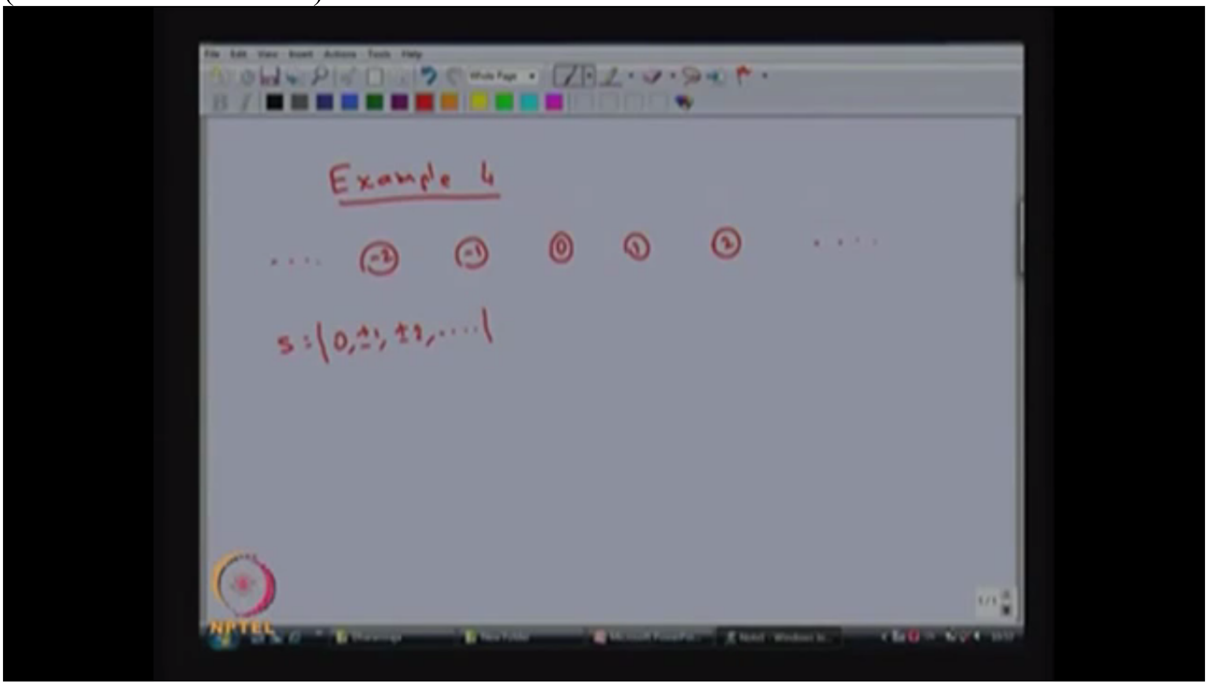
Example of Classification of states(Contd.)

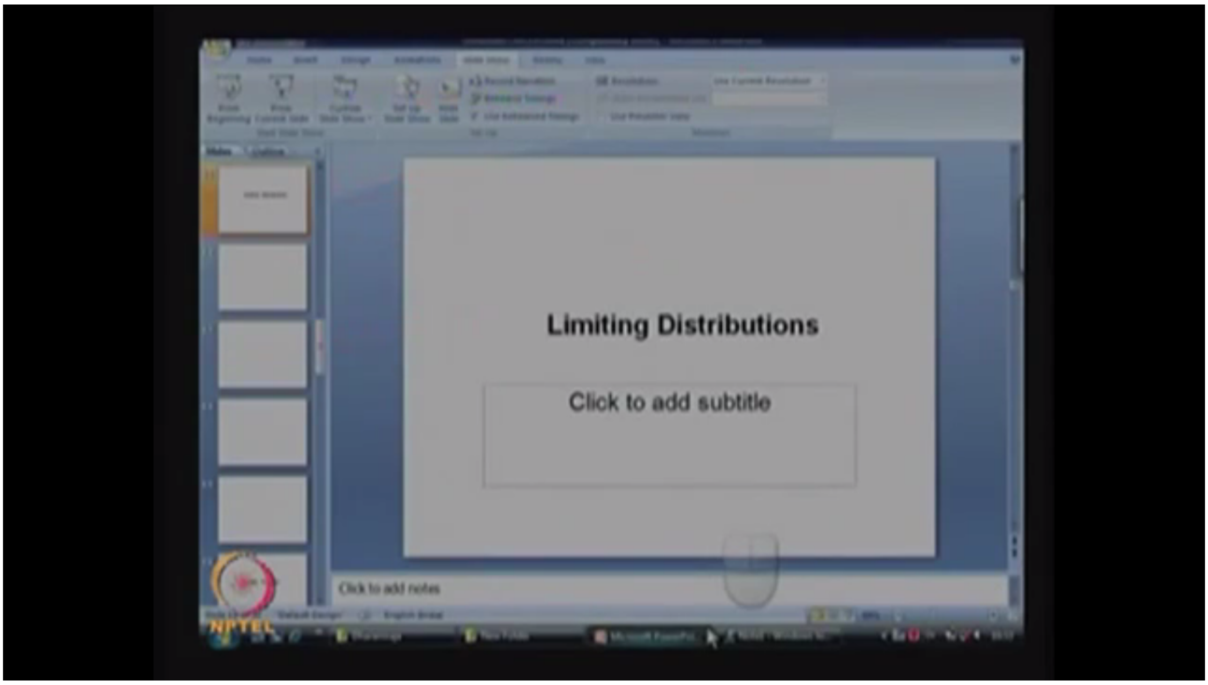
Dr. S Dharmaraja
Department of Mathematics, IIT Delhi

