Video Course on

Stochastic Processes-1

by

Dr. S Dharmaraja

Department of Mathematics, IIT Delhi

Module 3: Discrete-time Markov Chain

Lecture # 1

Stochastic Processes

Module 4: Discrete-time Markov Chain

Lecture 1:

Introduction, Definition and Transition Probability Matrix

Dr. S. Dharmaraja

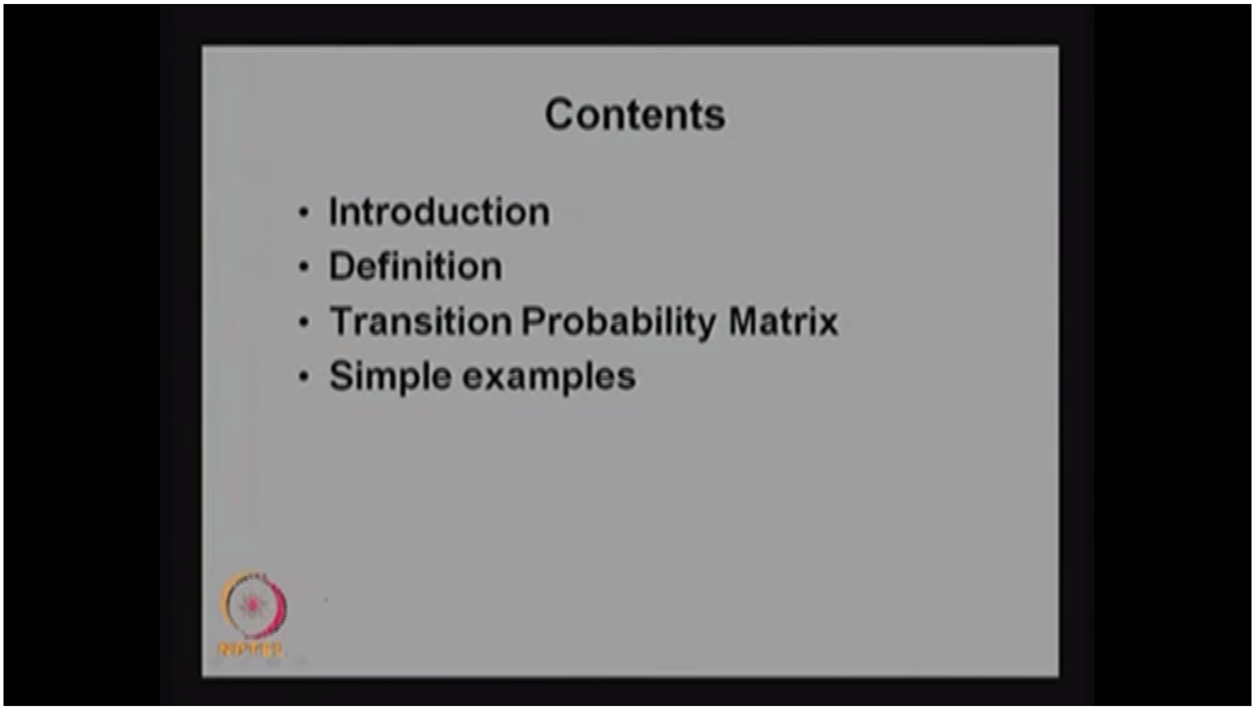
Department of Mathematics

Indian Institute of Technology Delhi

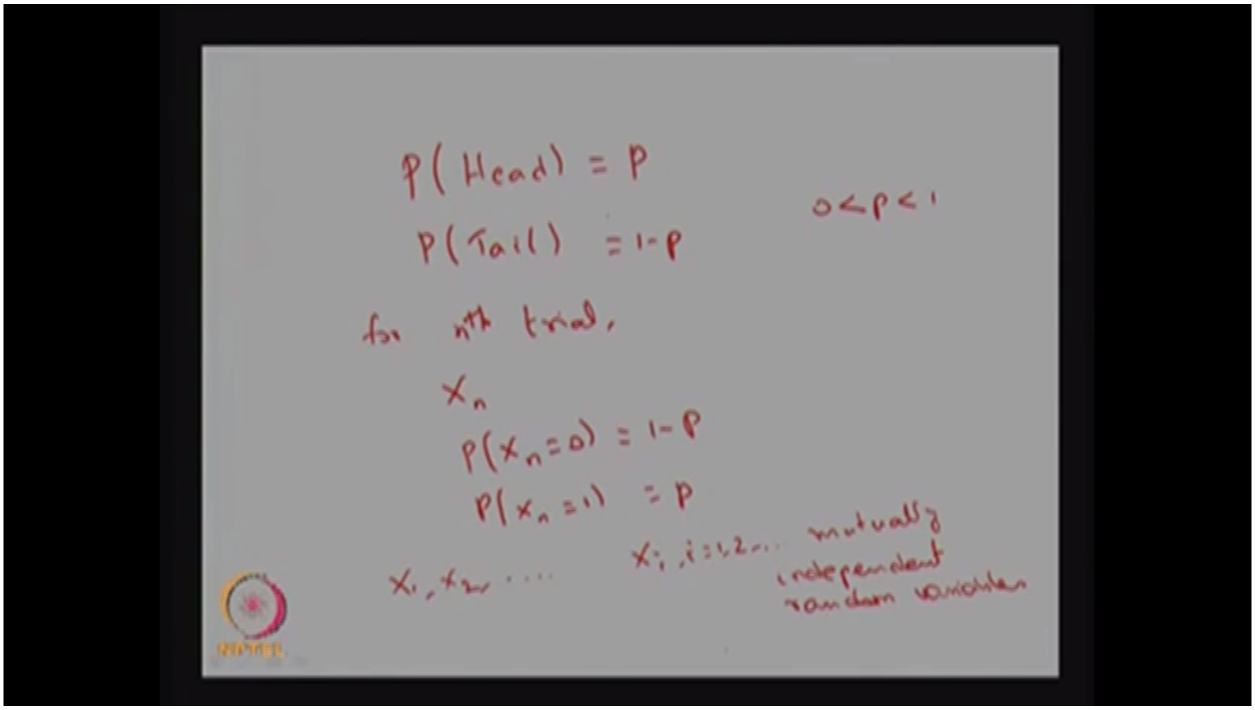
New Delhi, India

http://web.iitd.ac.in/~dharmar

This stochastic process in this we are going to discuss the model for a discrete-time Markov chain and this is a lecture 1. In this lecture I am going to discuss the interaction about the discrete-time Markov chain. Then followed by the definition and the important one concept called the one step transition probability matrix.

