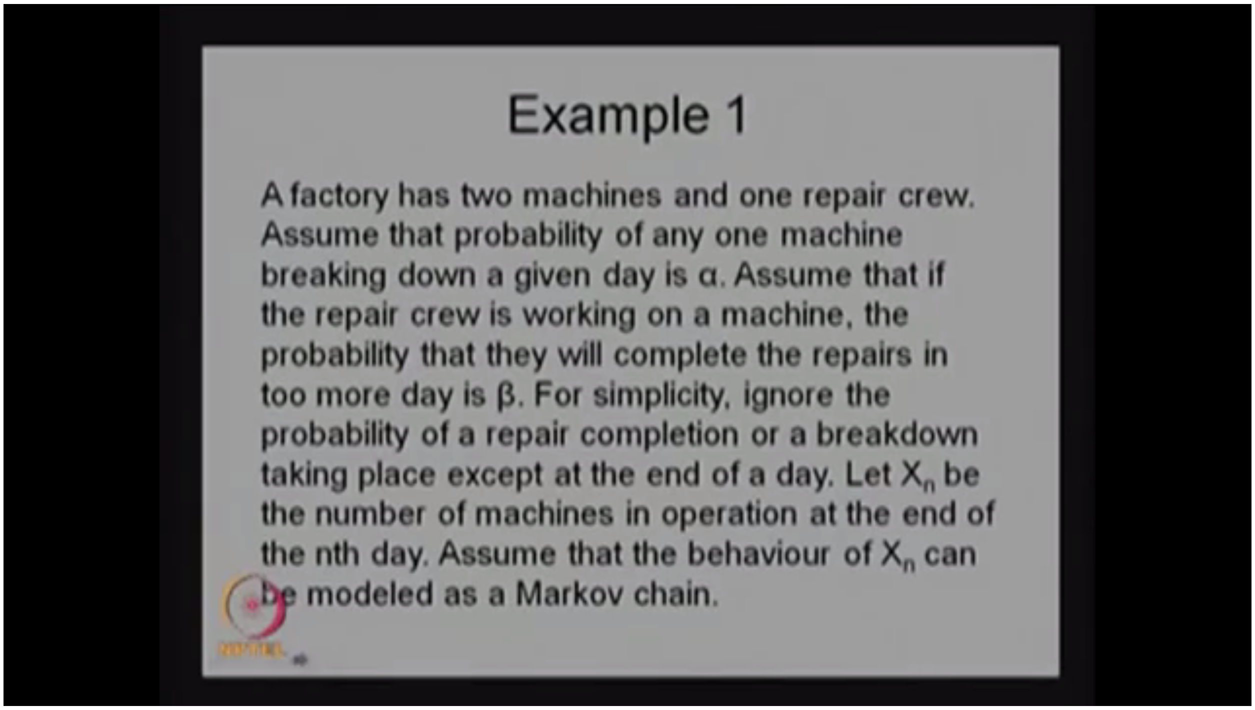
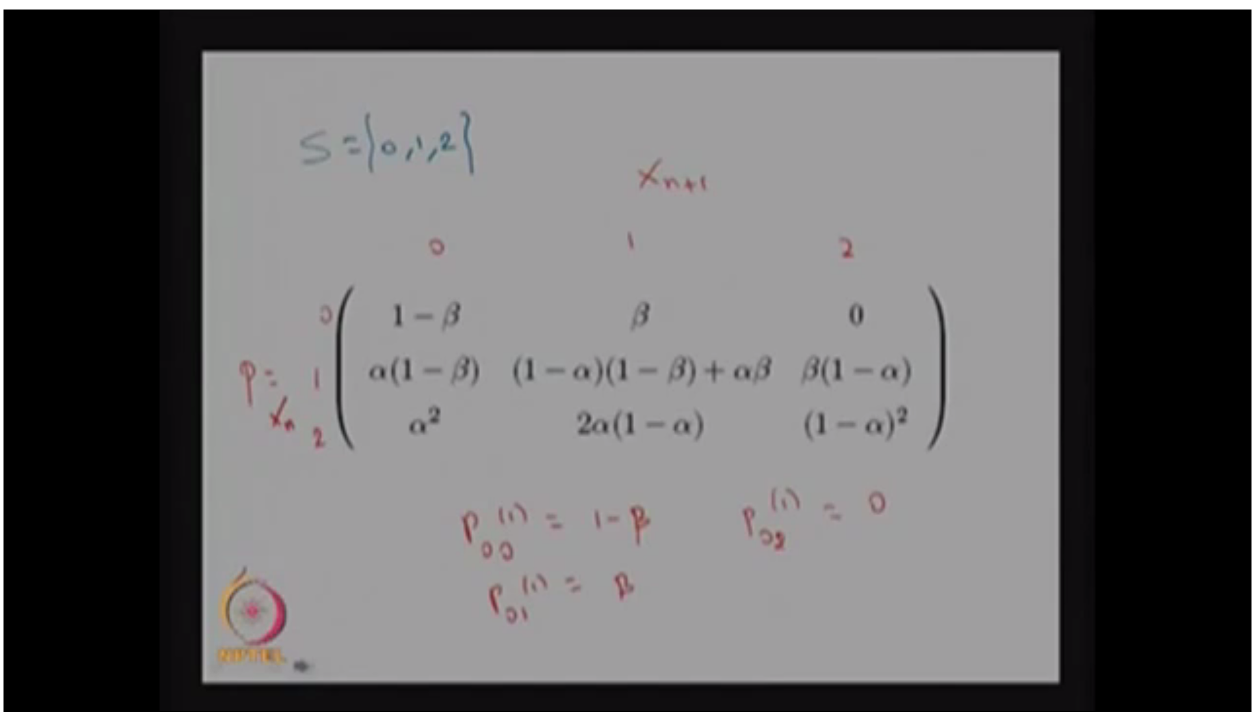
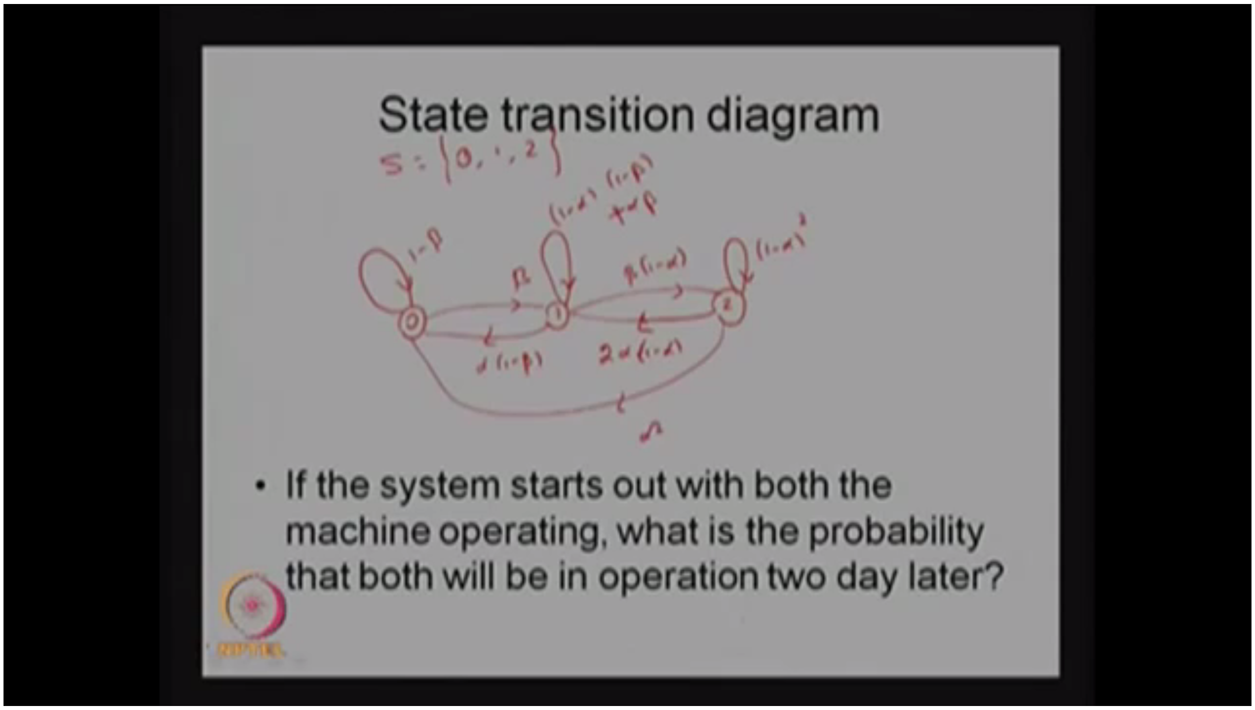
Now we are moving into simple examples using the answer transition probability matrix and the one-step transition probability vector, how to find the distribution of Xn for some simple example.

