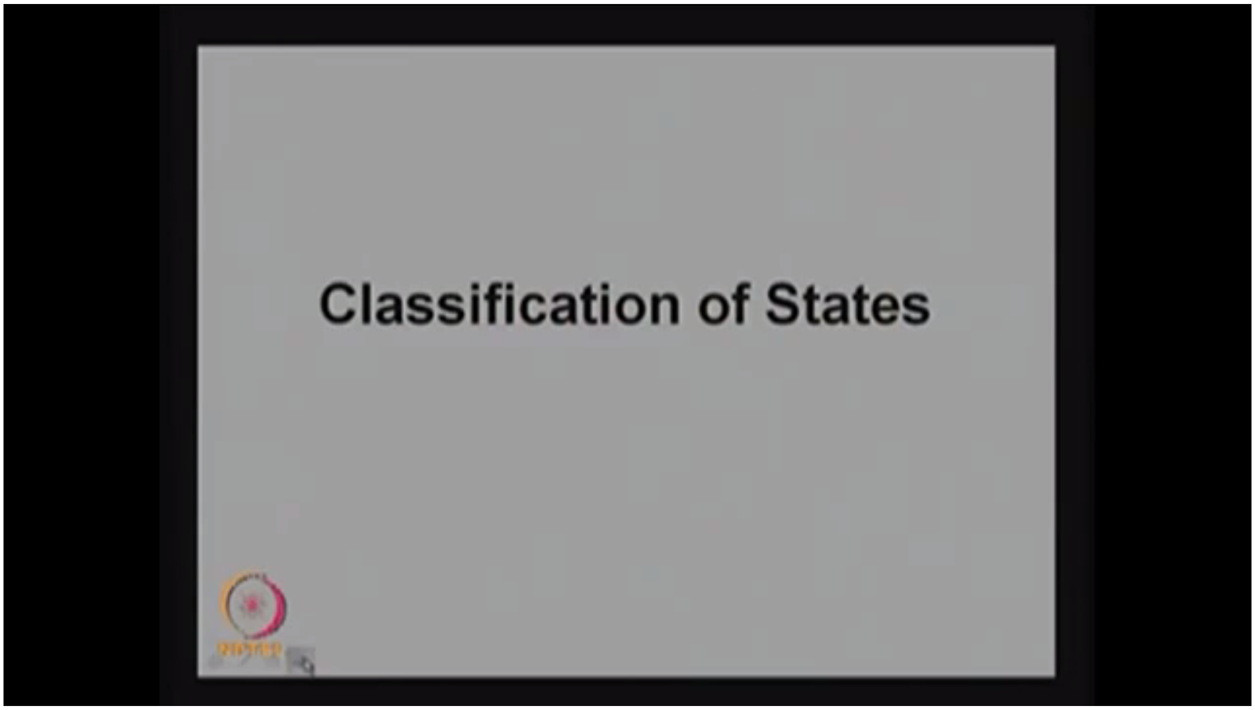
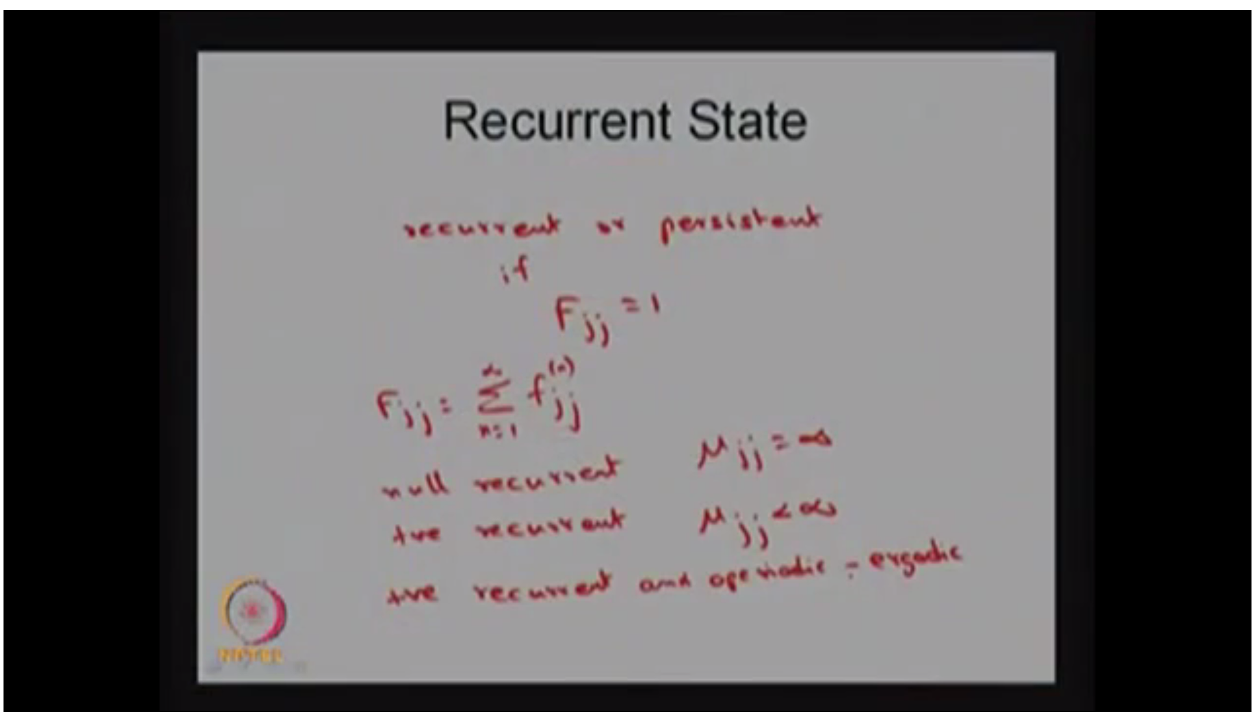



