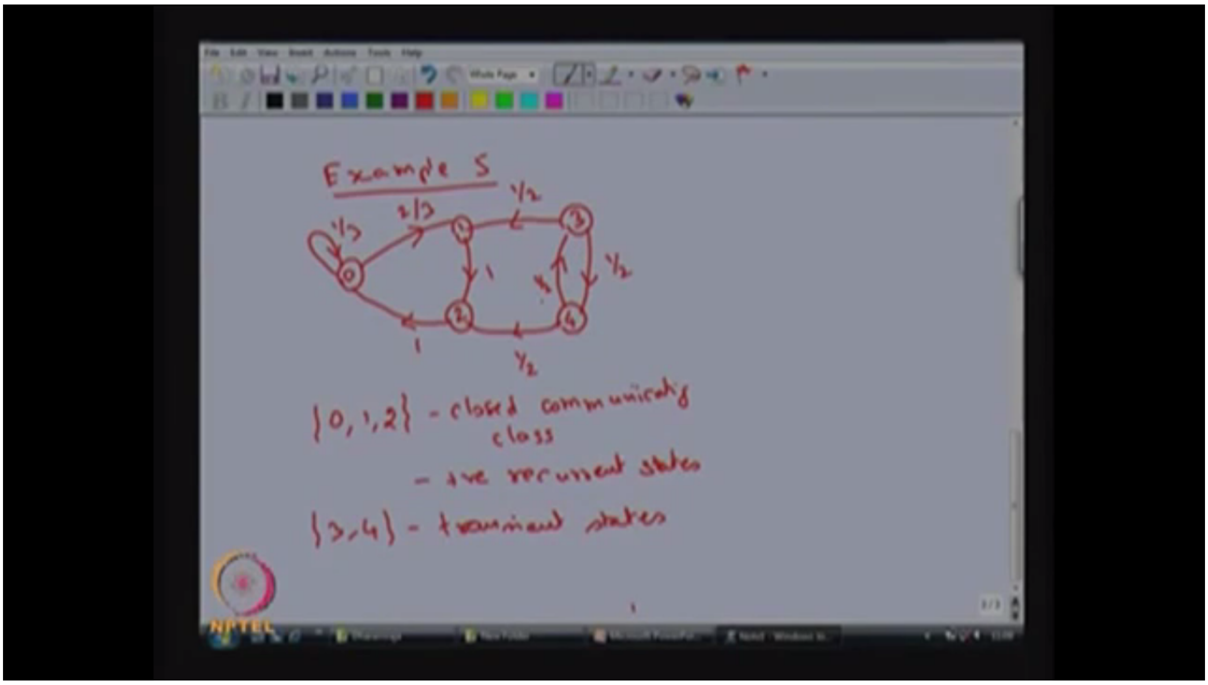

Even though the state 3 and 4 are communicating each other, even though the state 3 and 4 are communicating each other, these states will form a transient state because F_{33} and F_{44}, it is going to be less than 1.

And if you try to find out the periodicity of this state, you can find out the periodicity of any one state. Then that is going to be the period for all other states in the same class. Therefore, if you find out the periodicity of the state 0 that is d0, that is the greatest common divisor of what are all the steps the system will be come back to the same state. So either it can take one step or 1, 2, 3, 4 or it can take only three steps not making a self-loop 1, 2 and 3. So it can make a one or three steps or four steps or five steps. Five steps means it makes a two steps self-loop, then third step going from 0 to 1 and 1 to 2, then 2 to 0. Therefore, it's a five steps and so on. Therefore, the greatest common divisor is going to be 1.