Now we are moving into the fourth example. This has the infinite number of states. Suppose the state space, let me draw the state transition diagram, state 0, 1, 2 and so on. The left hand side it has the states minus 1, minus 2 and so on. So the state space of this Markov chain has a countably infinite number of elements with the state 0 plus or minus 1 plus or minus 2 and so on. Let me draw -- give the transition rates, transition probabilities.

(Refer Slide Time 00:51)

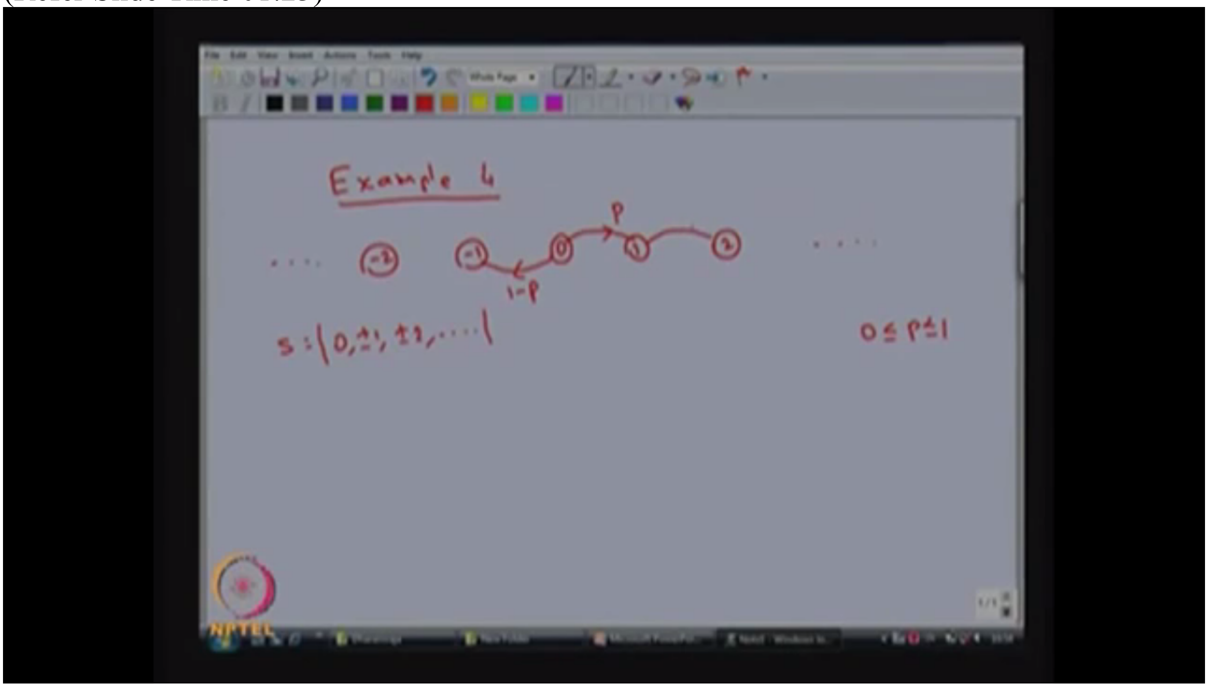


(Refer Slide Time 00:52)



