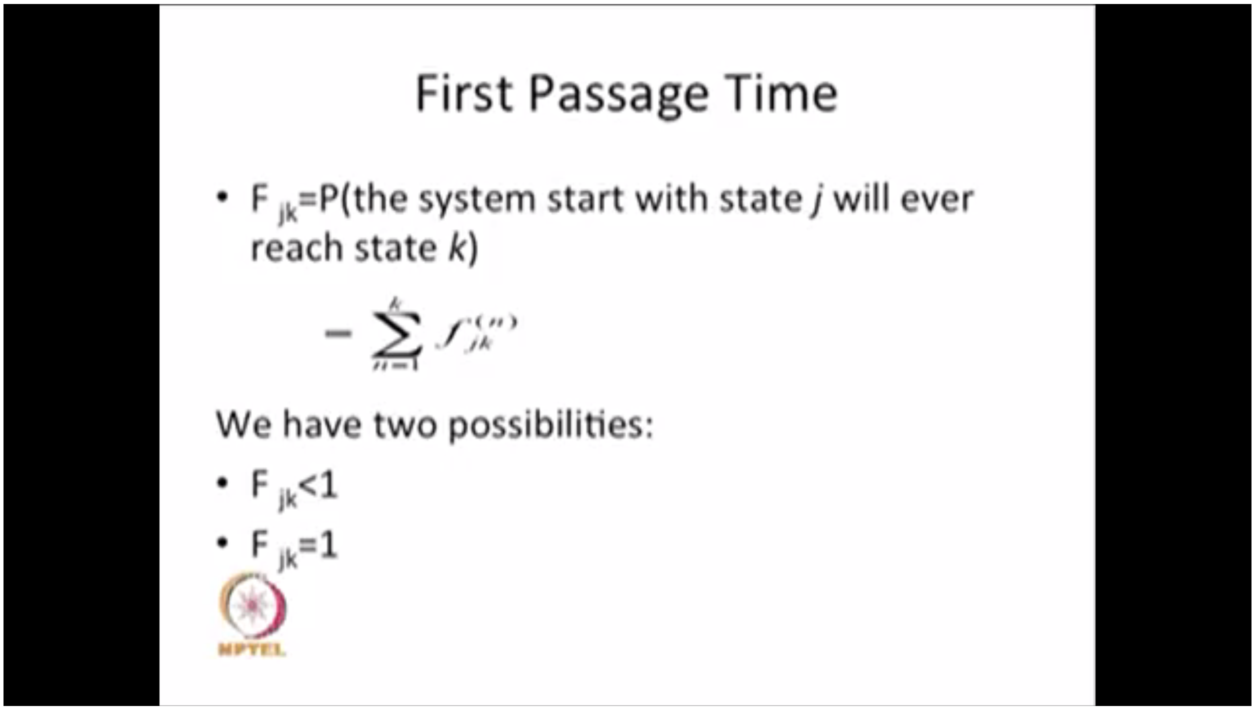
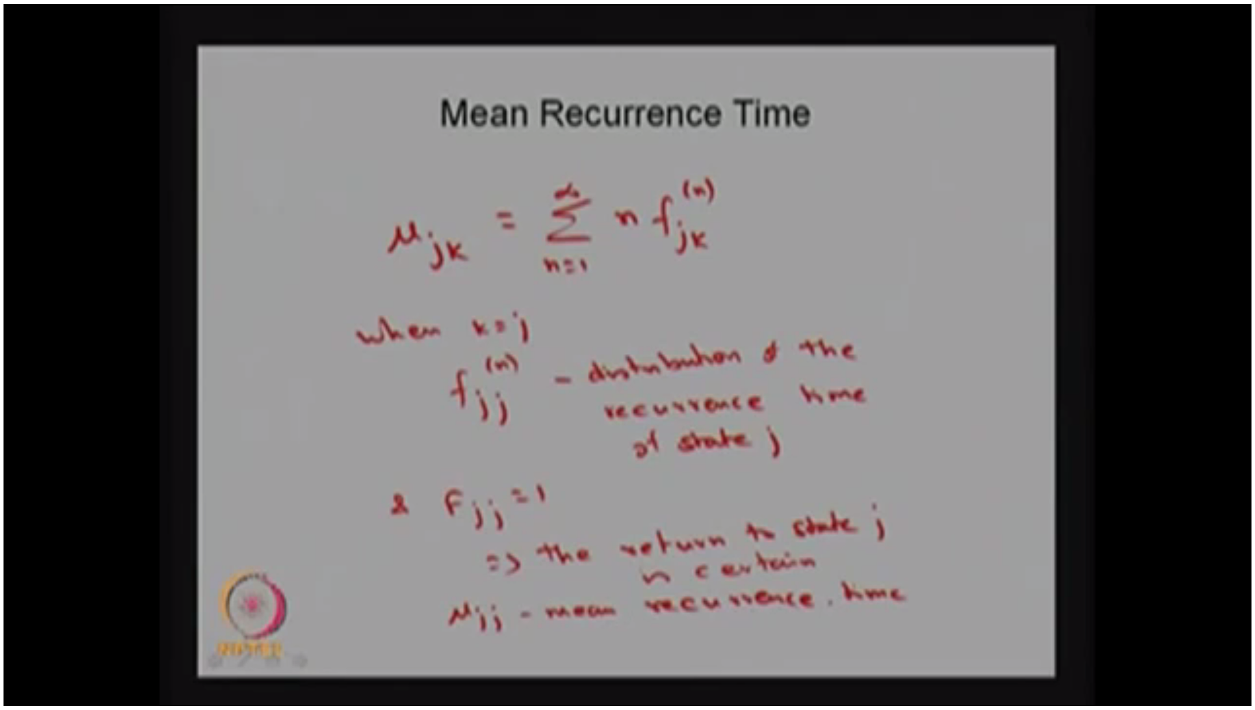


