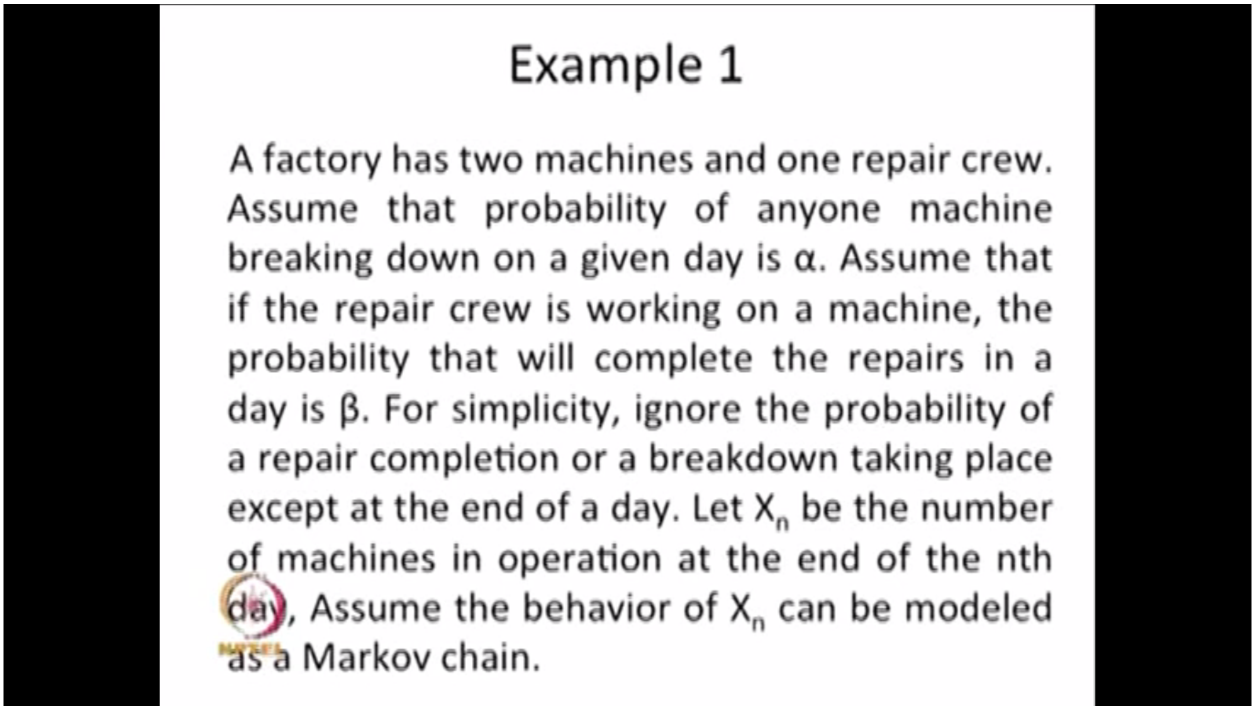


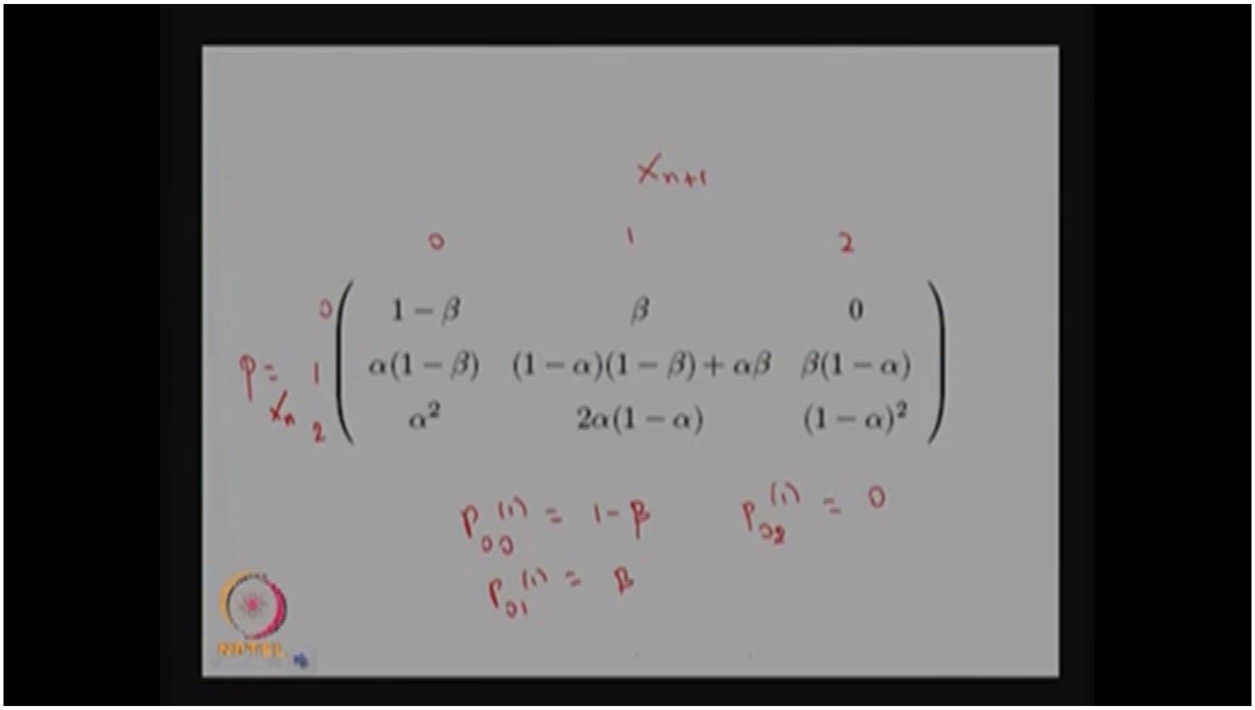
I am going to explain the discrete time Markov chain with the three simple examples the first example is as follows. A factory has a two machines and one repair crew. Assume that probability of any one machine breaking down here given day is alpha. So the alpha is the probability. Assume that if the repair crew is working on a machine the probability that they will complete the repairs in two more day is beta. For simplicity ignore the probability of a repair completion or a breakdown taking place except at the end of a day that means we observe the system at the end of the day. How many working machines in the system. Let Xn be the number of machines in operation at the end of the nth day. Assume that the behavior of Xn can be modeled as a Markov chain.

