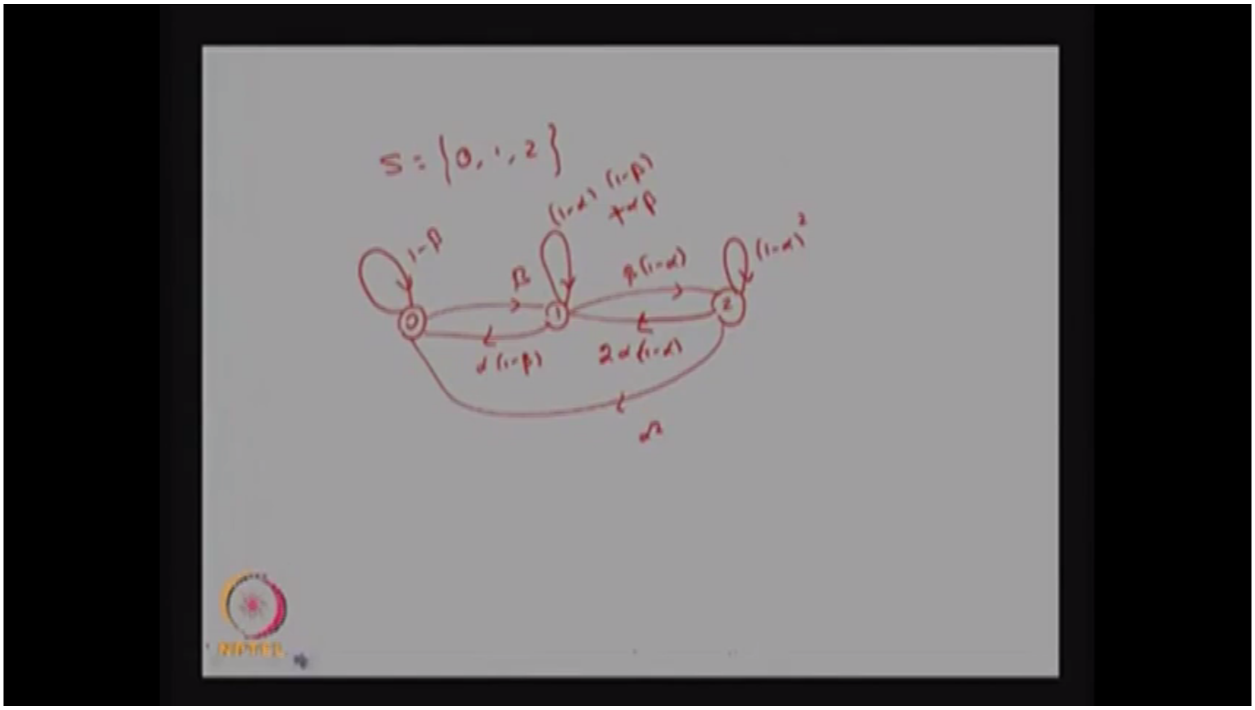
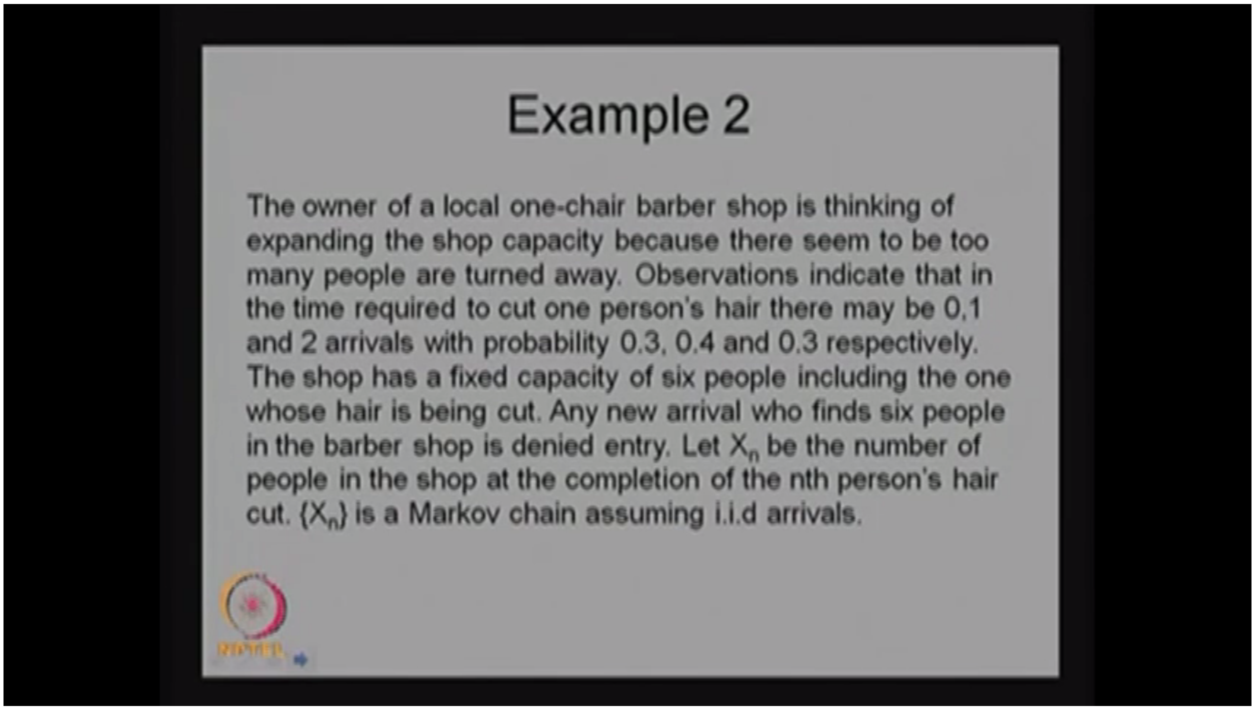