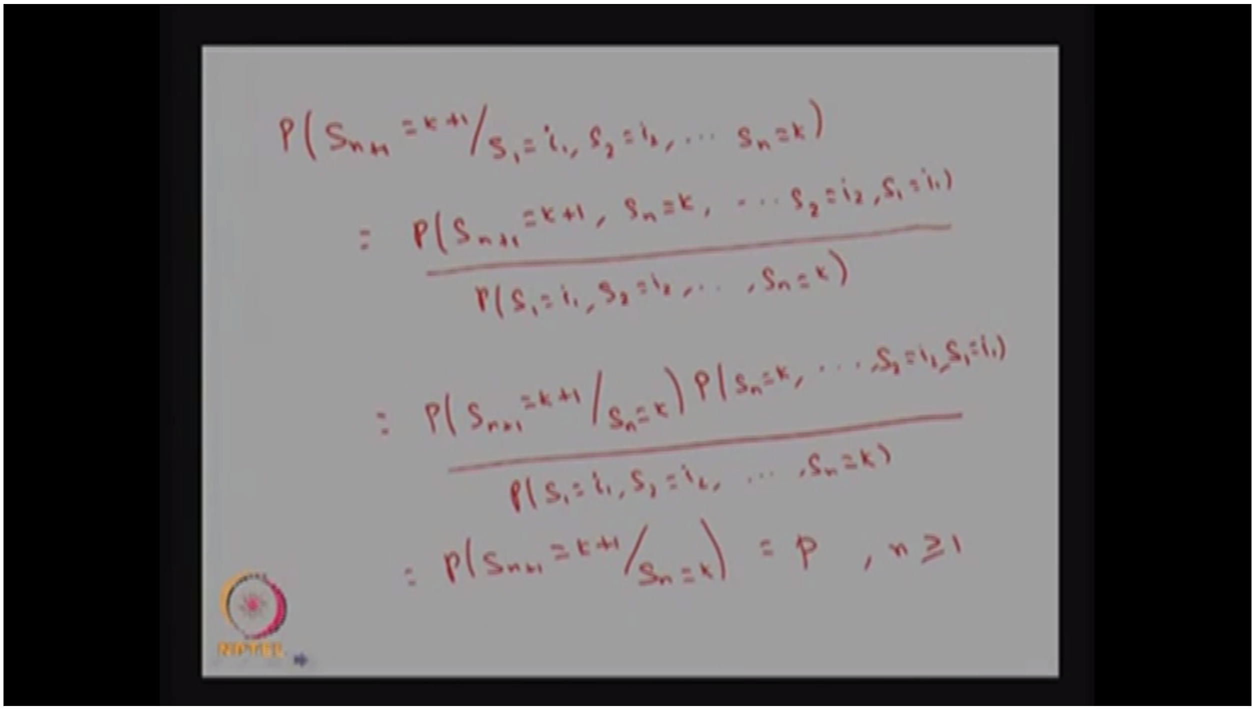


So this lecture I am going to cover the introduction definition transition probability matrix and few simple examples also. Consider a random experiment of tossing a coin infinitely many times. Each trial there are two possible outcomes namely head or tail. Assume that the probability of head that probability you assume that that is P and the probability of tail occurring in each trial that you assume it as 1 minus P. You assume that the P is lies between 0 to 1. Denote for the nth trial because you are tossing a coin infinitely many times for the nth trial you denote the random variable Xn is the random variable whose values are 0 or 1 with the probability, the probability of Xn takes the value 0 that is same as in the nth trial you are getting the tail that probabilities 1 minus P and the probability of Xn takes a value 1 that probability is make it as P for the head up yes and there already you assume that and the probability is lies between 0 to 1. Thus you have a sequence of random variable X1, X2 and so on and this will form a stochastic process and assume that all the Xi's are mutually independent random variables. So this is a random experiment in which we are tossing a coin infinitely many times and for any nth trial you define the random variable Xn with the probability it takes a value 0 with the probability 1 minus P and it takes a value 1 with the probability P and that is equivalent of appearing a head with the probability P and occurring the tail with the probability 1 minus P.



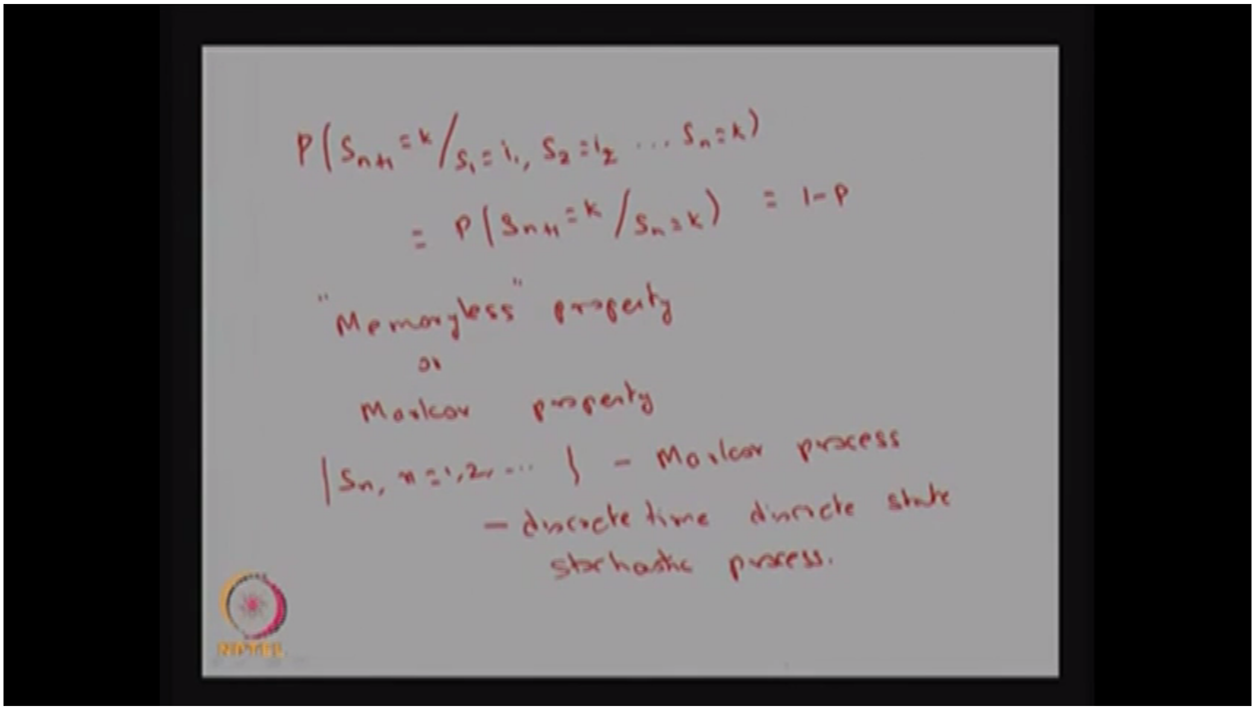