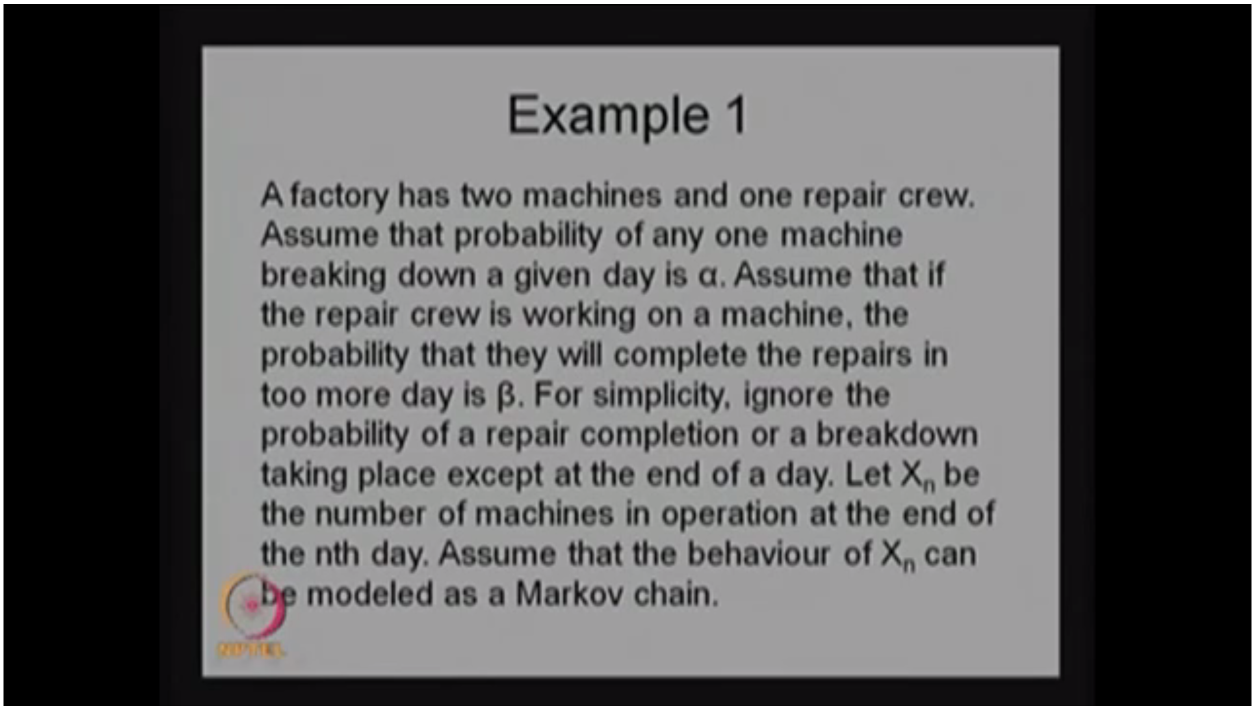




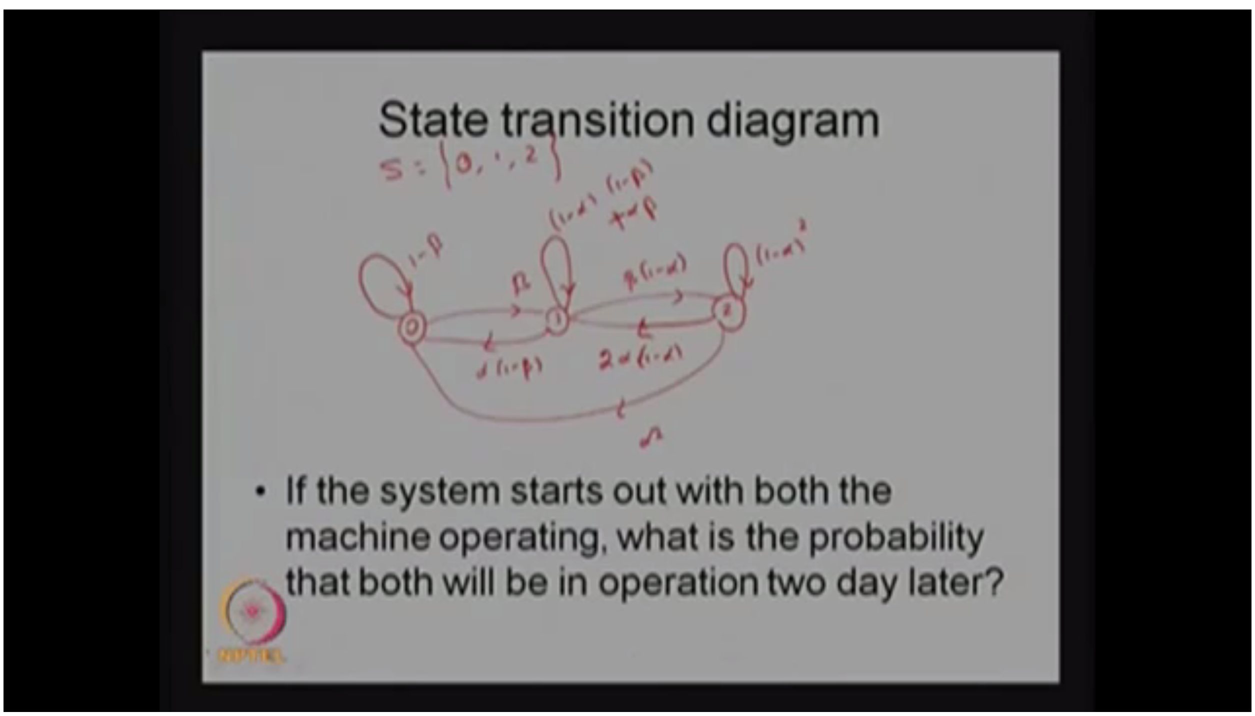
The first example, which I have discussed in the lecture one, and this is a very simple example in which the underlying stochastic process is the time homogeneous discrete time Markov chain with the state space S is 0, 1, and 2, so this is the state space, and the information which we have based on that we can make a one-step transition probability matrix that is nothing but what is the possible probability in which the system