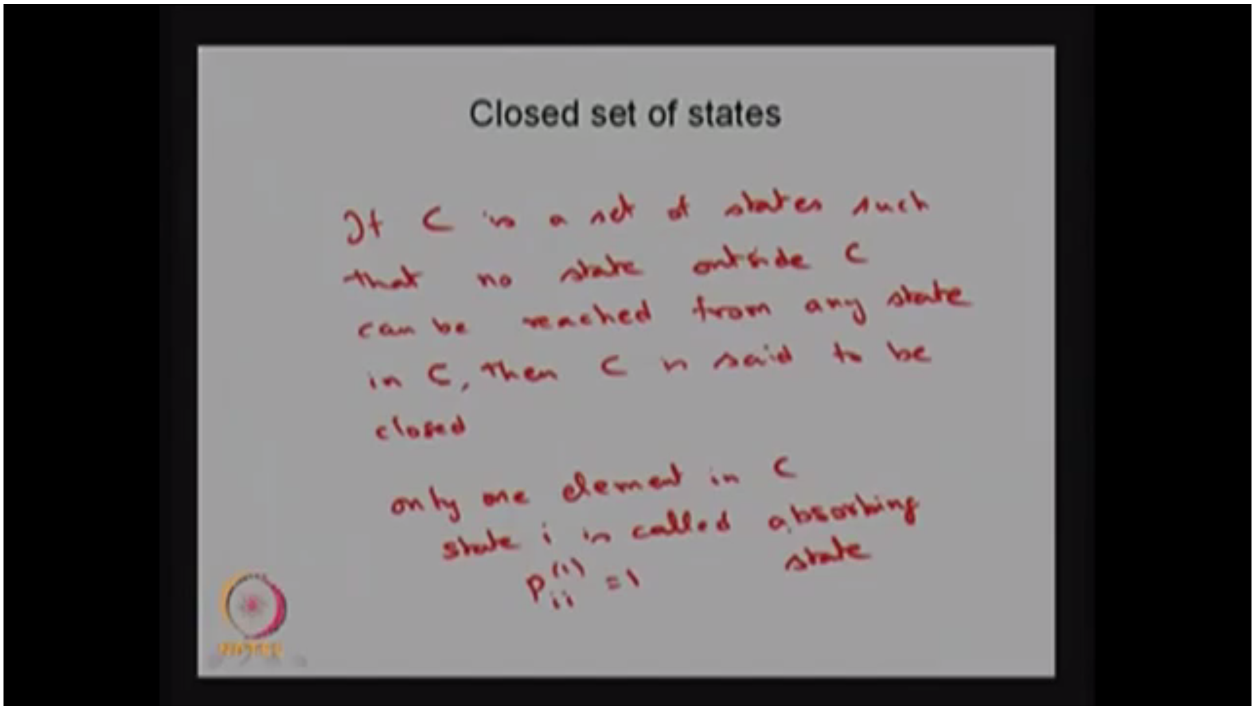
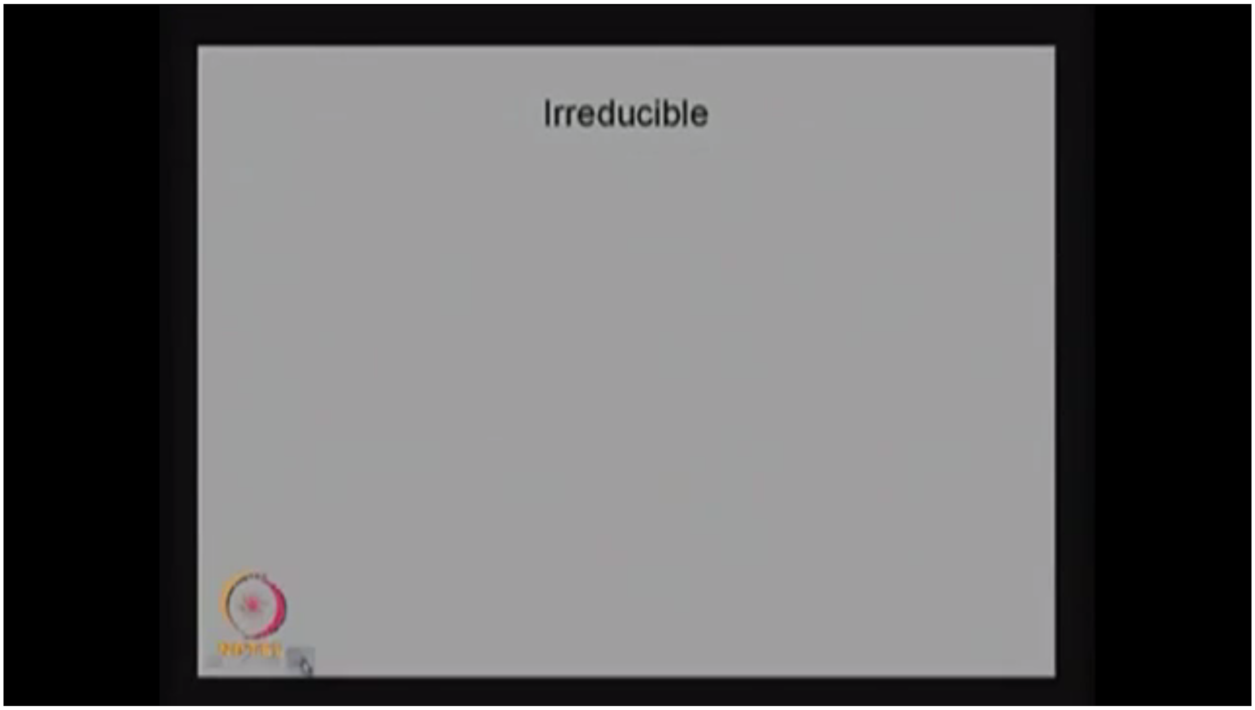


