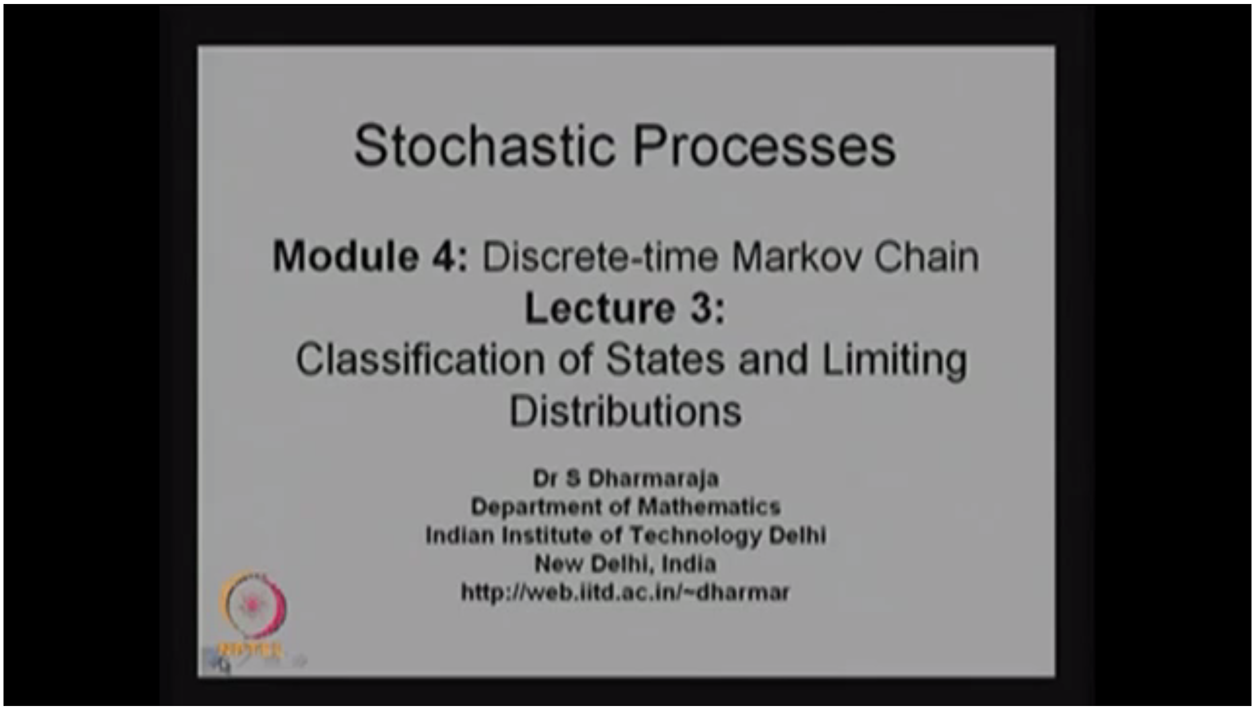
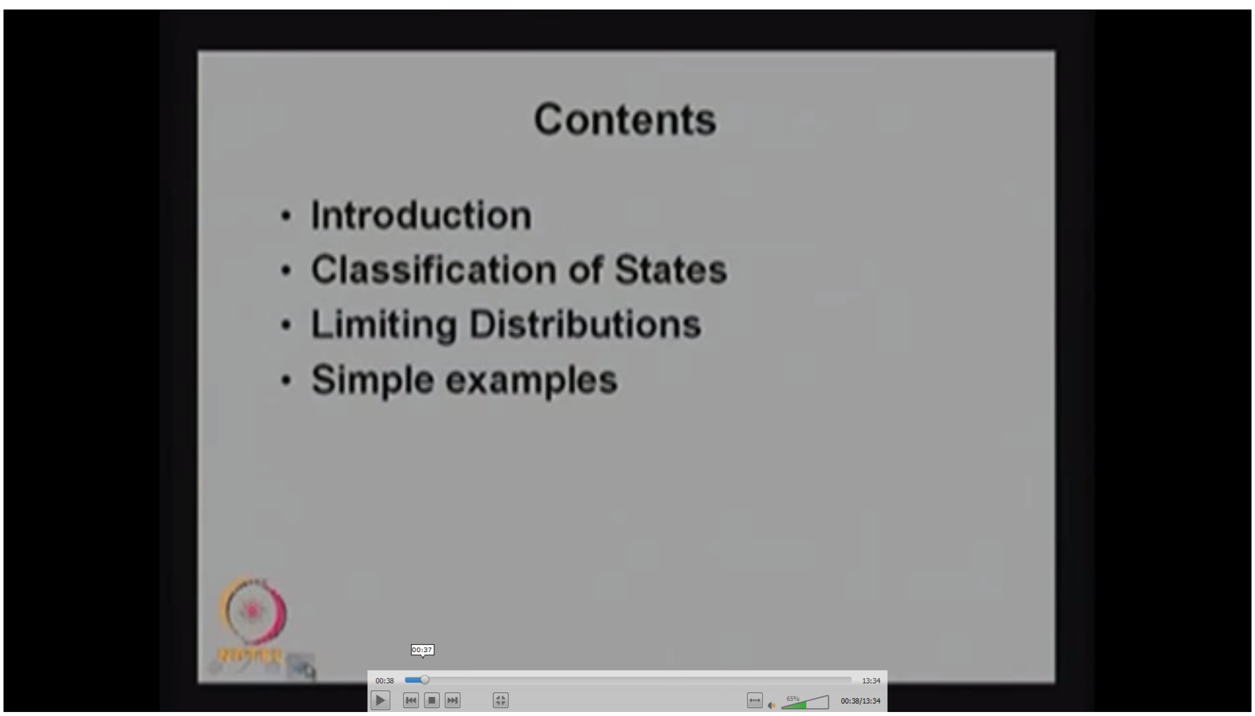