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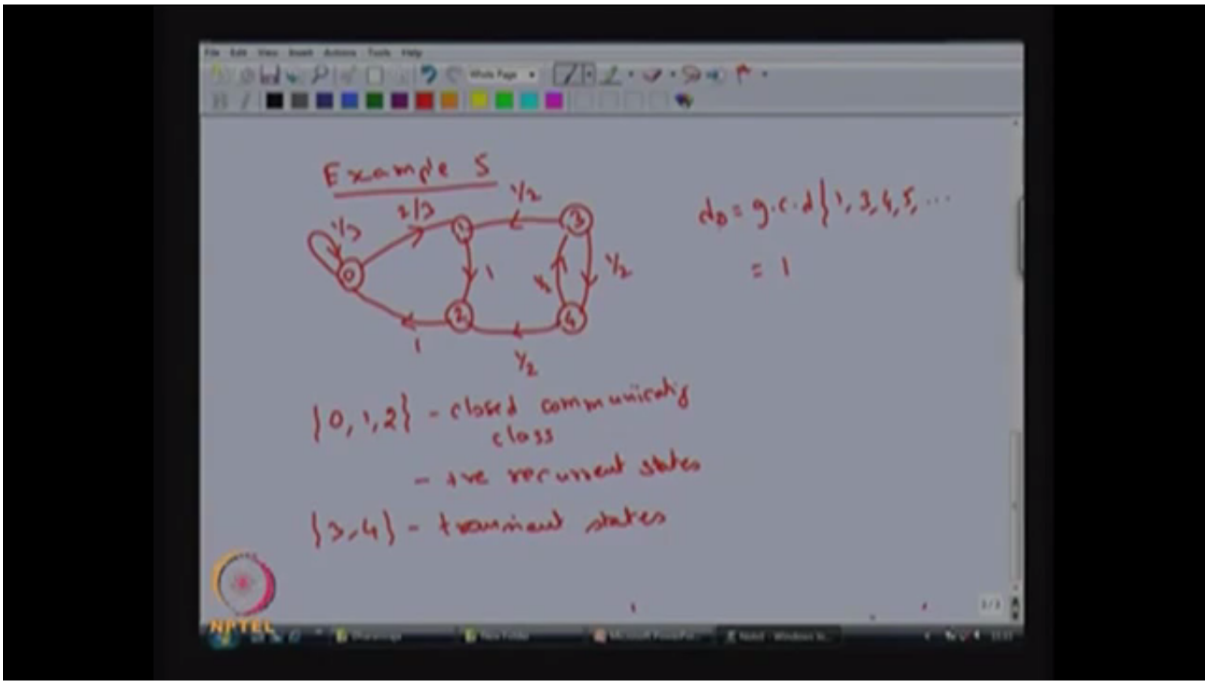

So since the period for the state 0 is going to be 1, this is going to be aperiodic state and all the states are going to be aperiodic states. Since these states are the positive recurrent aperiodic, these states are also going to be call it as an ergodic states. These states are going to be an ergodic states. Since the state-space S has the union of the closed communicating class and these are transient states, therefore, this is going to be a reducible Markov chain.

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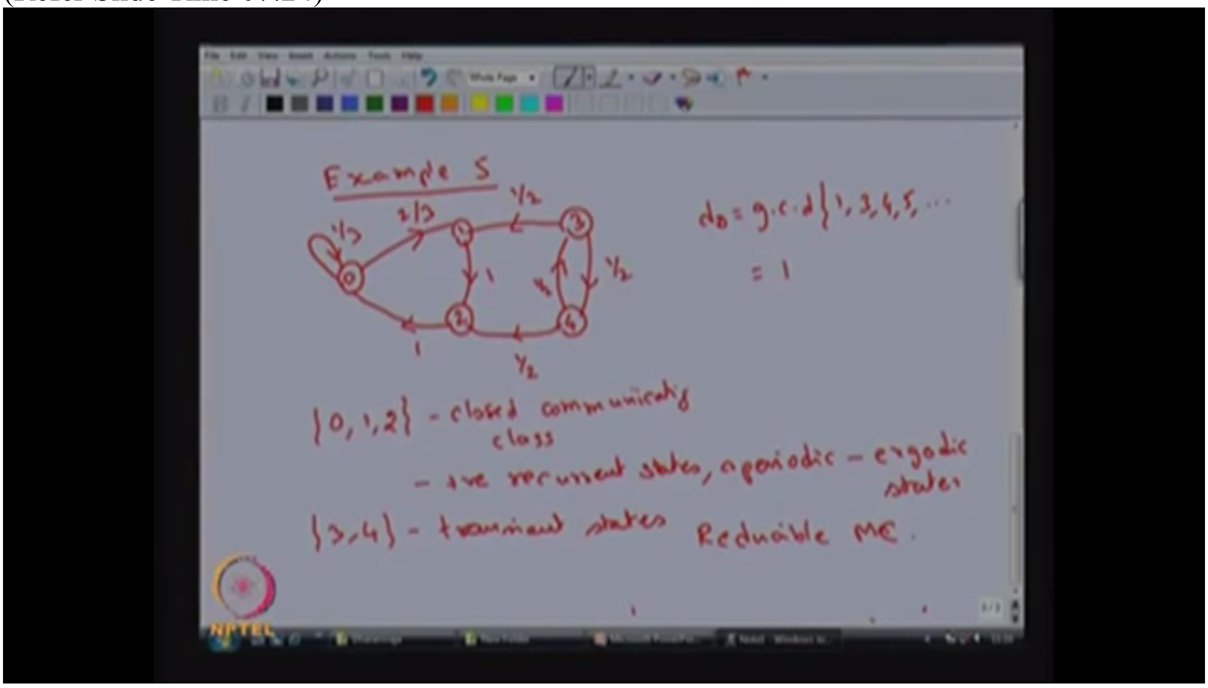