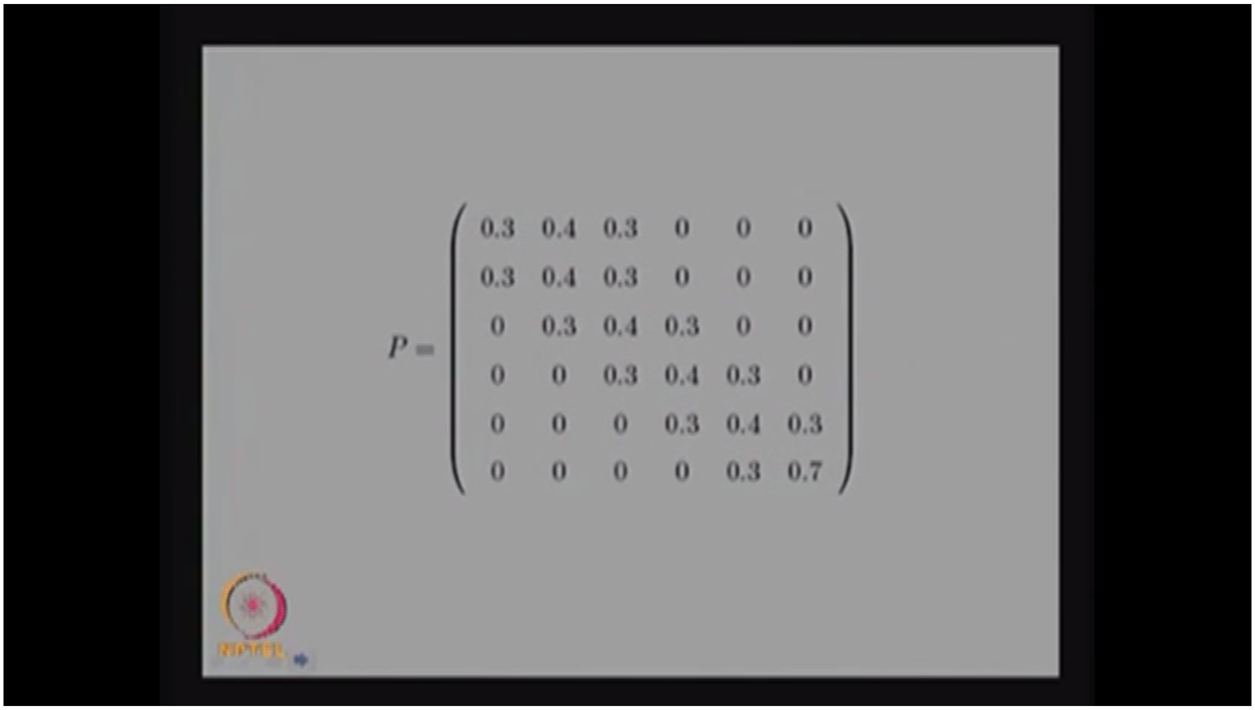
Now I'm moving into the second example. In this example I have taken the barbershop example which I have discussed in the model 1 also, the same example. So the owner of a local one chair barber shop is thinking of expanding the shop capacity because there seems to be too many people are turned away. Observation indicate that in the time required to cut one person's hair there may be 0, 1, and 2 arrivals with the probability 0.3, 0.4 and 0.3 respectively. So this information is very important that means during one person's a haircut what is the probability that no people turned up with the probability 0.3 and the one people may turned up with the probability 0.4 and there is a possibility two arrivals is possible during the one person's haircut with the probability 0.3 therefore the summation of probability is going to be 1. So during the one person's haircut these are all the only three possibilities are possible with the 0 arrival or 1 arrival or 2 arrival. The shop has a fixed capacity of six people including the one whose hair is being cut that means a maximum six people can be allowed in the system.