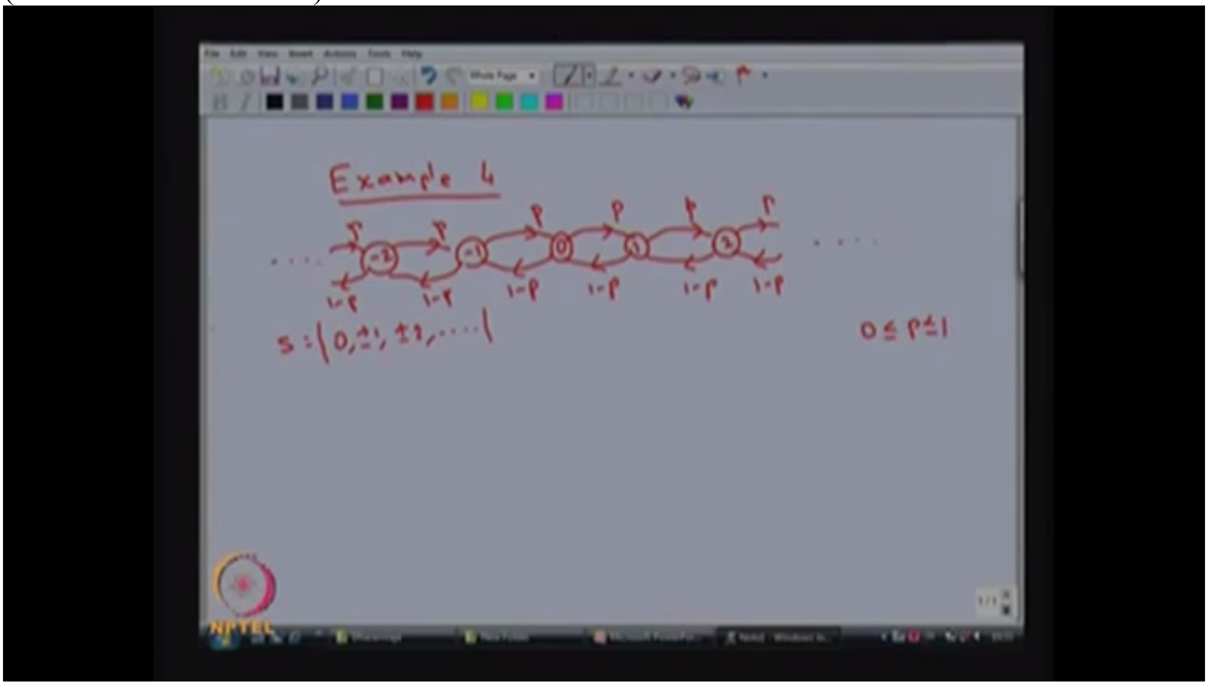
So the system is moving from state 0 to 1 with the probability P and the system is moving from the state 0 to minus 1 with the probability 1 minus P. Therefore, if you see the state transition diagram, the state one-step transition probability matrix, the row sum is going to be 1. So you keep P lies between -- P can lies between 0 to 1.

(Refer Slide Time 01:23)



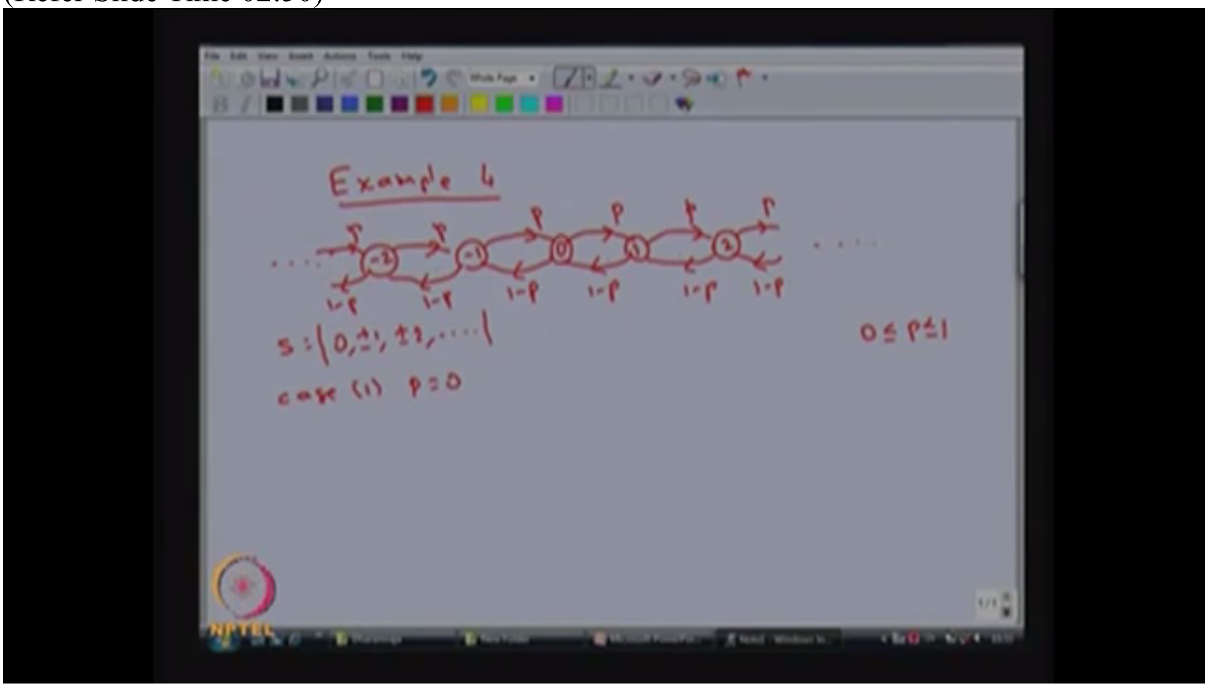
Similarly, you go for the all other states. The system is moving from the state to the forward 1 state that is it the probability P, backward state with the probability 1 minus P. With the forward is P and coming back to the one-step less 1 state less that is 1 minus P, so this is a way it goes for all the states, 1 minus P and this is one P and you have a countably infinite number of states.