Now I'm going to give the next concept called First Passage Time distribution, First Passage Time distribution. So that is written in the F_{jk} that is nothing but what is the probability that the system start with the state j will ever reach state k. So this probability I'm writing as a $F_{j,k}$, therefore this same as there is a possibility, it would have gone to the state k in n steps first time and all the possible steps for the first time, that union will give you $F_{j,k}$. What is the conditional probability that the system is starting from the state j and ever entering into the state k, that is all the possible of first time to reaching the state n and all possible n that will give the probability of ever visiting the state k starting with the state j.

Now we have two issues or two cases. One is what is $F_{jk}<1$? What is the situation corresponding to this probability is going to be less than 1? The other case of interest is when $F_{jk}=1$. That means with the probability 1, you will be ever visiting the state k by starting from the state j with the probability 1, or whether this probability is going to be less than 1. If it is less than 1, then it is not the correct one. That means with the 1 minus of this probability, there is a possibility you won't ever visit the state k if you start from the state j, the first case. The second case is, with the probability 1, you will always reach the state k whatever be the number of steps starting from the state j. So our interest is both less than 1 as well as equal to 1.

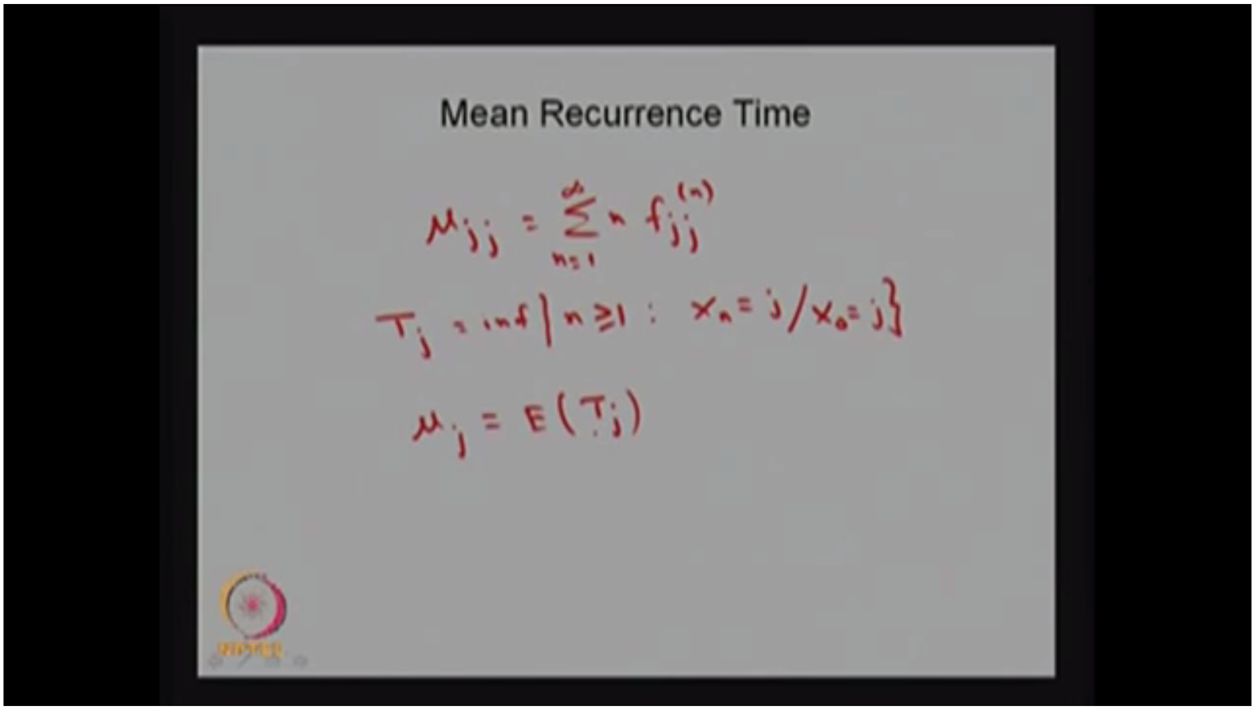
So the $F_{jk}=1$, that will give the probability distribution and that distribution is called First Passage Time distribution. So this case is our interest and this will give the First Passage Time distribution, because whenever the system is starting from the state j whatever be the number of steps reaching the state k with the probability 1, that means you have the whole mass as 1 and this is going to be the distribution of the first passage time.



So using this, I am going to give the next concept called Mean First Passage Time or Mean Recurrence Time. Mean First Passage Time is same as the Mean Recurrence Time. That is defined as μ_{jk}, that is nothing but what is the average first passage time or average recurrence time whenever the system start from the state j to the state k. That is how many steps you have taken and what is the probability that starting from the state j to k in n steps and for all possible values of n. That summation is going to give the mean first passage time or mean recurrence time.