Now we are going for the actual Classification of a State using the concept of accessible communicate closed set, then communicating class, then we have defined First Visit, then we have defined the mean passage time or mean recurrence time or mean first passage time. so using these concepts, we are going to classify the states.



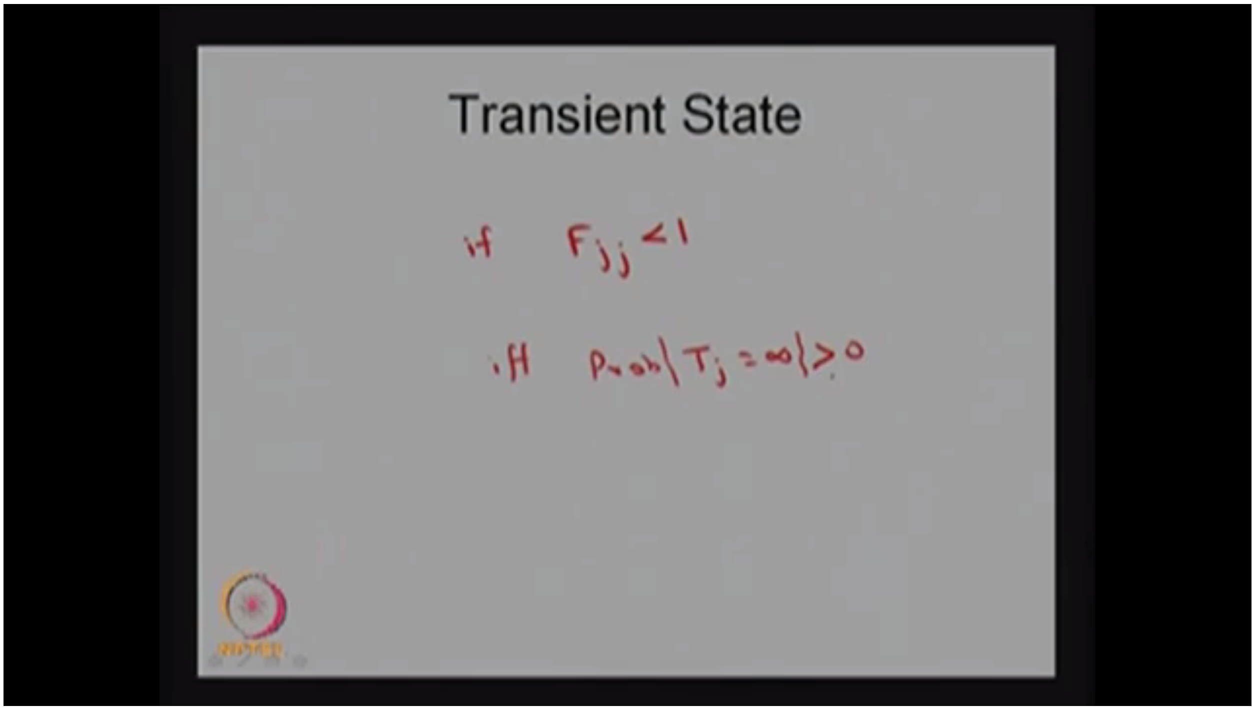
The first definition is a Recurrent State. State j is said to be recurrent or in the other word called persistent if the $F_{jj}=1$. If you recall what is $F_{jj}$, $F_{jj}$ is the probability of ever enter to the state j given that it was in the state j. So the $F_{jj}$ I have given in the summation form of small $f_{jj}^{(n)}$ using the first elicitor. So if you recall the $F_{jj}$ is nothing but what are all the possible ways the system can reach the state j as a first visit. You add all the combination all the probabilities that is going to be the $F_{jj}$. So if $F_{jj}$, that means the probability of returning to the same state j if that probability is certain. That means if the probability is 1, then that state is going to be the recurrent state.