(Refer Slide Time 02:05)



Now let me go for the case 1 in which the P is going to be 0. Suppose P takes the value 0. What happens or how to classify the states when P is equal to 0 in this time homogeneous discrete-time Markov chain? When P is equal to 0, there is no forward arc. When P is equal to 0 implies the system is always go to the 1 state, one step less, 1 state less with the probability 1 because P equal to zero.

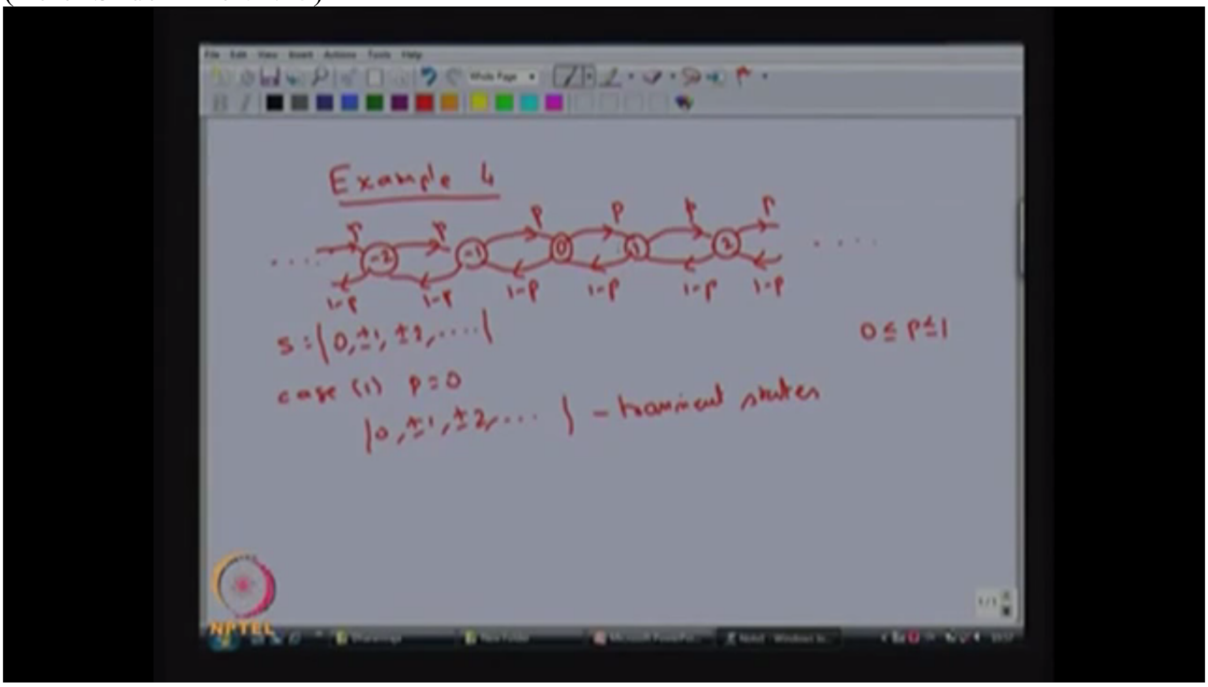
(Refer Slide Time 02:50)



Therefore, you should be able to visualize what is the state transition diagram corresponding to P is equal to 0. There is no forward arc arrows. That means whenever the system starts from some state, it will keep on going to the 1 state less in every step and you can visualize for a longer run where the system will be. Whether it will be in the positive side or in the negative side you can visualize whenever the system start from any finite state over the period, it may be in some state with the some positive probability for the finite number of steps. For a infinite number of steps or for a longer run, the system will be in the negative side for a longer run.