is moving from the state I to J in one step that you can fill it up. So this exercise we have done it in the lecture one.



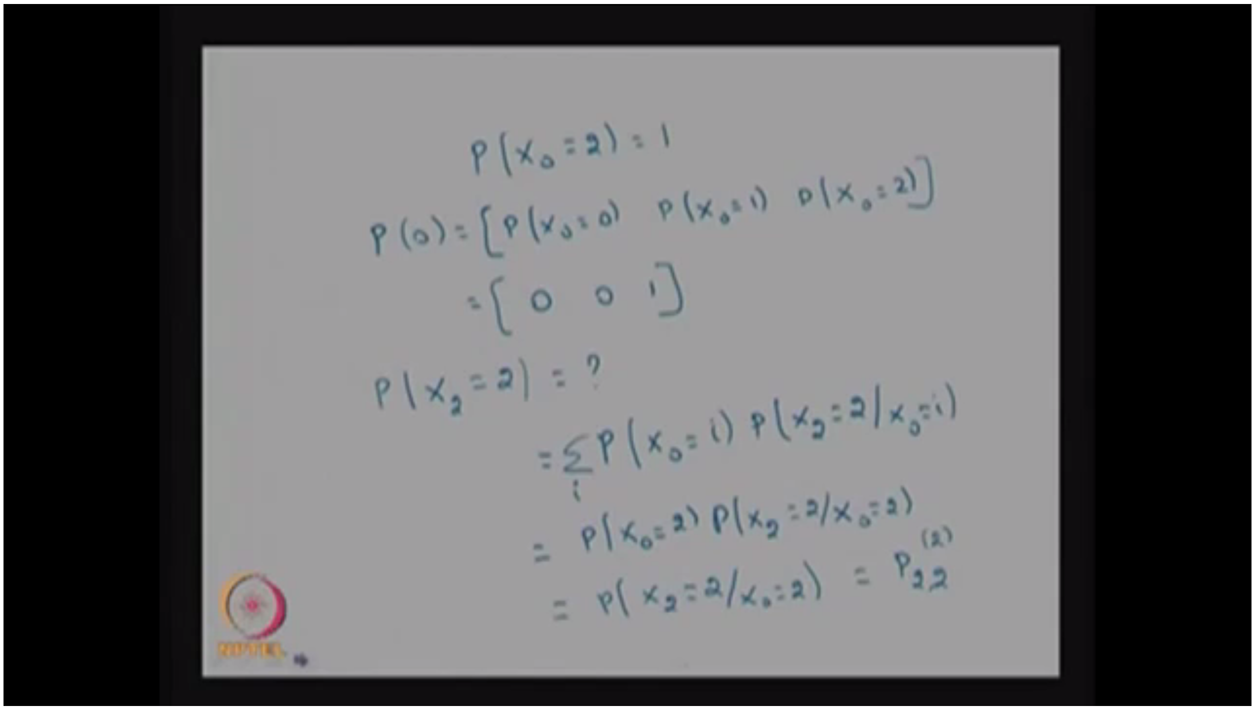
And now our interest is to find out and also we have made the state transition diagram which is equivalent to the one- step transition probability matrix and we have got the state transition diagram. Now the question is if the system starts out with both the machines operating what is the probability that both will be operation two days later.