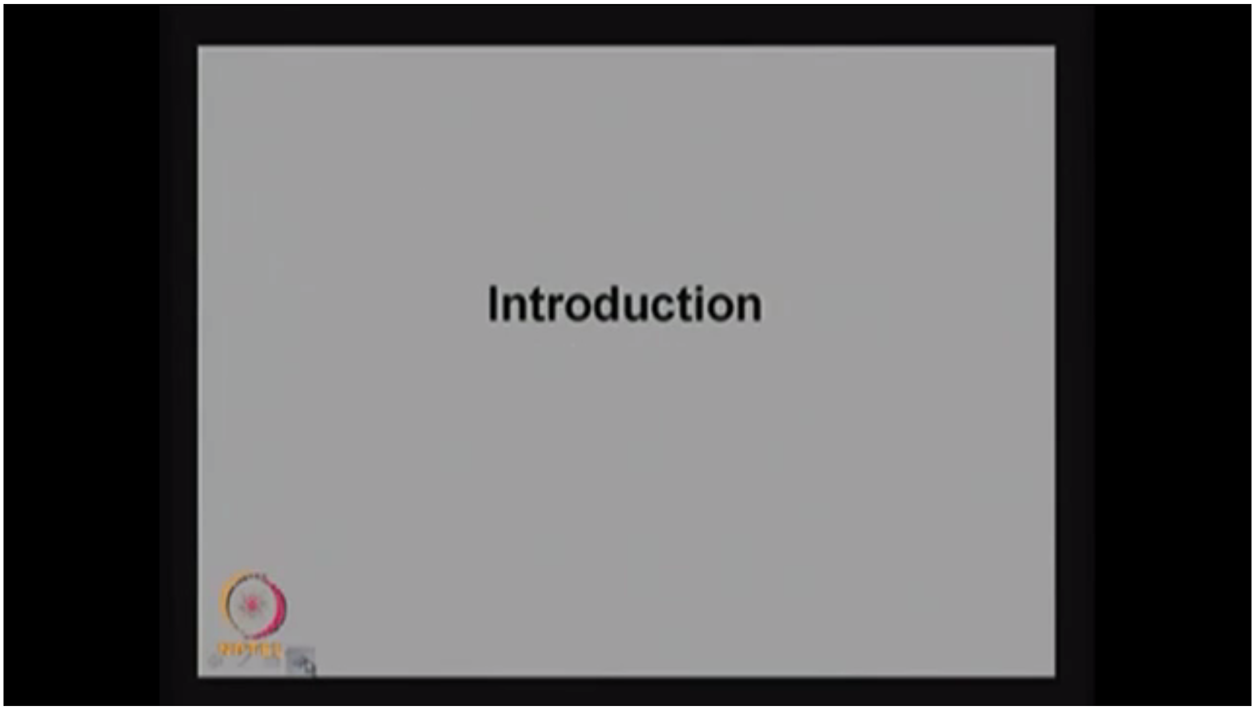