So that's a limiting distribution, but here we are discussing the classification of the states. Therefore, with the probability 0, it won't be back at all. If the system starts from any state, it won't be back to the same state with the probability 0. Therefore, all the states are going to be -- all the states are going to be the transient states.

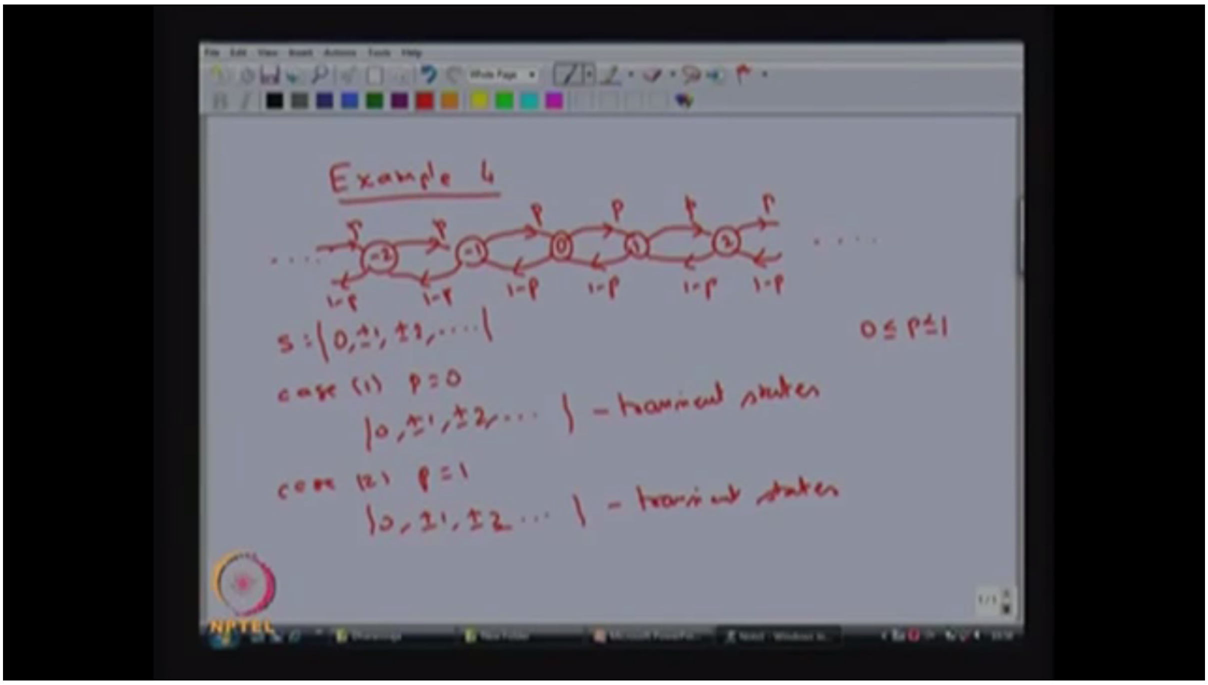
(Refer Slide Time 04:13)



If you calculate F for -- you take any finite state 1 or something, then F_{11} of 1, F_{11} of 2 and so on if you calculate, then you may land up F_{11} capital one, that is going to be always less than 1. Therefore, if you start with the 1 state, you can conclude it's a transient state and all other states also are of the same way. Therefore, all the states are going to be same.

Suppose we discuss the case 2 with the P equal to 1 what happen? If P equal to 1, then you have all the forward arcs, not the backward arcs. That means whenever the system starts from any state, then the system will go to the forward all the states in subsequent steps with the probability 1. In a longer run, the system will be in the positive side, positive infinite each side in a longer run. Therefore, with the probability 0, it will be in any one of the finite states in a longer run whereas for the any finite steps, the system will be in some of the states and it will be keep moving forward straight over the number of steps. Therefore, here also you land up all the states are going to be transient states.

(Refer Slide Time 05:46)



The both the two cases, the situation for the limiting distribution may change. One is in the left side. The other one is in the right side whereas all the states are going to be the transient states.

But our interest is for the P is lies between that is our third case, our interest is P lies between 0 to 1, open interval. That means if you see the previous state transition diagram, you have both the forward arcs as well as the backward arcs because the probability P lies between open interval 0 to 1. Therefore, the 1 minus P is also lies between 0 to 1 in the open interval. Therefore, whenever the system starts from any state, it will come back to the same state with the even number of steps.