So based on the information available here the machine can be break down and we have only one repair person and the probability of he can do the repair in a day that probabilities beta. And 1 minus beta is the probability that he cannot be able to complete the repair of machine in a day. And the random variable Xn is it denotes how many machines are in the operation at the end of the day. Therefore, the possible values of Xn since we have a two machines the possible values of Xn will be 0, 1, or 2 so this will form a state space S. So the S consists of the element 0, 1, and 2 and the Xn over the n it is going to form a discrete time Markov chain because it is a discrete time a discrete state stochastic process and also the based on the clue the number of machines are working in any day depends on how many machines are working on the previous day and how many things are under repair and so on.

So the dynamics of the number of machines in operation depends only on the number of machines working in the previous day not all the previous earlier days. Therefore, the memoryless property is satisfied by the stochastic process Xn therefore this is called a discrete time Markov chain.

Our interest is to find what is the one step transition probability matrix with the assumption that the Xn is the time homogeneous also. Since it is a time homogeneous at DTMC therefore we are trying to find out what is the one step transition probability matrix for the given time homogeneous DTMC. So this is the one step transition probability matrix Pn and the possible states are 0, 1, and 2 and suppose the system was in the state 0, 1, or 2 in the nth step where the system will be in the n plus 1th step. Therefore this is the possible values of Xn plus 1 and this is the possible values of Xn and this one-step transition probability matrix will give suppose the system was in this state at the nth step what is the probability that it will be in these states in the n plus 1th step.

So the first index will give what is the probability that 0,0 in one step that means in the nth step number of working machines are 0 and what is the probability that in the n plus 1th step also 0 machines are in working condition. That means all are under repair. All two machines are under repair and the probability of a crew is going to be not repair that is going to be 1 minus beta therefore the probability is 1 minus beta. In one step what is the probability that the number of working machines going from 0 to 1 that is because of the crew is finishing the repair in a day and that probability is beta and since he can do the only one repair in a day therefore the possibility of repairing more than one machine in a day it is not possible. It's a rare event and the probability is going to be 0 therefore P02 is going to be 0. Similarly now we can visualize what is the probability that number of working machines is 1 in the nth step and what is the probability that in the 0 machines will be working in the n plus 1th step that is possible with the two independent things. The one machine can be failed and the other machine cannot be – finishing the repair. Therefore the crew is not finishing the repair that probability is 1 minus beta multiplied by one machine is going to be repaired therefore the total number of machines working will be 0 in the n plus 1th step that is alpha times 1 minus beta. And similarly you can evaluate the other element also and for example the system is going from the state 2 to 0 that is nothing but at nth step two machines are in the working condition and the n plus 1th step zero machines are the working condition that means both the machines got a repair got failed in the same day. Therefore that probability is alpha times alpha that is a probability both the machines got failed in the same day therefore in the next day the number of working machine is going to be from 2 to 0 like that you can visualize the other elements also.



The same one-step transition probability matrix can be visualized with the state transition diagram and the state transition diagram you have to make with this state space [Indiscernible] [00:07:31] and the weights of the [Indiscernible] [00:07:35] are nothing but the one-step transition probability of system is moving from one state to other states. Those are going to be the weights. If the probabilities are zeroes then no need to draw the directed arc from that particular node to the destination node.

So first you start with the nodes as the possible values of the state space. So you list out all the state space as a node. Now by seeing the one-step transition probability matrix you should make the arc from 0 to 0 self-loop is allowed if the probability is going to be greater than 0. So you should draw the self loop from 0 to 0 with the arc value 1 minus beta and you should draw the arc from 0 to 1 with the arc weight beta and you should not draw any arc from 0 to 2 because that probability is going to be 0. Therefore 0 to 0 that probability is 1 minus beta and 0 to 1 it is going to be beta and there is no arc from 0 to 2 because that probability is 0 and similarly now we can go for filling the second row. So 1 to 0 is alpha times 1 minus beta 1 to 1 is 1 minus alpha times 1 minus beta plus alpha beta and 1 to 2 so you have all three probabilities are greater than 0 therefore 1 to 0 that arc is alpha times 1 minus beta and 1 to 1 is 1 minus alpha times 1 minus beta plus alpha beta. 1 to 2 is beta times 1 minus alpha. Similarly you can draw the arc for 2 to 0 that is alpha square and 2 to 1 and so on. Therefore, 2 to 0 that is alpha square and 2 to 1 is 2 alpha times 2 to alpha times 1 minus alpha and 2 to 2 that is 1 minus alpha whole square.

So the state transition diagram is a pictorial view of a one-step transition probability matrix. This is nothing to do with the initial probability distribution. It gives only information about whenever the DTMC is a time homogeneous suppose the system start from one particular state what is the probability that the system will move into the another states with probability and it won't give more than that information but this much information is useful when you are going to study the properties of the discrete time Markov chain as well as when you want to find out the limiting distribution that is the distribution of Xn as n tends to infinity the diagram will be very useful to conclude whether the limiting distribution exists or not if it takes this whether it is going to be unique or not and so on. So those things can be visualized easily by seeing the state transition diagram.