So in this example, we come to the conclusion, we have five states and this is going to be the reducible Markov chain because of the closed communicating class is consists of element 0, 1 and 2 and the transient states are 3 and 4.

Moving to the next example, Example 6. In this example, I have six states and the one-step transition probability values are 1/4 and 3/4 it goes to the state 4 and for the state 1, it is 1/2 and this is also 1/2. For the state 2 with the probability ¾, it goes to the state 5 and with the probability 1/4, it goes to the state 4 whereas for the state 3, there is nothing.

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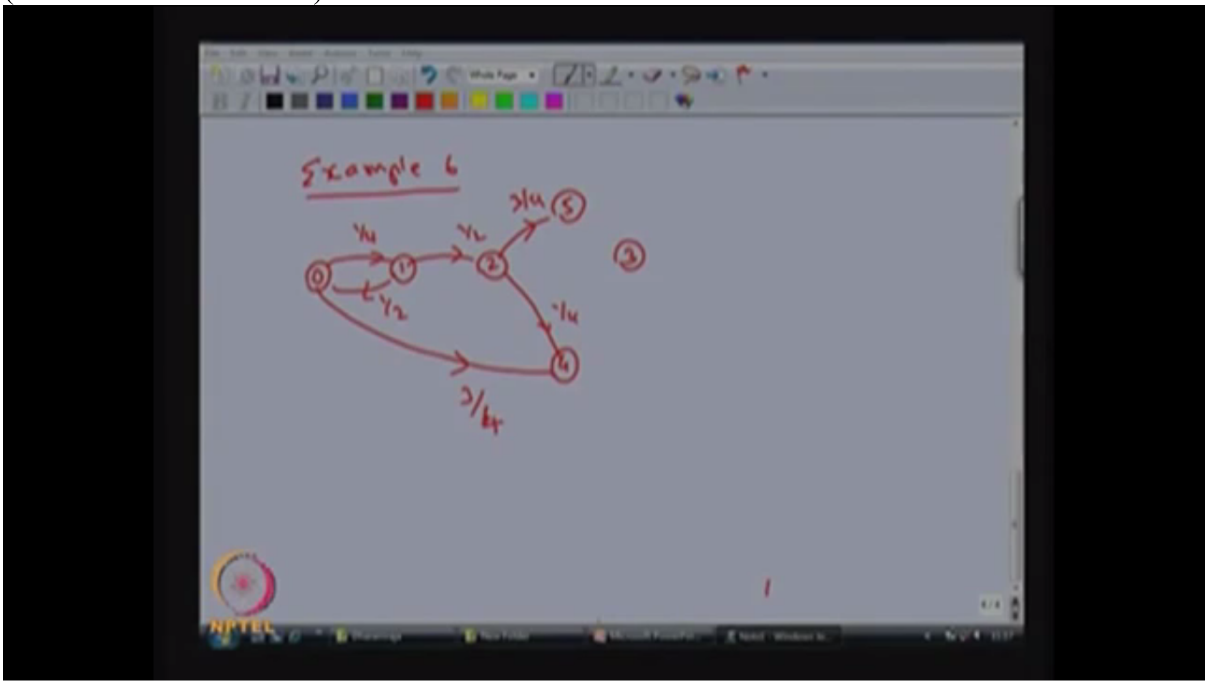

For the state 4, with the probability 1/3, it goes to the state 2 and 2/3, it goes to the state 3. Since there is no outgoing arc in the state 5 and 3, you can make out the self-loop has the probability 1 or you can draw also with the probability 1.