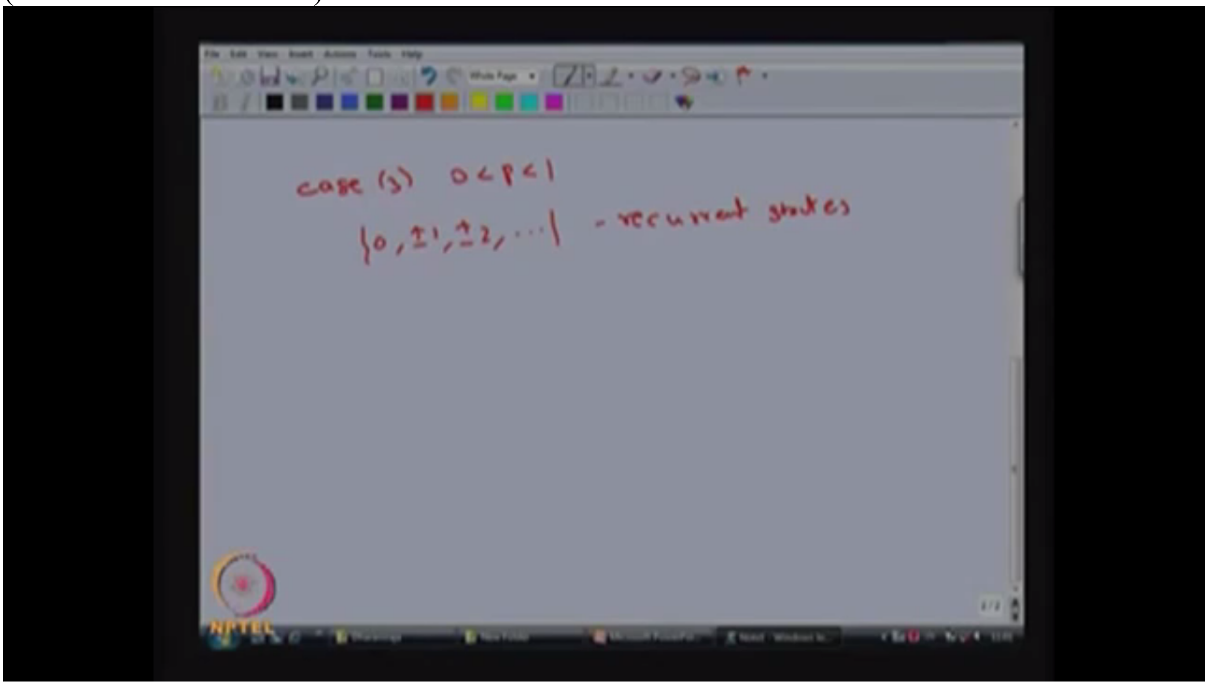
Suppose you visualize with the state 1, it can come back to the same state 1 not in the odd number of steps, but in the even number of steps suppose if the system moves to the state 0 in the first step and in the second step, it can come back to the state 1.

Similarly, suppose the system would have moved from 1 to 2, then in the second step, it would have come to the state 1. Therefore, it is two steps it can come back either via going to the state 0 or going to the state 2.

Suppose you go for think of four steps it coming back to the same state. That is possible. Need not be the first visit means it can make a two times loop the left side or it can make a two times loop in the right hand side or it can make a one step forward and one more step forward. Then it can come back. Therefore, all the possible steps, if you include all the possible steps, you will come to the conclusion it will take an even number of steps to come back to the same state.

So if you do the simple exercise what we have done it in the earlier case, you can come to the conclusion 0, plus or minus 1, plus or minus 2 and so on, all the states are going to be a recurrent state.