We can classify the recurrent state into two forms. One is called a null recurrent; the other one is called a positive recurrent based on the mean passage time value. So based on the $F5_{jj}$, that's the probability we classify the state is going to bear a current state now based on the first passage time distribution, the mean first passage time, we are going to classify. That recurrent state is going to be a null recurrent or positive recurrent. Accordingly, the $\mu_{jj,}$ if it is a finite value, then we say that recurrent state is going to be the positive recurrent state. If $\mu_{jj}$ is going to be an infinite value, that means on average the first passage time is going to be infinite, then that corresponding recurrent State is going to be call it as a null recurrent state.

So whenever any state is going to be called as a recurrent state, if the probability of ever entering into the state j starting from the state j, it is certain, or the probability is 1, then that is a recurrent state. And the recurrent state is going to be called as a null recurrent if the mean first passes time or mean recurrence time or mean return time is infinity. If that is going to be a finite quantity, then the recurrent state is going to call it as a positive recurrent state.

If any state is going to be a positive recurrent as well as a periodic, then that state is going to be called as ergodic state. Any state is going to be called as an ergodic whenever that state is a positive recurrent as well as a periodic. A periodic means the periodicity of that recurrent state is 1. That means the greatest common divisor of all possible steps in which the system coming to the same state that value is 1. If the period is 1 and as well as the positive recurrent, it should be recurrent as well as positive recurrent. That mean the mean recurrence time is going to be a finite quantity and then it is going to be called as ergodic state.