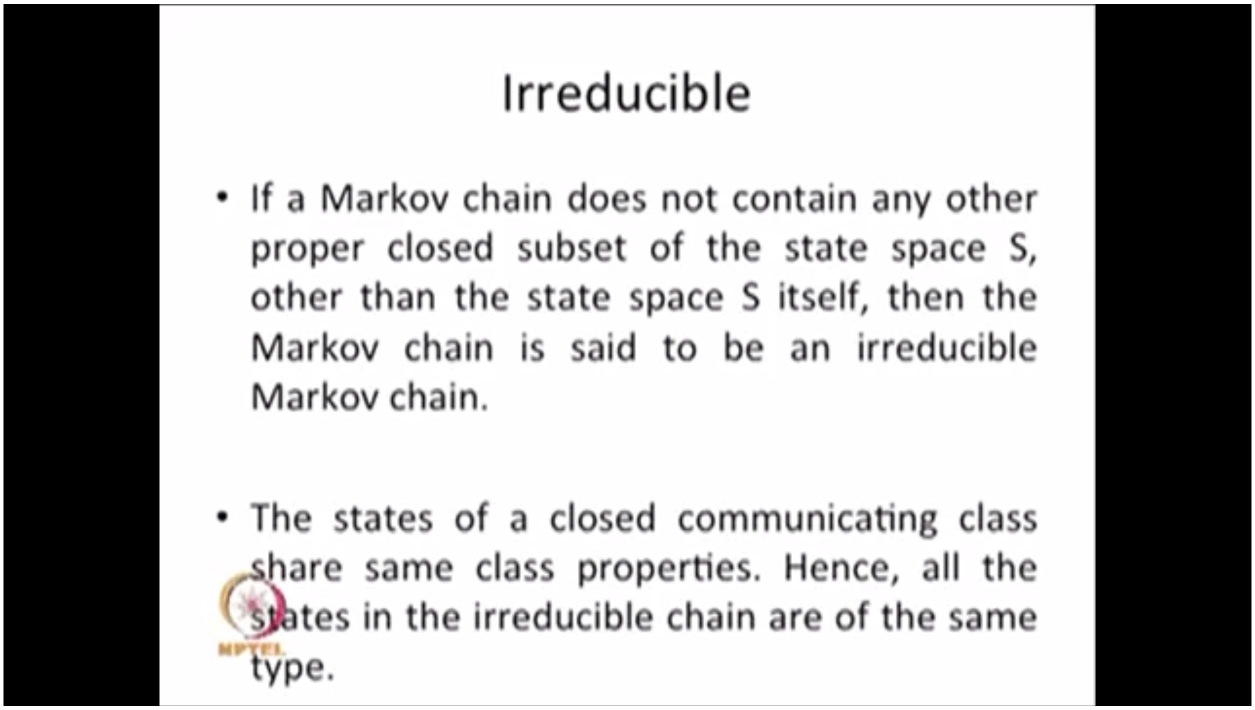
Now I'm going for the next concept called closed set of states, closed set of state. If C is a set of states such that no state outside C can be reached from any state in C, then we say -- then the collection or the set C is said to be closed. So whenever you create a collection of states and that set we call it as a C, if it satisfies this property, then we say that a set is called the closed set. So we can combine the class property with the closed set property, if both properties are satisfied, they communicate with each other as well as the closed property satisfied, then we can say that a closed communicating class. So any subset in the states space S, if it satisfies each element within the set is communicating each other and satisfies this property, then we say that collection is going to be a closed communicating class.