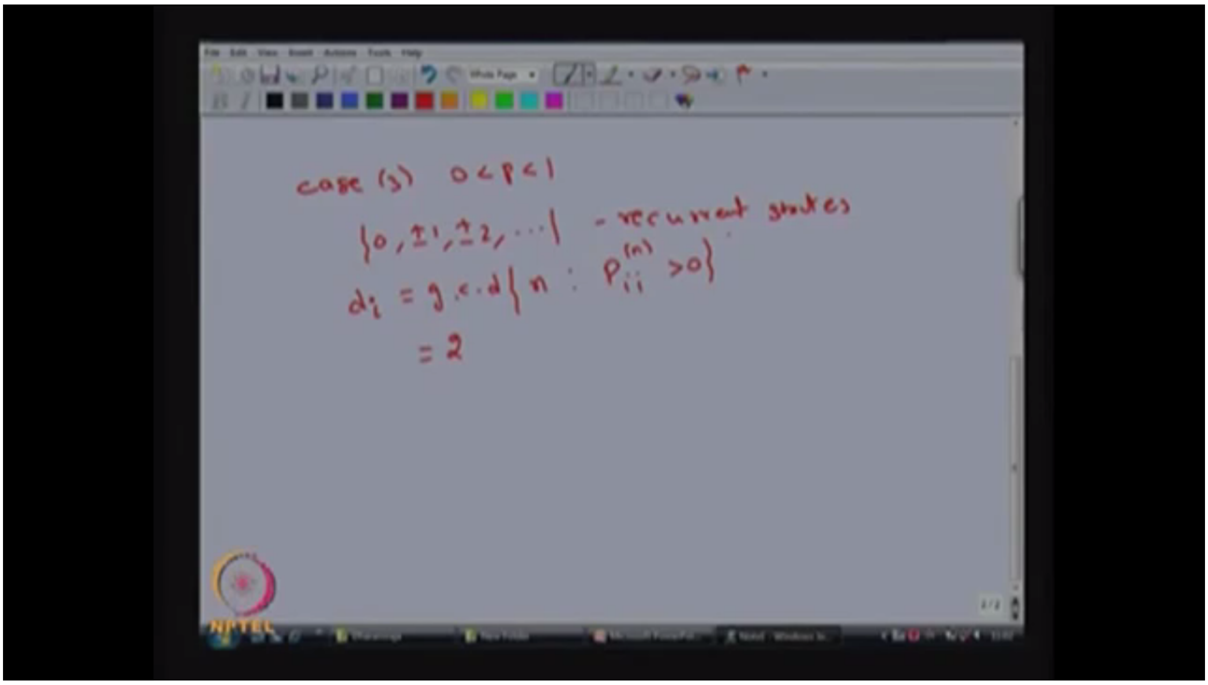
(Refer Slide Time 08:19)



Without fixing the value P, you can conclude F_{ii}, capital F_{ii} is going to be 1 for all these states. Therefore, now we come to the conclusion all the states are going to be the recurrent state.

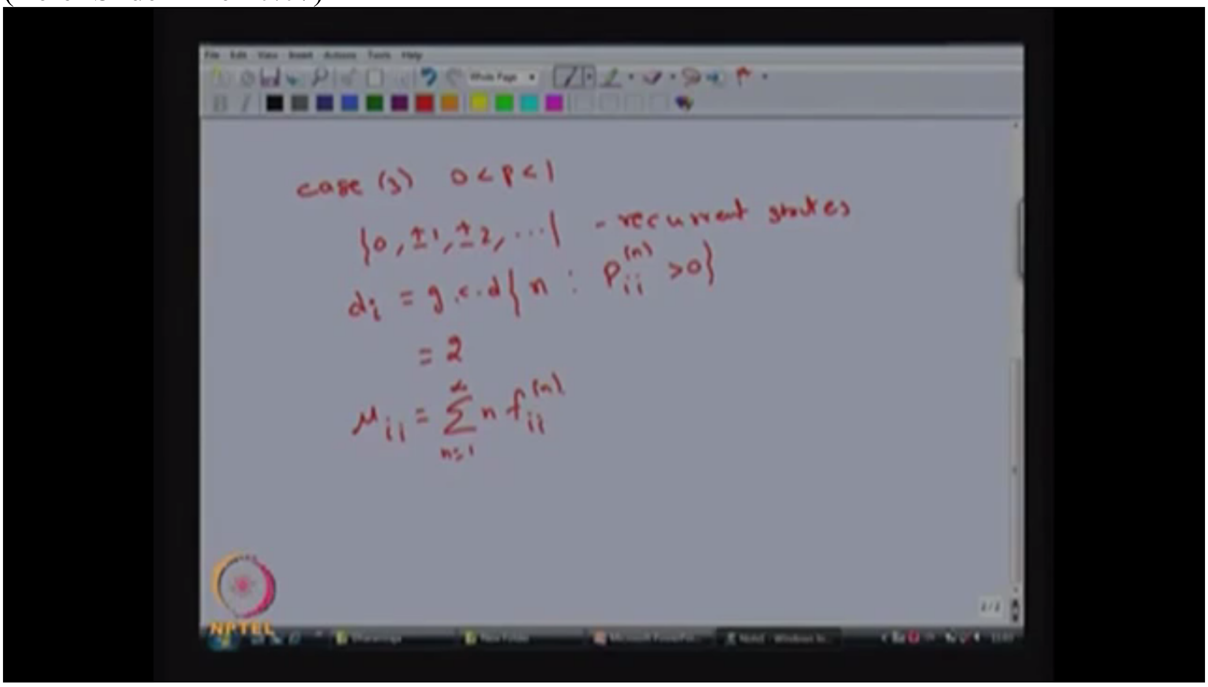
If you try to find out the periodicity for any state, the way I discussed the greatest common divisor of coming the greatest common divisor of n such that the P_{II} of n which is going to be greater than 0 and this is possible for all the even number of steps. Therefore, the system will be come back to the same state two steps, four steps, six steps and so on. Therefore, the GCD is going to be 2 for this particular Markov chain. So the period is going to be 2 and the recurrent state.