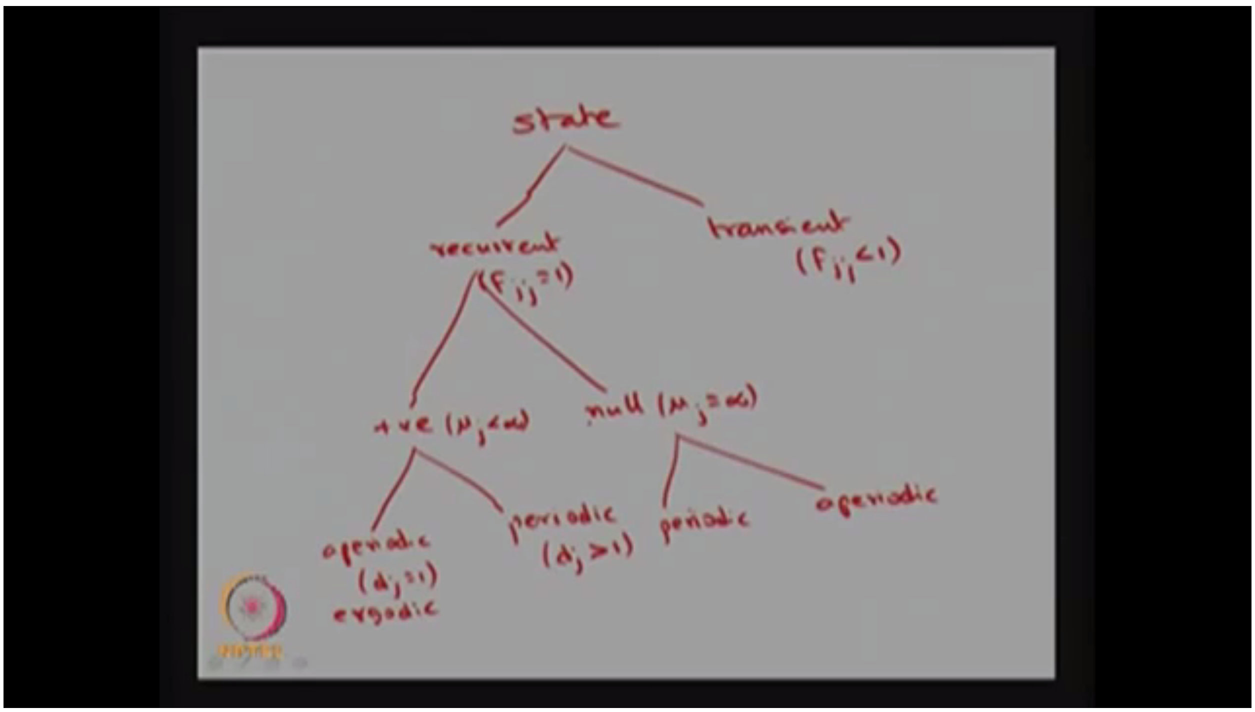
In a Markov chain, if all the states are going to be ergodic one, that means all the states are going to be a positive recurrent as well as a periodic, then we call that Markov chain itself ergodic Markov chain. That means that there is a possibility the Markov chain may be irreducible. That means you will end up with only one class in which all the states are going to be form one closed communicating class. Suppose each one state is going to be a positive recurrent and a periodic, then all other states are also going to be of the same type and the same period, therefore all the states are going to be the ergodic states, then that Markov chain is going to be called as an ergodic Markov chain.




Now I'm going to classify the state as a Transient State whenever the $F_{jj}$ <1. If you recall we have considered only two cases, whether the $F_{jj}$<1 or $F_{jj}$=1. Equal to 1 land up recurrent state and $F_{jj}$<1, that gives the transient state. That means the probability of returning to the state j starting from the state j is not certain. That means 1 minus of this probability with that much probability, the system may not return to the same state j if the system start from the state j. That means with some positive probability, because 1 minus -- this value is less than 1, therefore 1 minus of $F_{jj}$ is going to be greater than 0. So with some positive probability the system may not return to the same state if it start from the stage j, then that corresponding state is going to be called as a transient state.

By seeing the one-step transition probability matrix or by seeing the state transition diagram of a discrete-time Markov chain, you can easily come to the conclusion the state is going to be a recurrent state or transient state. Whenever it is going to be a finite number of states, it is easy to come to the conclusion. If it is an infinite number of states, then we need some work to be needed to come to the conclusion whether it is a positive recurrent or null recurrent, but easily you can make out the given state is going to be a transient state that you can make out from the state transition diagram or one-step transition probability matrix.