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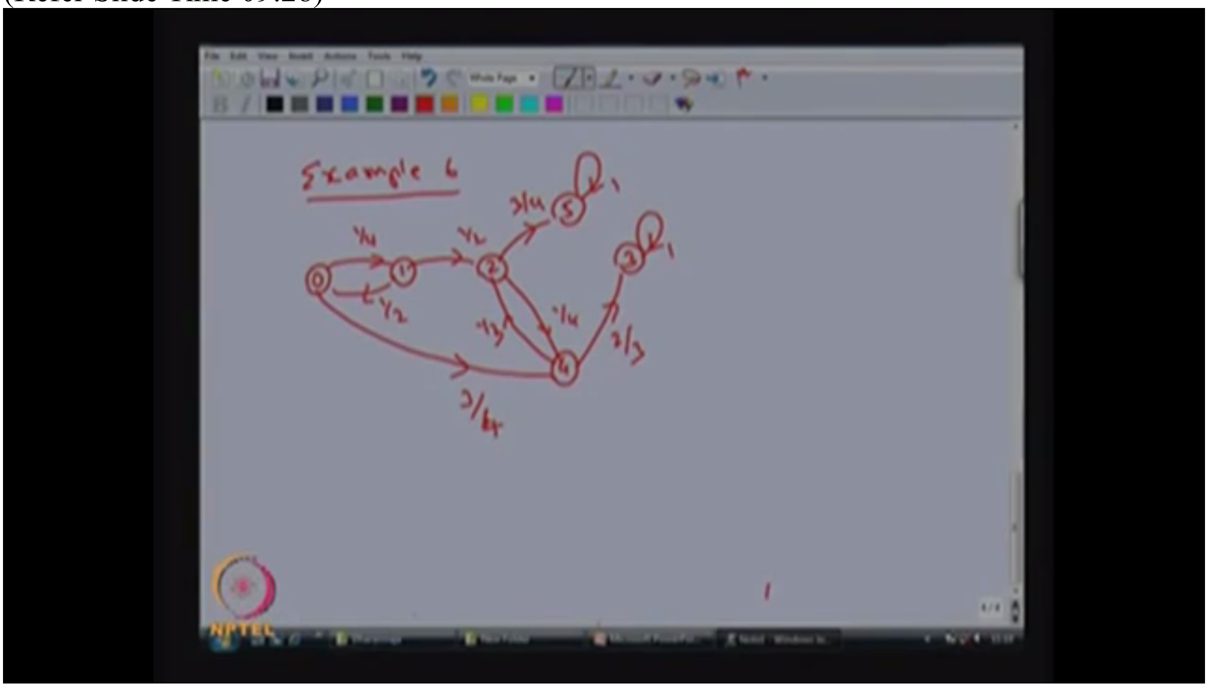

So now we can go for classifying these states because of the self-loop with the probability 1 for the state 3 and 5, you can directly make out the state 3 is going to be absorbing state and this is going to form a one class, C_1. This closed communicating class has only one element, which is state 3.

Similarly, I can go for the second class, which has the only one element that is state 5, that is also absorbing state. That is also absorbing state.

Now I can go for classifying the state 0, 1, 2 and 4 because it is a finite state discrete-time Markov chain, if the system starts from the state 3 or 5, it will be in the state 3 or 5 forever because both are absorbing state. If the system start from other than the state 3 or 5, ultimately it comes to the state 3 or 5 via these 2 to 5 or 4 to 3, then it won't be back. Therefore, all these states 0, 1, 2 and 4 will form a transient state. So this is a collection capital T, that is 0, 1, 2 and 4 are going to be form a transient states.