There is a possibility in a set you can have more than one elements, more than one state in that collection. The class may have only one element or it may be more than one element. If any closed set or the closed communicating class has only one element, that means you cannot include one more state to make it as the closed or communicating class, then that closed set is called or that state is called only one element in C, then the state i is called absorbing state. Yeah, state i is said to be absorbing state, then it is going form a closed communicating class, which has only one element in that class. There is a possibility more than one element is also possible in the closed communicating class.

So we can define the absorbing state through the closed communicating class or we can make it in the same absorbing state using the definition $P_{ii}^{(1)} = 1$. That means if you see the one-step transition probability matrix, the diagonal element of that corresponding state, that corresponding row, the element is going to be 1. That means the system starting from the state i and in 1 step the system moving to the same state i, that probability is 1. If this probability is 1, then we say that state is going to be absorbing state. The other way around, we can go for defining the absorbing state via closed communicating class as only one element also. So there are two ways you can see the absorbing state.



Using these concepts, I am going to develop the next concept called Irreducible Markov chain. We are discussing a time homogeneous discrete-time Markov chain. This concept is called Irreducible, that is valid for the discrete-time Markov chain as well as the continuous time Markov chain, so that we are going to discuss later. Now I am defining the irreducibility for a time homogeneous discrete-time Markov chain.