Then our interest will be when k=j, return to the same state. So that means $F_{jj}^{(n)}$ that will give the distribution of the recurrence time of the state j. And if F_{jj}=1, this corresponding $F_{jj}^{(n)}$ is going to be the distribution, so correspondingly F_{jj} is going to be 1. This implies the return to the state j. Whenever the system start from the state j, that is certain, because that probability is 1. Whenever F_{jj} is 1, that means with the probability 1 if you start from the state j, you will definitely come to the state j. Therefore, that is corresponding to F_{jj}=1 and the μ_{jj} that will give what is the mean recurrence time. μ_{jj}, the μ_{jj} will give mean recurrence time for the state j.

So we are considering the second case in which a F_{jj}=1, so that is nothing but the return to the state j whenever the system start from the state j is certain, and the small $f_{jj}^{(n)}$ will give the distribution of the recurrence time, and our interest is also for the mean recurrence time that can be calculated by using μ_{jj}.



So earlier we have given μ_{jj} that is same as $\sum n f_{jj}^{(n)}$. By knowing $f_{jj}^{(n)}$, we can find out the mean recurrence time for the state j. The same thing can be obtained by using another concept by introducing the random variable that is T_j, that is nothing but inferior of $n \geq 1$ such that the X_n is state j given that X_0 was state j. This is a random variable, denoting the faster return time to the state j. The faster return time, time is here it is the step, nth step, and you find out what is the first time you return to the state j, starting from the state j, reaching the state j, so whatever be the first number, that integer, and that is going to be the T_j, and this is going to be a random variable. So using this random variable also you can give that definition of a mean recurrence time.

Now I can define the mean recurrence time, μ_j, you don't want two suffix j,j, one suffix is enough. So μ_j is nothing but what is the expected or expectation of the random variable T_j. So the T_j will give the step that denotes the first return time. Therefore, the expected first passage time that you can write it as the μ_j. So this μ_j and μ_{jj} both are one and the same, and here you are finding the distribution and using the distribution you are getting, and here you are finding that time and finding the average time using the expectation of T_j. So in both ways one can define the mean recurrence time.