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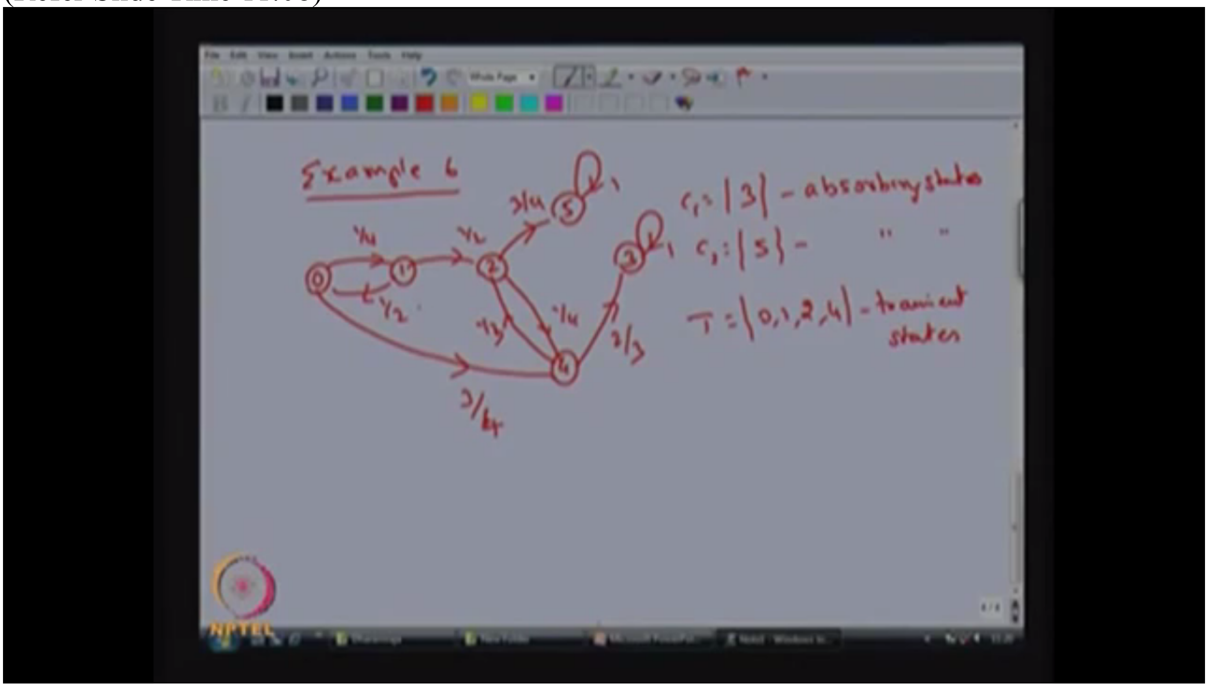

I have not computed what is F_{00} or F_{11}, F_{22}, F_{44}. Since it is a finite Markov chain and these two states are going to be absorbing state, whenever the system start from the state 0 or 1 or 2 or 4, either it will make a loop or ultimately land up to the state 5 or 3 with these arcs. Therefore, these three -- these four states are going to be a transient states and this will make a reducible Markov chain.