The conclusion of the state is going to be the transient state that can be given via the random variable $T_j$ also. So the state j is a transient if and only if the probability of the $T_j=\infty$ and that probability is 0. Sorry, I made a mistake. If this probability is strictly greater than 0, the probability of the mean -- the probability of the system return to the first passage, the first passage return time, that is infinity, if that probability is greater than 0. That means there is a certainty over the system returned to the state j with the infinite amount of time going to take. If that event is going to be with the positive probability, then that state is going to be the transient state. So there are through two ways you can conclude the given state is going to be the transient state, either $F_{jj}>1$ or the probability of $T_J=\infty$, which is greater than 0.




So based on this, I can come to the conclusion, any state could be recurrent or transient. That means this is corresponding to $F_{jj}<1$ and this is corresponding to $F_{jj}=1$. I can classify the recurrent state into two forms, either it could be a positive recurrent or null recurrent. Positive recurrent corresponding to the $\mu_j$ or $\mu_{jj}$ both are one and the same. That is going to be finite value, or null recurrent is corresponding to $\mu_j=\infty$. That means based on the mean recurrence time, you can conclude whether it is a positive recurrent or null recurrent.

Again, I can classify the positive recurrent into two, one is a periodic and the other one is a periodic. Periodic means that corresponding positive recurrent state that period is a greater than 1. Periodic means that $T_j$ is 1. So the periodic positive recurrent state, that is going to be called as an ergodic state. Similarly, I can classify the null recurrent state into two, one is a periodic and the other one is a periodic. The absorbing state is a special case of positive recurrent state where the transition probability from a state to itself is 1. So this is the way you can classify the state is a recurrent state or transient state, positive recurrent state, null recurrent state. Again, each one could be a periodic or periodic state.

So, in this lecture, we started with the few concepts of accessible, then communicate, then closed set, then we have discussed communicating class, then we have discussed what is the meaning of first visit, then we have given the first passage time, then we have given the mean first passage time distribution or mean recurrence time distribution. So based on those concepts we have classified the state as a recurrent state or transient state. So this is related to the probability whereas the conclusion of the positive recurrent or null recurrent is related to the average time. So here it is, only it involves the probability that whether in a certain probability, the system will come to the same state with the probability 1, whereas here there is an uncertainty, the system may not come to the state j if the system starts from the state j. If there is uncertainty of returning, that means with some positive probability, the system won't be back, then that state is going to be called as a transient state.

So this, you can easily visualize in the state transition diagram of any discrete-time Markov chain. You can see it whether that's from -- by seeing the state transition diagram, you can come to the conclusion whether the state is going to be the transient or recurrent, but through these diagrams, you cannot come to the conclusion whether it is going to be a positive recurrent or null recurrent unless otherwise you evaluate this quantity, $\mu_j$ is going to be n times $C_{jj}^{(n)}$, so you find out that summation. So based on the summation, value is going to be a finite one or infinite one. Accordingly, that means whether the mean recurrence time or means return time or mean first passage time is going to be a finite quantity or infinite quantity. Accordingly, you can conclude whether that recurrent state is going to be a positive recurrent or null recurrent. So here you need a computation, whereas by seeing the state transition diagram, sometimes you can come to the conclusion it is positive recurrent -- sorry. Sometime you can come to the conclusion whether it is a transient state or recurrent state.

Now the issue of a periodicity, the periodicity is important to conclude whether the limiting distribution exists or not whether that is going to be unique. So you need to find out the aperiodic or periodic. So if the period is going to be 1, then that state is going to be called as aperiodic; if the period is greater than 1, then it is a period with that integer. Then it is going to be a null recurrent, then also you can come to the conclusion whether it is a periodic or aperiodic.

Whenever you have a Markov chain with the finite number of states, then it is easy to find out whether it is going to be a positive recurrent or transient. So you need, a quite good exercise is needed whenever the Markov chain have an infinite number of states, then you need some work to be done to come to the conclusion, it is a null recurrent and so on.

In today's lecture with this classification, I stop here, and all the simple examples and the limiting distribution, that I'll explain in the fourth lecture. Thanks.