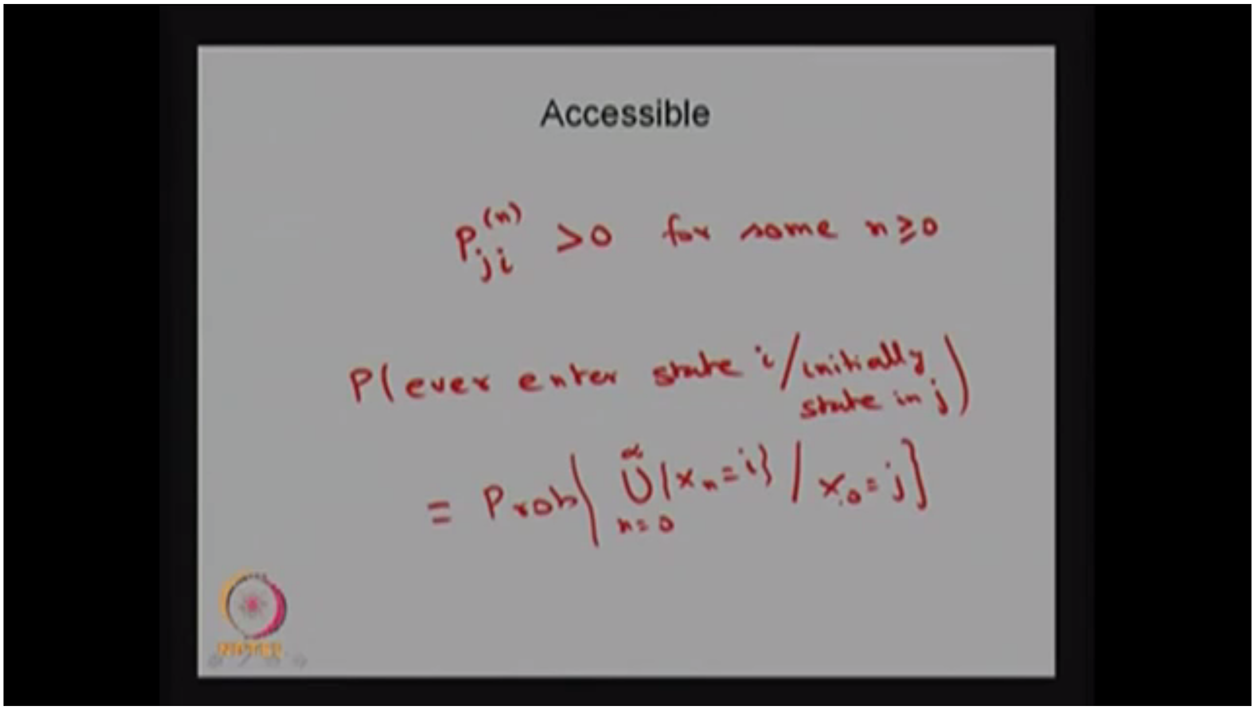
In this, we are discussing Discrete-time Markov Chain, and we have finished already two lectures on this model and this is the third lecture, Classification of States and Limiting Distributions.