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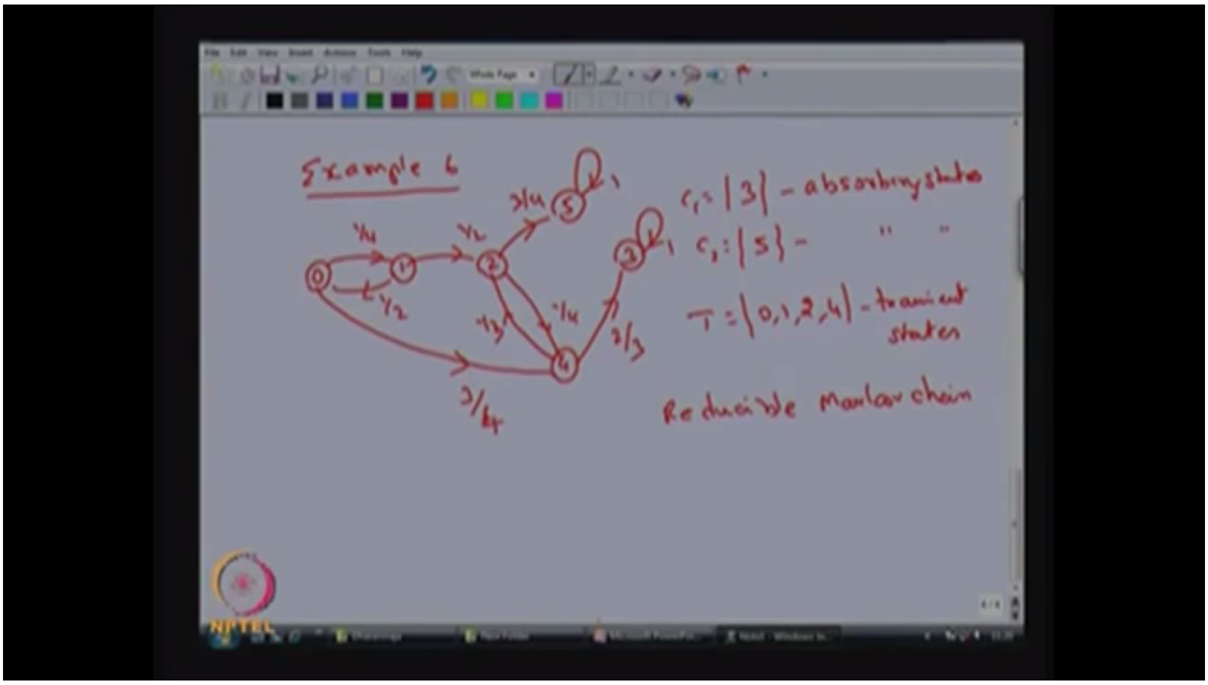

Suppose the system start from 0 or 1 with the arc 1 to 2 or 3 to 4, either the system can go -- either the system can go to the state 2 or state 4. If the system starts from the state 0 or 1, either the system can go to the state 2 via 1, 2 or state 4 via 0, 4. Then after that it will be keep roaming here 2 to -- 2 and 4, but with the positivity probability 3/4 and 2/3, it can go to the state 5 or state 3. Therefore, these states are going to be absorbing state. Therefore, ultimately, the system will land up the state 3 or 5. Therefore, these states are -- therefore, this is the one type of reducible Markov chain in which you have transient states and few absorbing states.

Now I'm moving into next example. This is another type of reducible Markov chain that is Example 7, which has the finite states, which has the finite N plus 1 states and the transitions are like this. With the probability 1 minus P, the system goes to the state 1 to 0. With the probability -- with the probability P, it can go to the state 1 to 2, and this probability is 1 minus P and so on. So all the forward arcs are P and the backward arcs are 1 minus P whereas here this is P and there is no forward arc. There is no forward arc. Therefore, the state 0 and N is going to be absorbing states.