So five people can wait maximum and one people under the service. Any new arrival who find six people in the barber shop is denied entry. That is the meaning of a capacity of the system is finite with the size six. Now I am going to define the random variable. Let Xn be the number of people in the shop at the completion of the nth person's haircut. This is very different random variable or this is very different stochastic process. Usually the parameter space is a time but here the parameter space is the number of people in the shop. The parameter space is the person who leaves after the haircut. So it's a nth person who leaves the system that becomes the parameter space. Whereas the random variable is how many people in the system when the nth person leave the system. That means you should not count at that person then you are finding the values of X. That means this number is counted at the departure time point.

So when the nth person leaves how many people in the system and the system is maximum six people allowed therefore he cannot see more than five people in the system when he leaves. So because of this constraint, because of during the one person arrival either 0 or 1 or 2 arrivals can take place and so on based on this information the stochastic process Xn is going to be a discrete time discrete state stochastic process as well as the Markov property satisfied. That means the probability of Xn plus 1 takes some value given that all the previous values are known that is same as the conditional probability distribution of Xn plus 1 takes some value given that Xn was some value. So all so the future distribution given that present as well as the past information is same as the future distribution given the present; not the whole past information.

So this Markov properties will be satisfied by this stochastic process therefore this Xn will form a discrete time Markov chain. Obviously, it is the time homogeneous discrete time Markov chain also. So in this example our interest is to find out what is the one step transition probability matrix.