In this lecture, I am going to give the information about the classification of the states for the time homogeneous discrete-time Markov chain. Then I'm going to give the definition of a limiting distribution. Then I am going to discuss a few simple examples so that we can understand the classification of states as well as the limiting distribution.

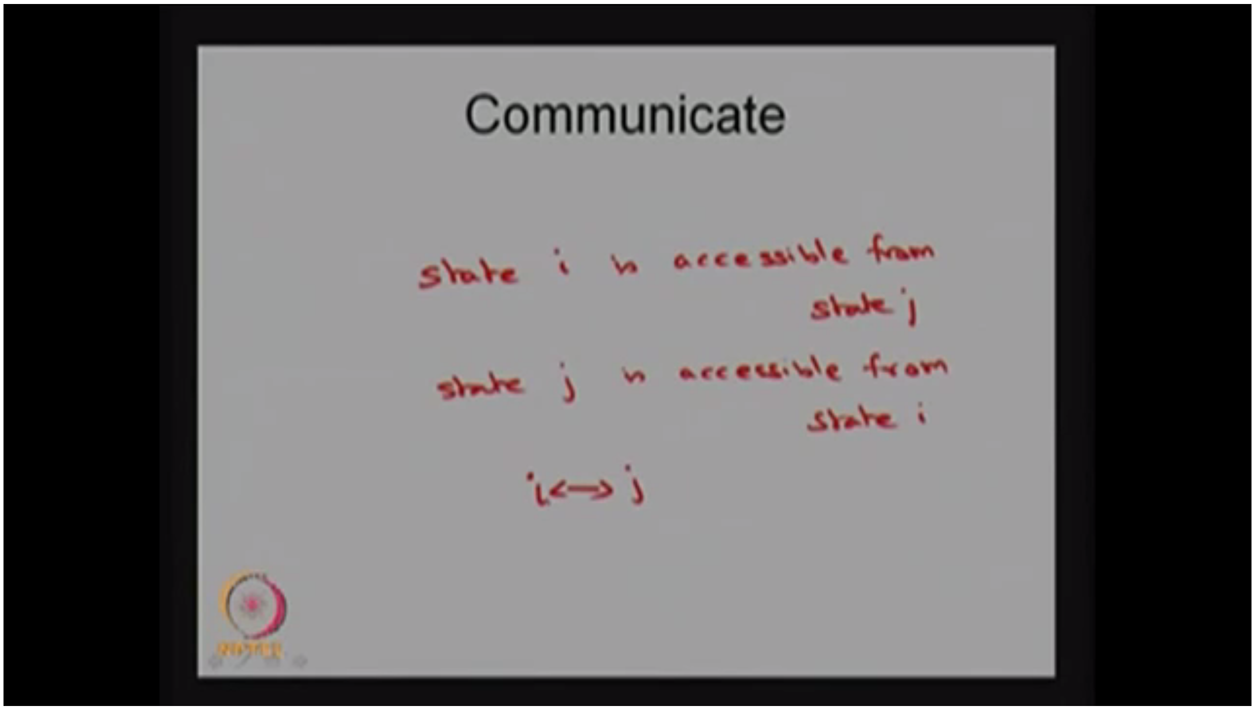


Why do you need classification of states? Whenever we study the time homogeneous or discrete-time Markov chain, our interest is to find out the limiting distribution of the random variable Xn. To study the limiting distribution for a stationary distribution, later we are going to use a word called equilibrium distribution, for all those things you need the classification of a state. Without the classification of a state, we cannot come to the conclusion whether the limiting distribution exists, whether that is going to be unique and so on, so for that we need a classification of states.