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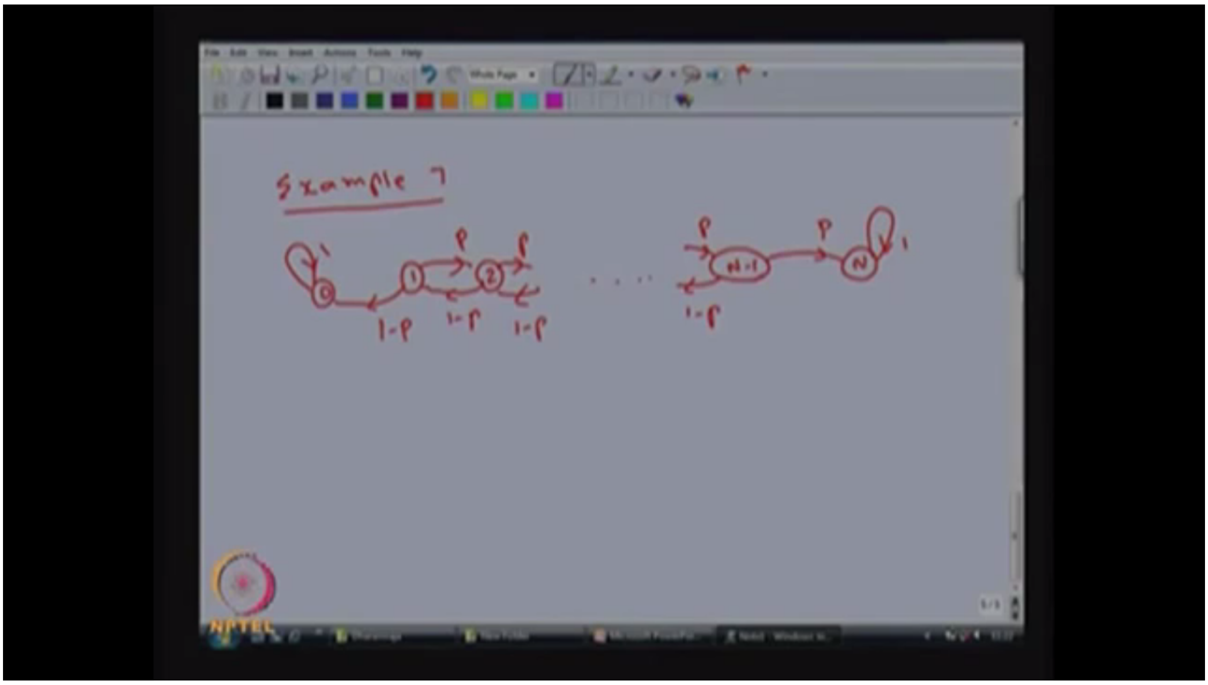

So here the P can lies between -- later I'm going to explain this same DTMC for the Gambler's Ruin Chain Problem and here we have N plus 1 states with the state as 0 and 1 are going to be -- 0 and 1 are going to be absorbing states. We usually write absorbing states individually because each one will form a closed communicating class, but here I have written both the states are absorbing states and all other states 1, 2 to till N minus 1, those will form a transient states because if the system starts from these states 1 to N minus 1, it can keep move between these states over the number of steps, but with the positive probability of 1 minus P, it can go to the state 0. With the positive probability of P, it can go to the state N in this group of transient states.

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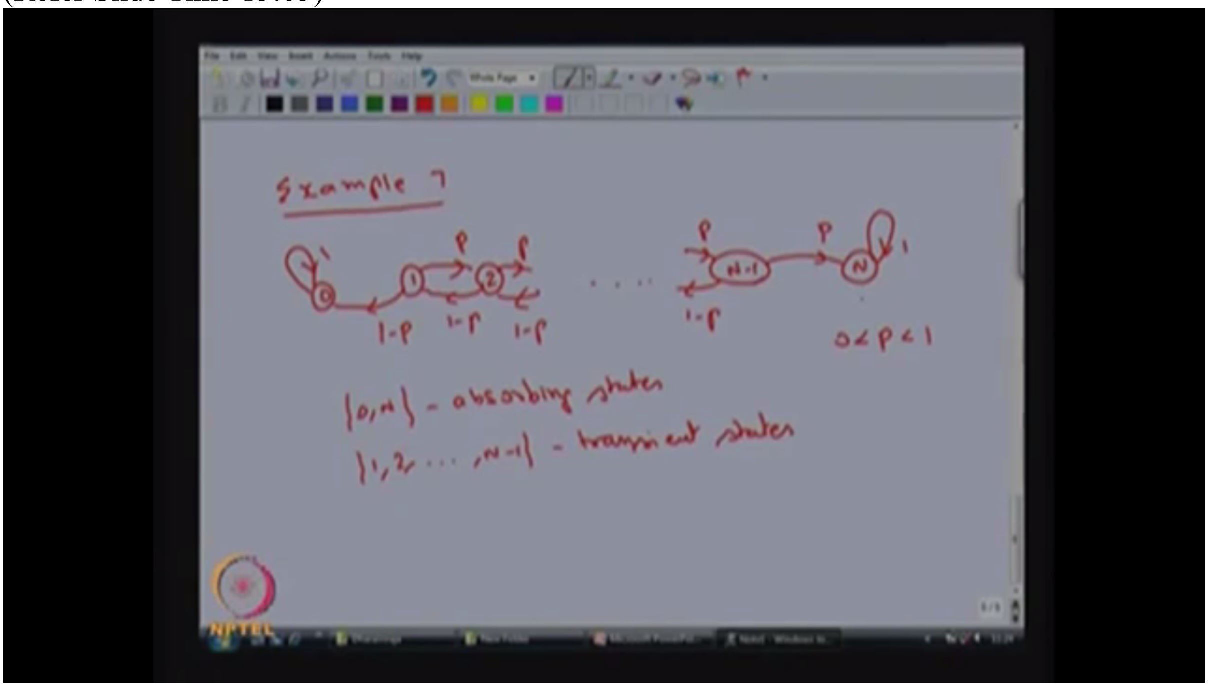