So if you recall what is a random variable the X n be the number of machines in operation at the end of the end of the Nth day. So the random variable is how many machines are in the operation at the end of Nth day.



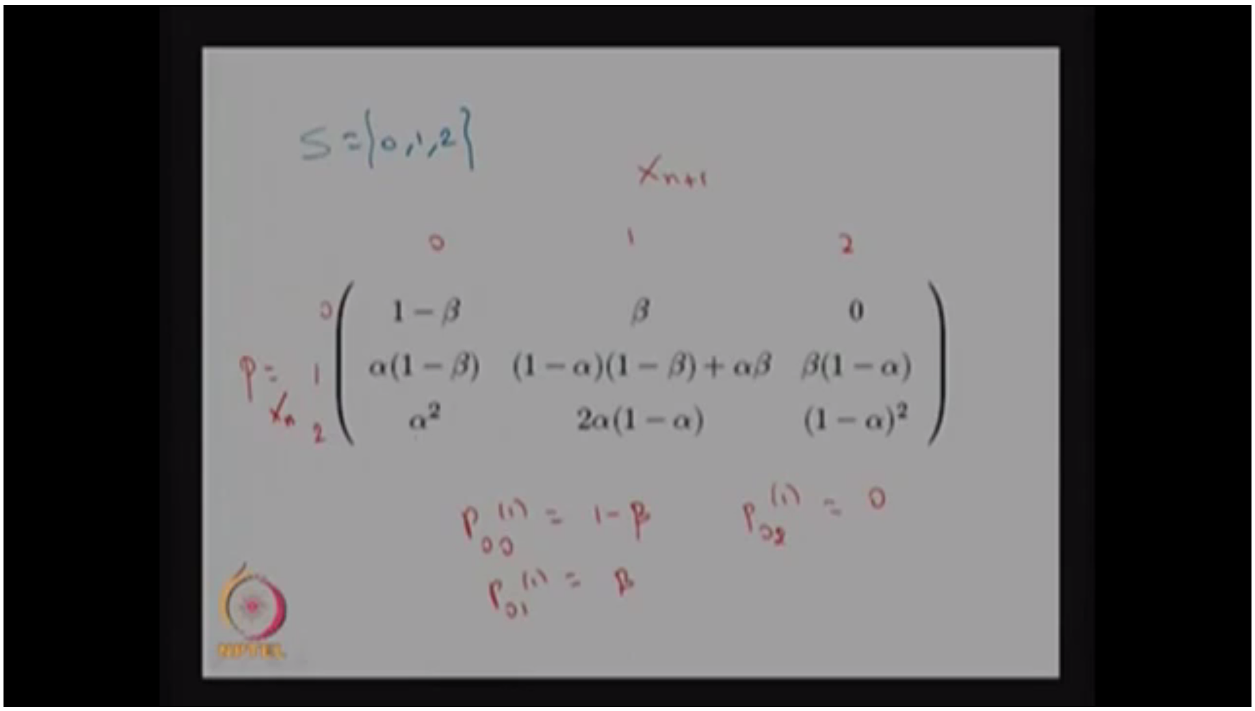
So here the clue is at the time 0 or the 0th step both the machines are operating. Therefore, X 0=2 with the probability 1, so the given information with the probability 1, both the machines are working at the 0th step.