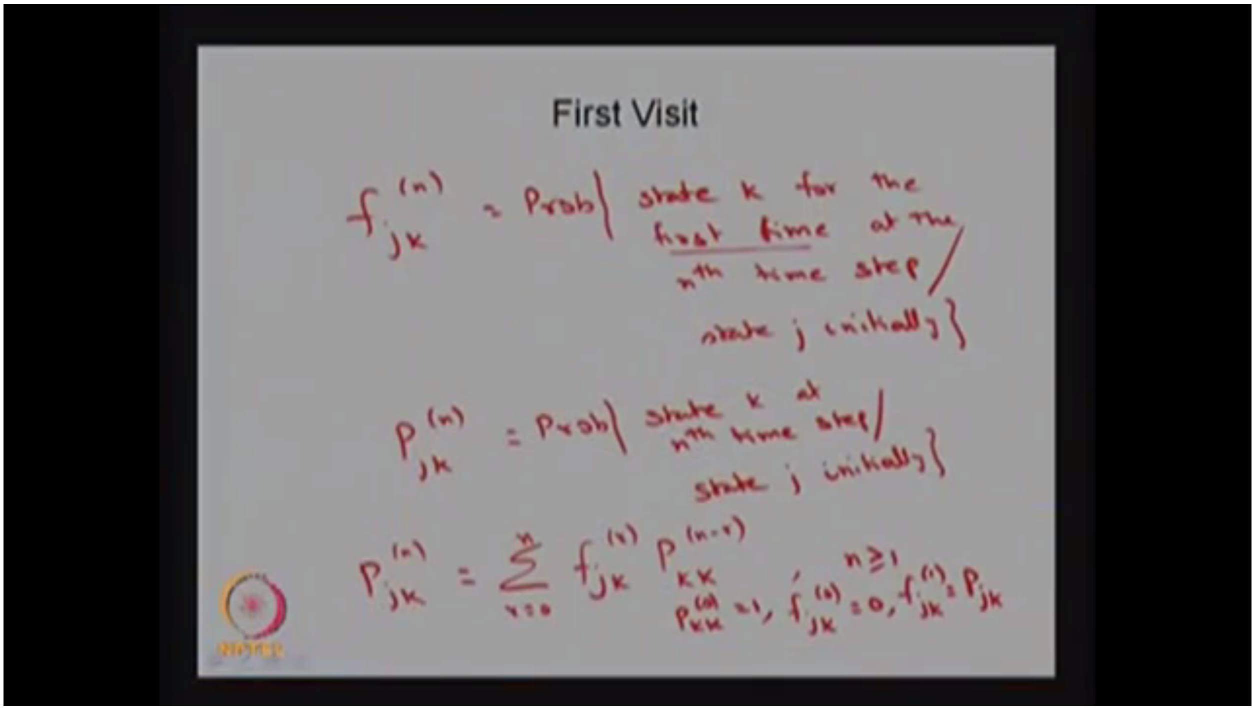


If the Markov chain -- since the irreducible concept comes for the discrete-time Markov chain and the continuous time Markov chain, we will use the word called Markov chain that is valid for both. If the Markov chain does not contain any other proper closed subset other than the state space S, then the Markov chain, in short we can use the word MC for Markov chain, then the Markov chain is called irreducible Markov chain. Whenever the state space cannot be partitioned into more than one closed set, the proper set, that means you can have only one closed set and that is same as the S. All the elements in the state space is going to be former only one closed set. In that case, that Markov chain is going to be called as an irreducible. Irreducible means you cannot partition, you cannot partition the state space.

If more than one closed proper, proper closed subsets are possible from the state space, then that Markov chain is going to be called as a reducible Markov chain. If more than one or we can able to make the partition of the state space into more than one closed set as well as a few transient states and so on, that I am going to discuss later. So whenever you are not able -- if you are able to partition the closed -- partition the state space, then that is going to be a reducible Markov chain. If you are not able to partition the state space and the whole state space is going to be only one proper closed subset, then that Markov chain is going to be called as an irreducible Markov chain.

In this case, all the states belonging to that class is going to be from one class, and since it is going to have only one class, all the states are going to have -- if one state has the period something, then all the other states are also going to have the same period. Because you are not able to partition, so you have only one class, therefore, if one state has the period some number, some integer, then that same period will be for all other states also.

So the Markov chain, which are not irreducible are said to be reducible or non-irreducible Markov chain.



Now I'm going to give the next concept called First Visit. We didn't come to the classification of the state, before that we are developing a few concepts. Using these concepts, we are going to if we are going to classify the states. The next concept is called First Visit. What is the meaning of first visit? I am going to define the probability mass function as the $F_{jk}^{(n)}$, that means what is the probability that the system reaches the state K for the first time that is important, for the first time at the nth step, nth time step, given that the system starts the state j initially. This is a conditional probability mass function of a system moving from the state j to k and system reaching the state k at the nth at the time step for the first time, that is important. So this is the first time the system reaches the state k at the nth step, exactly at the nth step, and this conditional probability mass function that I am going to write it as the $F_{jk}^{(n)}$.

This is different from the $P_{jk}^{(n)}$. This is also conditional probability, whereas this probability is defined what is the probability that the system reaches the state k at the nth time step given that it was in the state j initially. This is also conditional probability, the only difference is the first time. That means there is a possibility, the system here, the $P_{jk}^{(n)}$ means there is a possibility the system would have come to the state k before nth step also. So that probability is included. Whereas the F_{jk} at the nth step means, only the nth step, it reaches the state k. Therefore, the way I have given the time conditional this probability and this is not necessarily the first time -- this is also conditional probability, I can relate the $F_{j,k}$ with the $P_{j,k}$, both are in the N step transition probability, but one is for the first time, the other one is not necessarily.

I can relate both in the form of $P_{jk}^{(n)}$ that is a n step, that is same as Σ $F_{jk}^{(r)}$ steps and $P_{kk}^{(n-r)}$ and r can be vary from 0 to n for $n \geq 1$. This means if the system is moving from the state j to k in n step, not necessarily the first time, that can be written as the union of mutually exclusive events for different r in which the system moves from the state j to k in r steps for the first time, and the remaining n-r steps, there is a possibility, the system would have moved from the state k to k not necessarily, the first time, and a possible r can be 0 to n and this n can vary from 1 to infinity.

Obviously, we can make out -- I can give, the $P_{kk}^{(0)}$ step, that is going to be 1, and similarly, you can make out $F_{jk}^{(0)}$ that is 0 steps also 0, and $F_{jk}^{(1)}$ step, that is nothing but the P_{jk}. The first time the system is moving from the state j to k in 1 step, that is same as the one-step transition probability. The first time and one-step transition probability is same whereas for $n \geq 1$, then it is going to be the combination of the first time with not necessarily the first time n-r steps transition probability that all possible events that will give the altogether final probability.

So here we have used the total probability rule as well as the Chapman Kolmogorov equation for the time homogeneous discrete-time Markov chain to land up giving the relation between the P_{jk} with the F_{jk}.