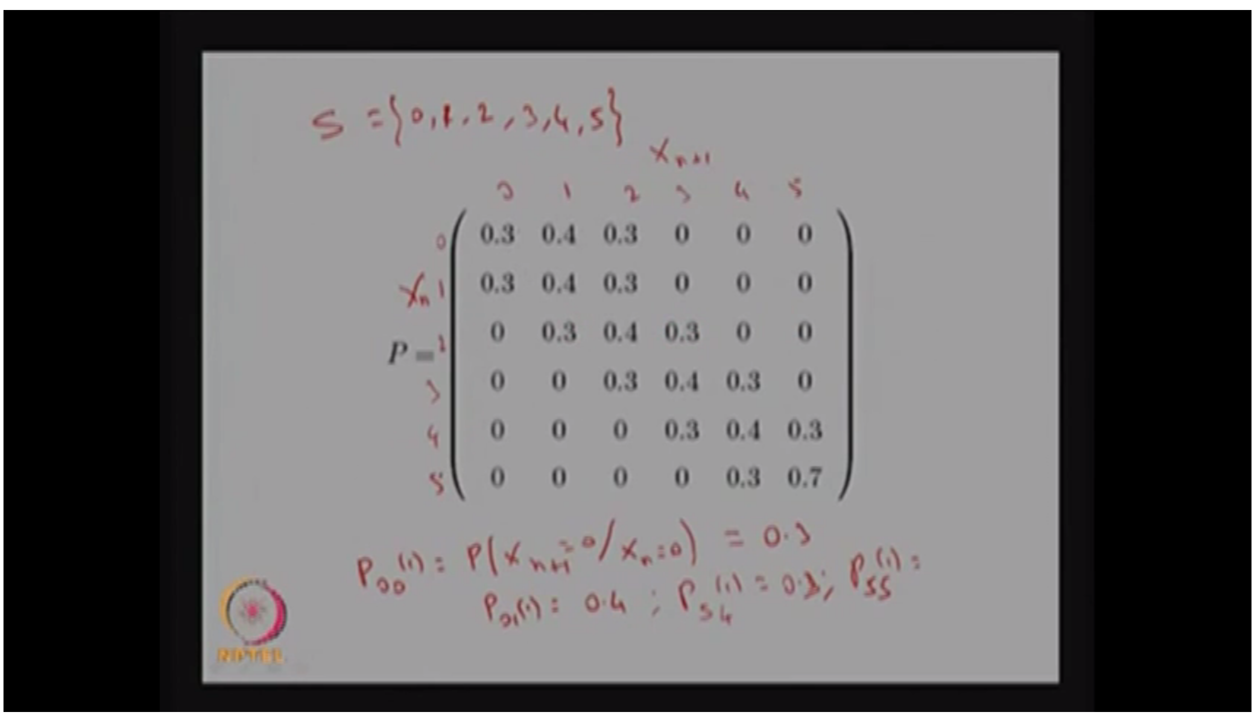
This is going to be the one step transition probability matrix and the possible states S is going to be 0, 1, 2, 3, 4, or 5 because the capacity of the system is 6 and whenever he – whenever the nth person leaves either first person, second person, third person leaves how many people are in the system. Therefore the maximum will be 5 and there is a possibility when he leaves known with in the system also. And this is the one step transition probability matrix and this is also going to be a square matrix because it is going to be countably finite number of elements and this is a 0, 1, 2, 3, 4, 5. Now we can discuss what is the probability that 0,0 in one step. That is nothing but in the nth – when the nth person leaves no one in the system and when the n plus 1th person leaves no one in the system. What is the probability for that? That is Xn plus 1 is equal to 0 given that Xn was 0. It's a one step transition probability. It's independent of n because it's a time homogeneous. It's a one step transition probability matrix.

So this is possible at some person leaves whatever be the n nobody in the system. When the next person leaves nobody in the system. So that is possible by when some person leaves the system was the empty for some time you don't know how much time it was empty. Then the n plus 1th person enter into the system and during his haircut no one turned up or no arrival takes place during his or n plus 1th haircut is going on.