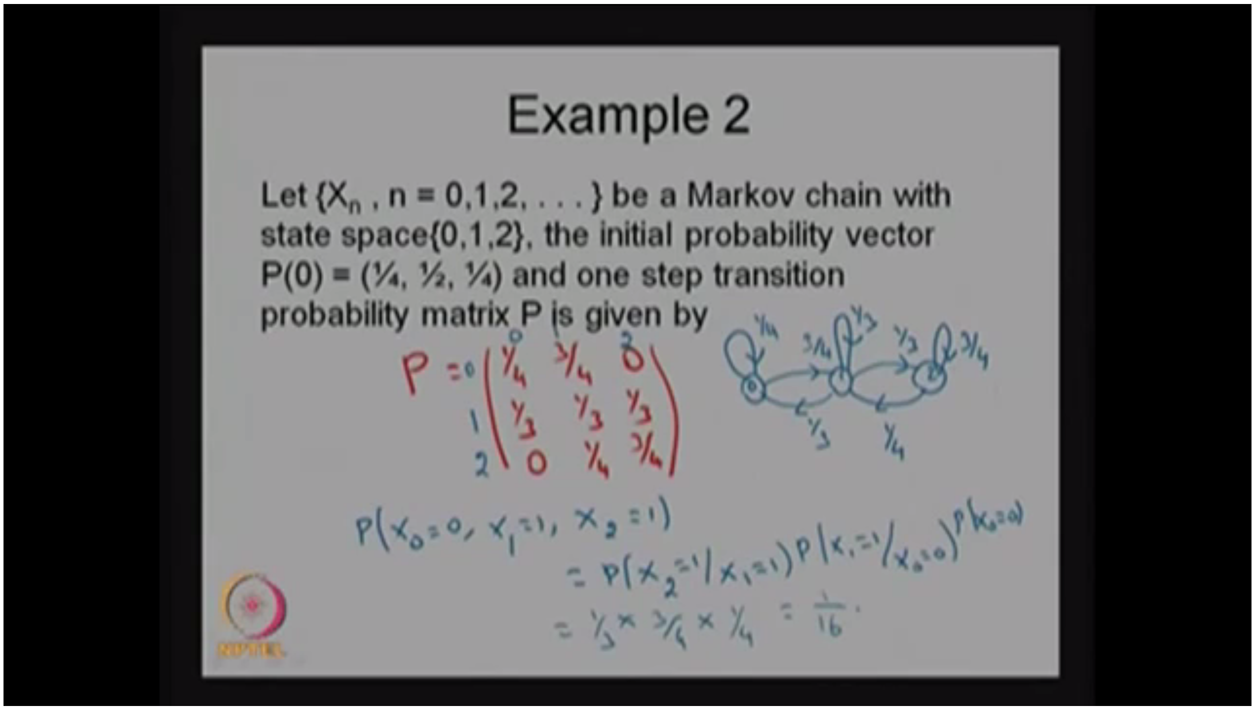
So this can be converted into the P X 0 takes a value 2, that probability is 1, or we can make it in the initial probability distribution or initial probability vector as what is the probability that at X 0 the system was in the state 0, at the 0th step the system was in the state 1, so this is the initial probability vector. So at time 0, the system was in the state 2; therefore, that probability is 1 and all other probabilities are 0. So this is the given information about the initial probability vector.

Now the question is what is the probability that both will be operation two days later, that means what is the probability that you can convert this into what is the probability that the X 2 in the second step, the system will be in the state 2, given that the system was in the state 2 at the 0th step. So this is what you have to find out, what is the conditional probability. If this is some starts with the both the machines what is the probability that both will be operationing two days later. So not even this is a conditional probability, the question is what is the probability that what is the probability that the system will be in the state.

So to find this, you can make what is the probability that with the given information is there X 2=2 given that X naught = I for all possible values of I, and this is same as since the initial probability vector is going to be 0 0 1 so this is land up, what is the probability that the X naught = 2 multiplied by what is the probability that X 2 = 2, given that X naught = 2 and all other probabilities are 0 that was 0 into anything is going to be 0, that's the same as what is the probability that X naught = 2 and the conditional probably, and X naught = 2 is 1, therefore this is same as what is the probability that X 2 = 2 given that X naught = 2. So this is same as what is the probability that 2, 2 in two steps. This is nothing but the system was in the state 2 add 0th step and the system will be in the state 2 after the two steps. So this is a two-step transition probability of system moving from the state 2 to 2.