Now I am going to define another random variable that is a partial sum of first n random variables, n Xi's. So the Sn will be sum of first n random variables therefore the sum Sn gives the number of heads appear in the first n trials. It can be observed that Sn plus 1 is same as Sn plus Xn plus 1 since Sn is the partial sum of first n trials outcome so the Sn n plus 1 is nothing but Sn plus Xn plus 1. You can also observe that since Sn is the sum of first n random variables and the Sn plus 1 is Sn plus Xn plus 1 and also all the Xi's are mutually independent random variables Sn is independent with the Xn plus 1. That means here the Sn plus 1th random variable is the combination of two independent random variables whereas the Sn is the till nth trial how many heads you appeared plus whether it is a head or tail accordingly this values is going to be 0 or 1. Therefore if you see the sample path of a Sn plus 1 it will be incremented by 1 if Xn plus 1 takes a value 1 or it would have been the same value earlier if this Xn plus 1 takes a value 0. And also you can observe that Sn plus 1 is depends on Sn and only on it. It is not a depends on Sn minus 1 or Sn minus 2 and so on because it is accumulated the number of trials values over the n therefore Sn plus 1 is depends on Sn and only on it. The Sn for different values of n this will form a stochastic process and now we can come to the conclusion the probability of this is a stochastic process. The probability of Sn plus 1 suppose this values is k plus 1 given that Sn was k that means the Sn plus 1 value would have been 1. Therefore the appearance of the head appears in the n plus 1th trial and that probability is going to be P.