So once the system come to the state 0 or N, then it will be forever. It's basically, therefore, we can come to the conclusion this is going to be the reducible Markov chain of the type transient states and the few absorbing states.

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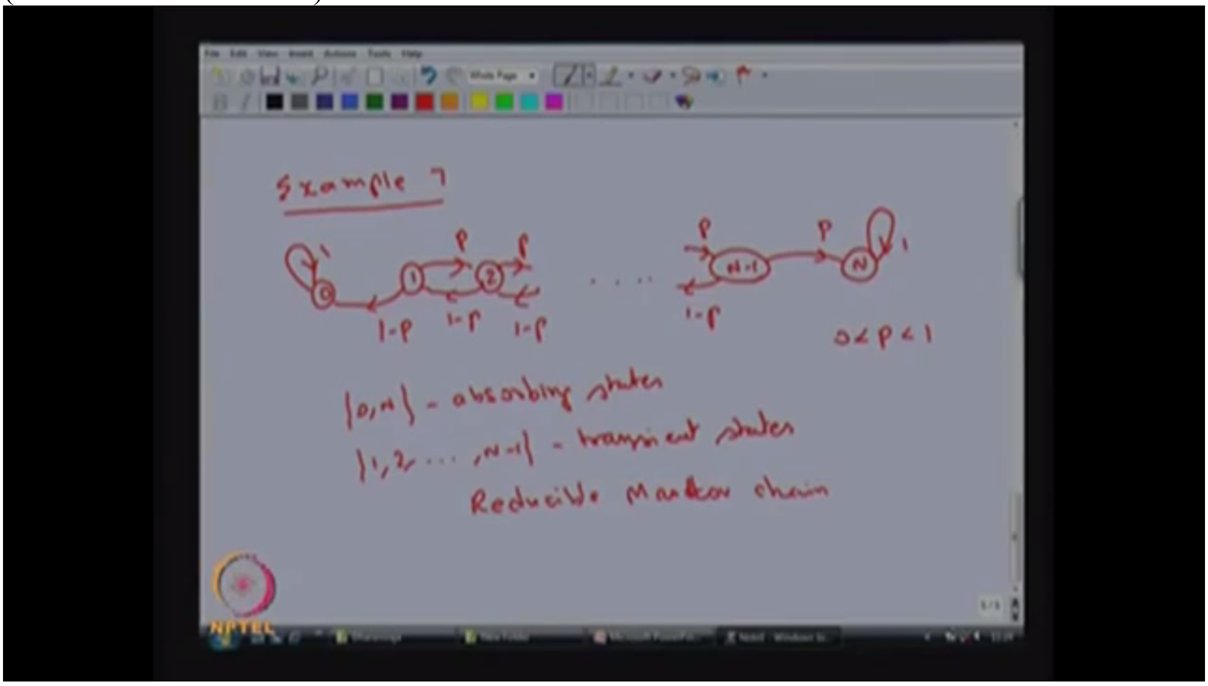

So with this let me stop the examples of classification of states. That means I have given the seven different examples in which it has the finite Markov chain as well as the infinite Markov chain and few Markov chains are the reducible, few are irreducible. In a reducible Markov chain, we have made two, three types of reducible Markov chain. That also I have explained. In the next class, we will discuss the limiting distribution and the stationary distribution. Thanks.

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