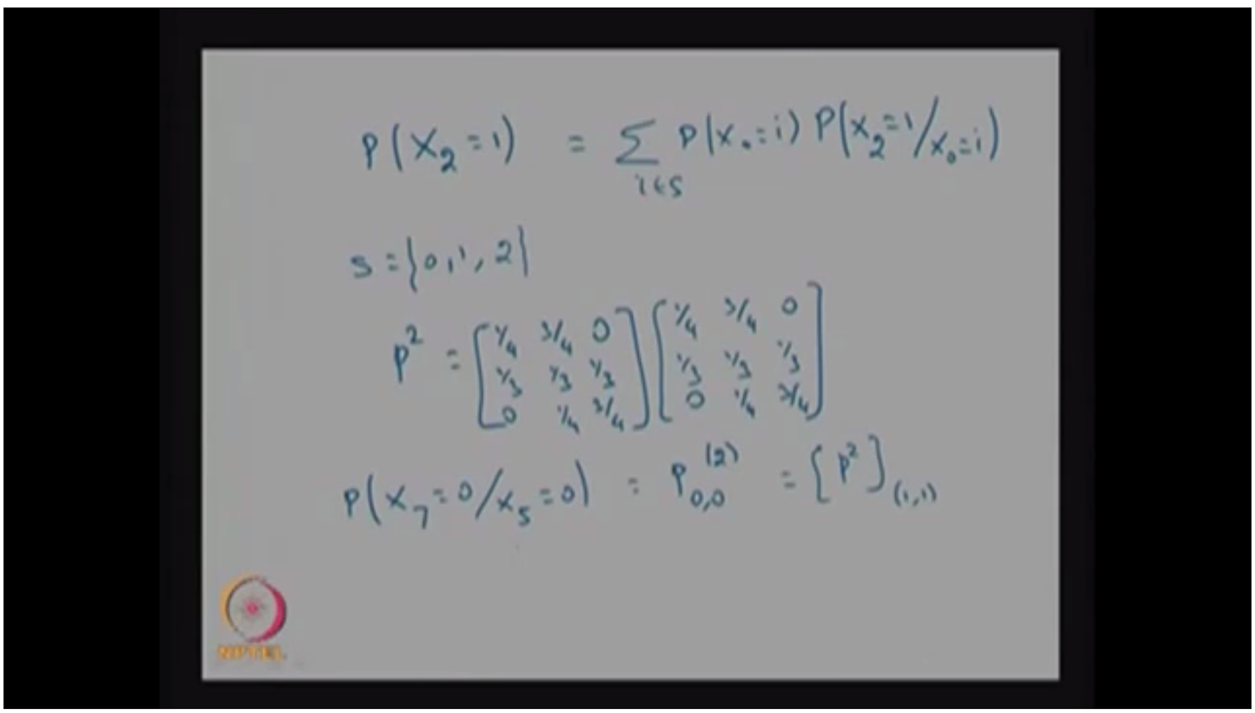
This is same as you find out the P square matrix and from the P square matrix, this is nothing but the 2,2 that is going to be the last element out of that 9 elements, the third row third column element, that is going to be the probable element for this probability. So what do you have to find out is find out the P square, find out the P square. So we have provided the P, so this is the P matrix. So from the P matrix you find out the P square. So the P square is also going to be a 3 x 3 matrix, so from the P square 3 x 3 matrix, you take the third row third element, third column element, and that is going to be the probability for two-step transition of system moving from the state 2 to 2, that is going to be the answer for the given question, what is the probability that both will be operation in two days later. Similar to this, we can find out the probability for any day, for any finite day, by finding the P power n matrix then pick the corresponding element and that is going to be the corresponding probability.