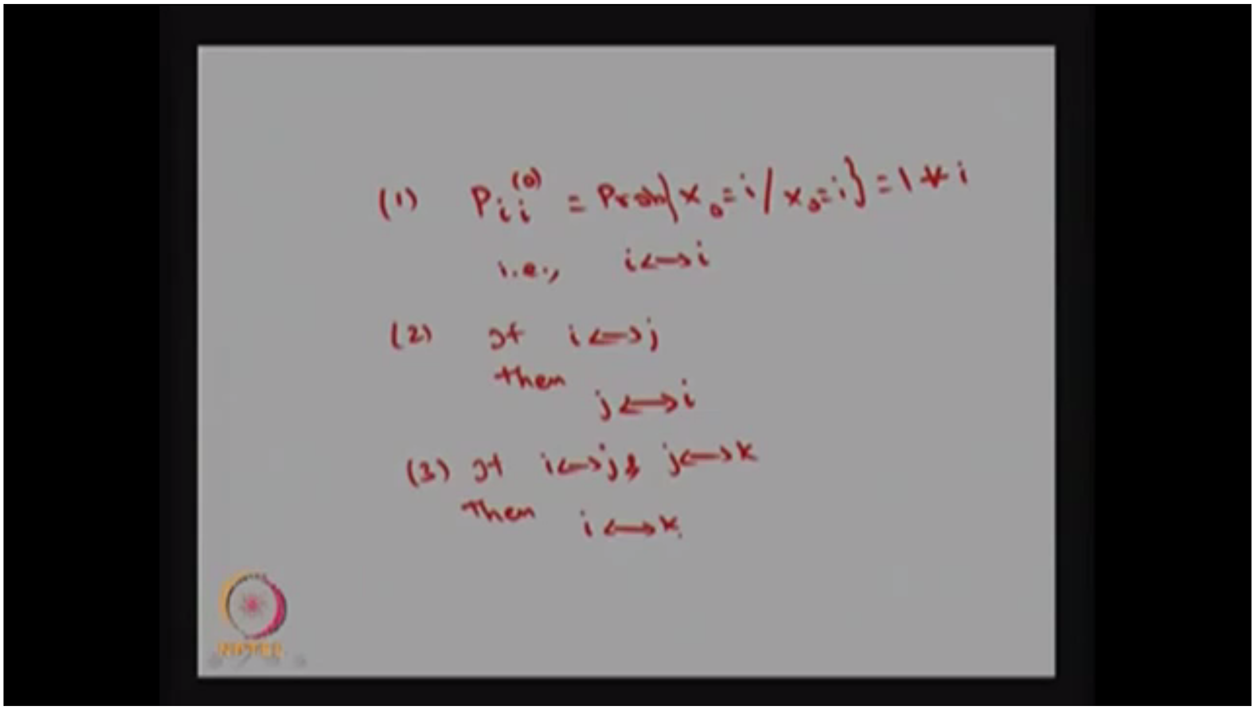


Before moving into the classification of states, we need some concepts, so that using those concepts we can classify the states. The first concept is called Accessible. When we say the state I is set to be accessible from the state and J whenever the P suffix j to i in n steps has to be greater than 0 for some n, which is greater than or equal to 0. We are including n = 0 for the safer side. Whenever we say the state I is set to be accessible from the state J if the P j,i in n steps has to be greater than 0. That means this is the transition probability from the N step transition probability matrix, and if that element is going to be greater than 0, then we say the state I is set to be accessible from the state J.

Using these we can write down what is the probability that ever he enter state I given that initially the system is in the state J. You can find out what is the probability of the system ever enter to the state I given that initially it was in the state J, that is nothing but the union of all the events corresponding to the X n takes a value i given that it was in the state J initially. You can find out what is the probability that ever entering the state I given that initially the system is in the state J that is the probability of Union of X n=1 given that X naught = J.