(Refer Slide Time 09:13)



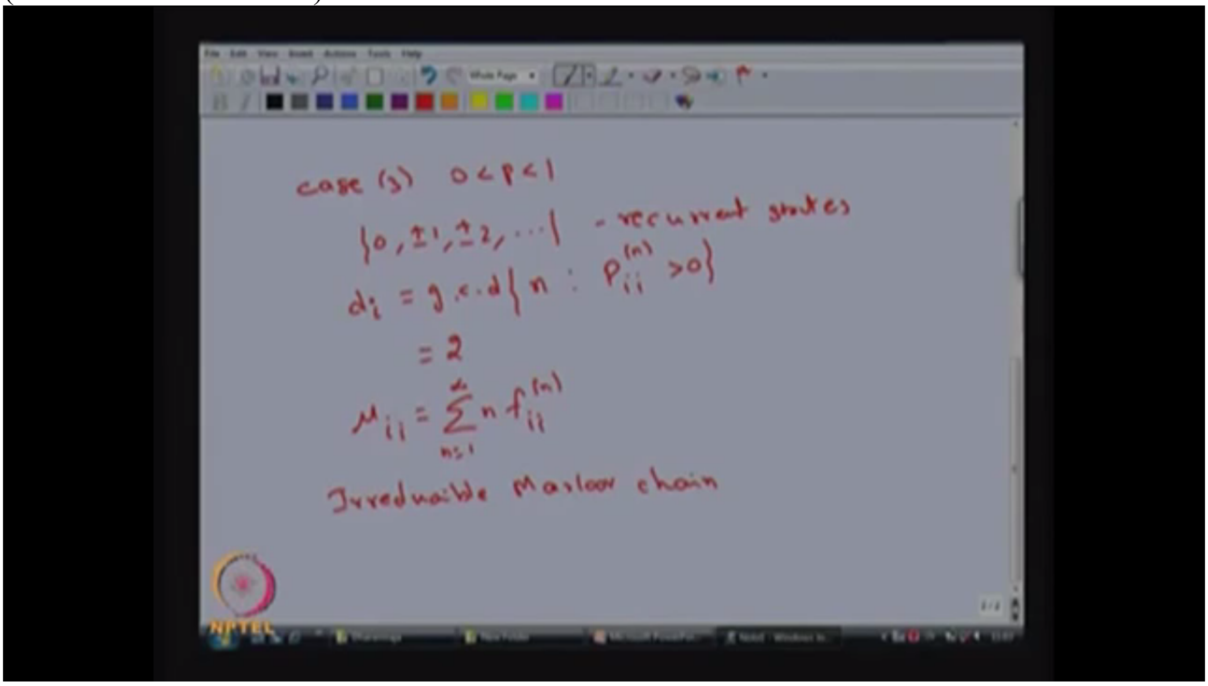
Now our interest is whether these states are going to be a positive recurrent or null recurrent? But for that you need what is the value of P because without P, without the value of P, you cannot come to the conclusion the new suffix ii that is going to be n times f_{ii} of n. You need the value, but some example it is not -- it is possible, but still by supplying the value of P or what is the range in which you can conclude whether this is going to be a finite quantity or going to be infinite quantity based on the range of P, you can conclude these recurrent states are going to be a positive recurrent or null recurrent.

(Refer Slide Time 10:07)



Since the state space is going to be 0, plus or minus 1, plus or minus 2 and so on and all the states are going to be a recurrent states, it will form a one closed communicating class. Both are communicate -- all the states are communicating with each other. Therefore, you will land up having only one closed communicating class, which is same as the state space. Therefore, this is going to be a irreducible Markov chain.

(Refer Slide Time 10:43)



These states may be a positive recurrent or null recurrent based on the range of P, but here we are just concluding this is going to be irreducible Markov chain with all the states are going to be a recurrent with the period 2. So since the period is 2, it won't be a ergodic states also. If you want a ergodic state, you need a positive recurrent as well as the aperiodic. Since the period is 2, we can conclude this is not going to be the ergodic state.