Now we'll move into the next example. This is abstract example in which they Xn be the discrete time Markov chain, the default discrete time Markov chain is always -- it's a time homogeneous. So this is a time homogeneous discrete time Markov chain with the state space 0, 1 and 2, and also it has provided the initial probability vector, that is a P 0 -- that is a vector that is 1/4 of ¼, so the summation is going to be 1. Therefore, this is the initial probability vector. That means that the system can start from the Nth 0th step with the probability 1/4 from the state 0, from the state 1 with the probability of, with the probability 1/4 it can start from the state 2, and also it is provided the one-step transition probability matrix. From the one-step transition probability matrix, you can you can draw the state transition diagram also, because the state space is 0, 1, 2, therefore, the nodes are 0, 1 and 2, and this is the one step transition probability, therefore, 0 to 0, that probability one step -- one step, the system is moving from the state 0 to 0 that is ¼, and the system is moving from the state 0 to 1 in one step that is a ¾, and there is no probability from this going from the state 0 to 2, therefore, you should not draw the arc.

From 1, the one step transition probability of 1 to 1 is 1/3 and this is 1/3 and similarly this is 1/3. From the state 2, 2 to 0 is 0, and the 2 to 1 is ¼, and 2 to 2 is ¾. This diagram is very important to study the further properties of the states. Therefore, we are drawing the state and diagram for the discrete time Markov chain. So this is a one-step transition probability matrix and this is the state transition diagram. Our interest is to find out the few quantities that is, what is the probability that X naught = 0 and X 1 = 1 and X 2 = 1. What is the probability that the system was -- it's a joint distribution of these three random variables, X naught = 0 and the X 1 = 1 and X 2 = 1. So this is same as the joint distribution, it’s same as, you can write it in the product of the conditional distribution. And the conditional distribution, again, you can write it using the Markov property, the conditional probability of only one step. Therefore, this is going to be -- by using the probability theory, you apply the joint distribution same as the product of conditional distribution by using the Markov property, you reduce it into another conditional distribution. So this is same as what is the probability that X 2 = 1 given that X 1 = 1, multiplied by X 1 = 1 given that X naught = 0 and the probability of X naught = 0. So the first term is nothing but the one-step transition of system is moving from 1 to 1, and this is nothing but the system is moving from the state is 0 to 1, and this is the initial -- you take the probability from the initial probability vector of X naught = 0.

Yeah, now we are going to label the one-step transition probability matrix with a 0, 1, 2 and 0, 1, 2. From this, we can find out ,this is a one-step transition probability of system moving from 1 to 1, so 1 2 1 is 1/3 into this is a system probability of system moving from 0 to 1, so 0 to 1 is ¾, and the system started from the state 0 in the 0th step, so that we can take it from the first element that is ¼. So if you do the simplification, you will get 1/16. So this is the joint distribution of the system was in the state 0 at 0th step, the system was in the state 1 and the first step, and the system was in the state 1 and the second step that probabilities 1/16.



Similarly, you can find out the other probabilities also. That is, suppose our interest is, what is the probability that at the end of a second step, the system will be in the state 1. That is nothing but what are all the possible states in which the system would have been started from the state I and what is the two-step transition of system is moving from the state I to 1. So the I is belonging to S. So here the S is a 0, 1, 2, so already we have given the initial probability vector that is 1/4 of and 1/4 using this and you need a two-step transition probability. That means you need to find out what is the P Square, so the P square will give the two-step transition probability matrix. Therefore, the P is provided to you so the P is 1/4, 3/4 and 0, 1/3, 1/3, 1/3, 0, 1/4, 3/4. So this is the P, so you multiply the same thing, 1/3, 1/3, 1/3, 0, 1/4, 3/4. You find out the P square. So from the P square, you pick out the element of X 0 zero is equal to four all possible I, then multiply this and this. That multiplication will give probability of X 2 = 1. So I am not doing the simplification, so once you know the P square, you can find out probability of X 2 = 1.

Similarly, one can compute the other conditional probabilities also. Suppose our interest is find out what is the probability of X 7 = 0 given that X 5 was 0. This is same as what is the probability that the system was in the state 0 at the 5th step given that what is the probability that the system will be in the state 0 in the 7th step. That is same as what is the probability of 2, what is the probability of -- what is the probability of 0,0 in two steps. That means you find out the P square from the P square, the 0,0 is the nothing but you take the first row first column element, and that is going to be the probability of X 7 = 0 given that X 5 = 0. Similarly, you can find out all other different conditional probability and what do you have to do is always you have to convert because of the given DTMC is a time homogeneous, so you convert it into find out the N step transition probability, and the N step transition probability is same as the P power n. So you pick the corresponding element to find out the condition probability.