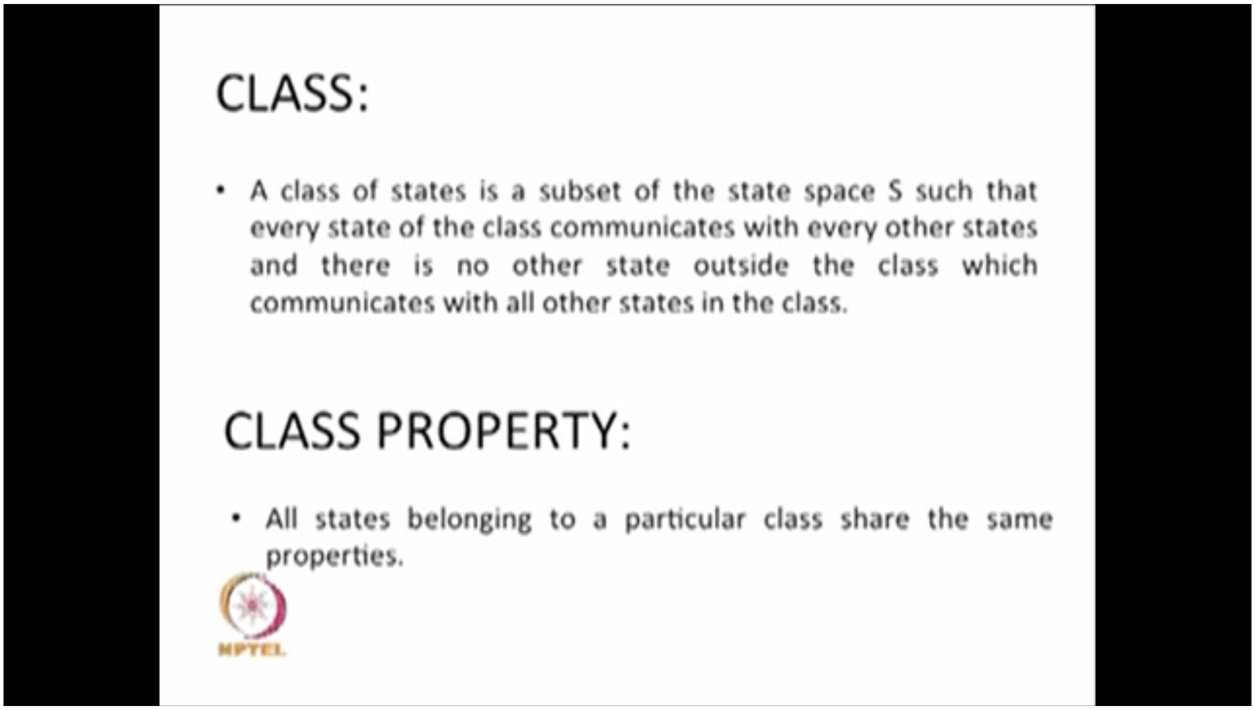


Now I am going to define, now I'm going to give the next concept called Communicate using the Accessible. Two states are set to be communicate, that means the state I is accessible from the state J as well as the state J that is accessible from state I. Whenever the state I is communicated with the state J that means the state I is accessible from state J as well as the state J is accessible from state I. In notation, we can use the notation I <-> J. State I is communicating with the state J means state I is accessible from state J as well as the state J is accessible from state I.



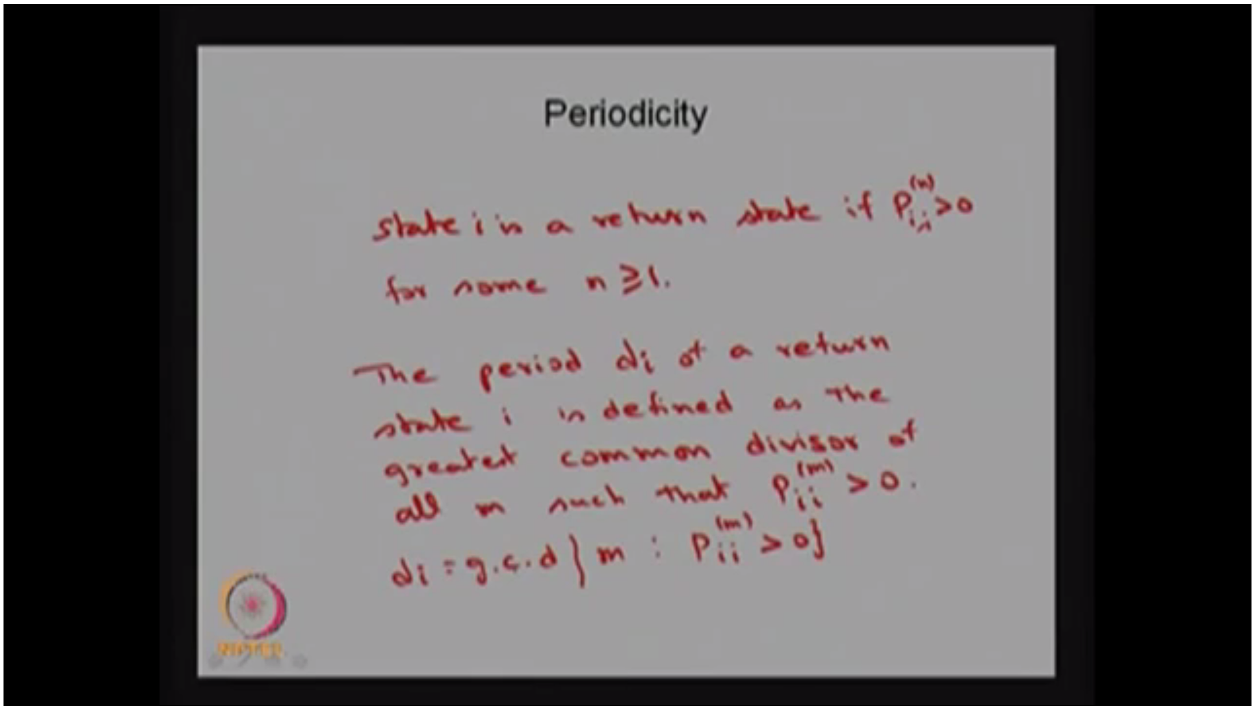
Since I use the concept of access to define the communicate, it is going to satisfy few properties. The first property, any state communicate with itself that means that the P i,i of 0 that is nothing but what is the probability that X naught = I given that X naught = I that is going to be 1 for all I, any state communicates with itself. That means in notation, I communicates with the I itself. The second property, if state I communicate with the state J, then the state J communicate with the state I also. That means it's a symmetric property. That means if I communicates with the J, then J communicates with I. The communicate satisfies the symmetric property.