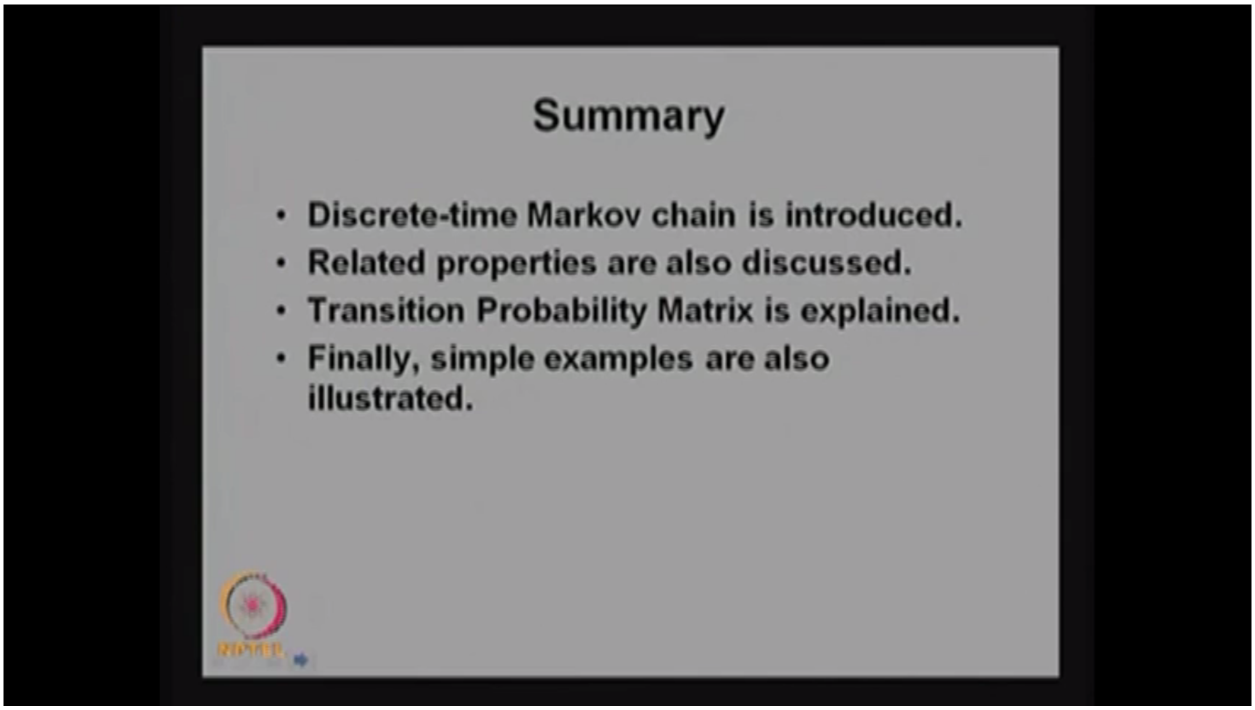
Therefore, when he leaves no one in the system. So we are not bothering when he entered into the system and so on. Our interest is how many numbers of people in the system when the n plus 1th person leaves and this probability is n plus 1th person leaves the zero people in the system and given that when the nth person leaves also zero person in the system. So that is possible with the explanation I have given; no one enter into the system during the n plus 1th person's haircut. And the information is provided indicate that that time required to the haircut one person haircut there may be a 0,1, or 2 arrivals with the probability 0.3. So no arrival takes place during the one person's haircut is 0.3. Therefore this probability is possible with the probability 0.3 whereas P01 of one step the same way you can write probability that Xn plus 1 is equal to 1 given that Xn is equal to 0 that is possible when the nth person leaves no one in the system when the n plus 1th person leaves one person in the system that means during his haircut one person enter into the system. That is possible with the probability 0.4. Similarly from 0 to 2 in one step that is going to be 0.3 with the probability two arrival takes place during the n plus 1th person's haircut.

Now the second row. Second row what is the probability that when the nth person leaves one person in the system when n plus 1th person leaves 0 person in the system. That is possible during the n plus 1th person haircut there is no one in the system. No arrival takes place therefore that probability is 0.3 and from one to one that is possible with one person arrived during the n plus 1th person haircult. Therefore that probability is 0.4 and going from the state 1 to 2 that is possible two persons arrived at during the n plus 1th person haircut. Whereas from 2 to 0 that is not possible because when the nth person leaves the two person in the system therefore n plus 1th person in the leaves definitely he will see one person in the system because of no arrival and one arrival and two arrival.

Therefore, it will be shifted by one column and it will be keep continuing till the end whereas the last one what is the probability that the five people in the system when the nth person leave and four people in the system when the n plus 1th person leave that is same as no arrival takes place during the n plus 1th arrival, n plus1th haircut going on. So therefore this is going to be 0.3 whereas P5 to 5 in one step that is possible with the combination of one person arrived the system or two person arrive the system this system size is going to be maximum 6 therefore when n plus 1th person haircut is going on if one person arrives then he will be entered. If two person arrives then he cannot be accommodated therefore he won't join the system. Therefore the system -- the number of customers in the system in the Xn that is going to take the value 5 and the combination of 0.4 as well as 0.5 therefore this probability of system is moving from 5 to 5 is 0.7 because of 0.4 plus 0.3.



Now I can give the state transition diagram for this example because S is going to be 0, 1, 2, 3, 4, 5 therefore the nodes are going to be 0, 1, 2, 3, 4, and 5 and the possible values from the one-step transition probability matrix I can make out so 0 to 0 that probability is 0.3 and 0 to 1 is 0.4 and 0 to 2 is 0.3. Similarly I can fill up the all other things and five to five that is very important and five to four that is possible with the probability 0.3 and the five to five is possible with the probability 0.7. So this is the state transition diagram. I didn't complete the state transition diagram. You have to fill up all the aux with the rates going from one or two other arc with the positive probability. Wherever there is a probability zero we should not draw the arc for it.



So in this lecture we have discussed the discrete-time Markov chain. Then we have given the few important properties followed by we have explained the one-step transition probability matrix and also we have given two simple examples with this the lecture one is over for the model 4. Thanks.