The third one, if I communicates with J and J communicates with the K, then we can conclude the I communicates with K. This relation is called a transitive, so the communicate that property satisfies itself and it satisfies the symmetric property, as well as the transitive property that is if I communicates with the state J and the state J communicates the state K, then the state I communicates with the state I. Communication is an equivalence relation on the set of states and hence this relation partitions the set of states into communicating classes. I'm not giving the proof here, so one can easily prove using the one-step transition probability and the N step transition probability matrix and the accessible concept one can prove these three properties.



Now I'm going to define the next concept Class Property. What is class property? A class of state is a subset of the state space S such that every state of the class communicates with every other states and there is no other state outside the class, which communicates with all other states in the class. It’s that the time homogeneous discrete-time Markov chain. Since it is a discrete-time Markov chain, you have a state space. The state space may be a finite number of elements or countably infinite number of elements. So that is a state space S.

In the state space S, you are going to create a subset that is going to be called as a class if within the subset of that collection, it satisfies the communicate. That means each state inside the class has to be communicate with each other state and also it has to satisfy the second property, there is no other state outside the class, which communicates with all other states in the class. That means you can start with one element, then you can include one more element, you can include one more element, once this property is satisfied. That means you cannot make including one more state and make it as a class, then you have to stop framing the class. So the subset will be created by including one more state, one more state, one more state in the state space as long as this property is satisfied. So once the second property violates, that means you should stop with creating that subset and that is going to be the class. We are going to discuss this, how to create the class by simple examples, so that I am going to do it later.



Next one is next concept is Periodicity. The definition of periodicity goes like this: The state I is a return state if the P i,i of n, which is greater than 0 for some n which is greater than or equal to 1. First, I am defining what is the meaning of return state. Any state is going to be called as a return state if the probability of starting from the state I, coming to this same state in the Nth step if that is greater than 0, then we say it is a return state. Now I'm going to define the periodicity only for the return state.

The period in notation it is D suffix i, suffix i, i is for the state of a return state, i is defined as the greatest common divisor of all m such that P suffix i,i of m, which is greater than zero. So the period of return state is going to be integer and that integer is computed by using the greatest common divisor of all the possible m such that at the P i,i of m should be greater than zero. That means you find out how many steps you will take to come to the same state if you start from the state I. You collect all the possible number of steps, you will be coming back to the state with the positive probability, and you find out the greatest common divisor of those integers, those positive integers, then that number is going to be the period or periodicity of the return state or the period of the state. That means you can write down in a short D suffix i is the greatest common divisor collection of m such that the P i i of m should be greater than zero.

If the greatest common divisor of collection of m such that greater than 0, if this D i is going to be 1, then we say that state is a periodic state. Otherwise, if it is greater than 1 and whatever be the integer you are going to get and that is going to be the period of the state I. If the period is going to be 1, then we call it as the periodic state.

Note that whenever you have a class in which we have more than one states, if one state has the period some number, then the other states of the same class also going to have the same period. That can be proved easily, so within the class all the states will be having the same period.