Similarly you can make out suppose Sn plus 1 value will be k such that Sn is also k then that is possible with n plus 1th trial you got the tail. Therefore that probability is 1 minus P. This is satisfied for n. So you can make out this is satisfied for n is greater or equal to even I can go for n is greater than or equal to 1.



Not only these similarly I can come to the conclusion the probability of SN plus 1 is equal to K given that s1 was i1, S2 was i2 and so on . Sn was k that is also can be proved the probability of Sn plus 1 is equal to k given that Sn is equal to k that is same as what is the probability that the value was same k in the subsequent trials. That is possible of appearing tail in the n plus 1th trial therefore the appearance of the tail in the n plus 1th trial the probability is 1 minus P or I can use the notation Q. That means the probability of the n plus 1th trial that distribution given that I know the value till the nth trial that is same as the distribution of n plus 1th trial given with the only the nth distribution not the earlier distributions and this property is called a memoryless property. The stochastic process the Sn satisfies the memoryless property or the other word called Markov property. The distribution of n plus 1 given that the distribution of 1 first random variable, second random variable, the nth random variable that is same as the conditional distribution of n plus 1th random variable given that with the nth random variable only and this property is called memoryless or Markov property.

The stochastic process the Sn satisfying the Markov property or memoryless property is called Markov process. The stochastic process satisfying the memoryless property or Markov property is called a Markov process.



In this example the stochastic process Sn is the discrete time discrete state stochastic process. Now I can give based on the state space and the parameter space I can classify the Markov process or I can give the name of the Markov process in an easy way based on the state space as well as the parameter space. So when the state space S, the S is the state space this is nothing but the collection of all possible values of the stochastic process. If this is of the discrete type that means the collection of elements in the state space S is going to be a finite or countably infinite then we say the state space is of the discrete type. So whenever the stochastic process satisfying the Markov property then the stochastic process is called the Markov process or you can say whenever the state space is a discrete then we can say the corresponding stochastic process we can call it as Markov chain whenever the state spaces are discrete. Now based on the parameter space T parameter space is nothing but the possible values of T whether it is going to be a finite or countably infinite then it is going to be a discrete parameter space or discrete time or it is going to be uncountably many values then it is going to be call it as a continuous type. So whenever the T is going to be a discrete type then the Markov chain is going to be call it as a discrete time Markov chain. Whenever the parameter space is going to be of the continuous type that means the possible values of T is going to be uncountably many then we say continuous time Markov chain.

So in this example the Sn the possible values of Sn is also going to the state space is going to be a discrete type and the parameter space is also going to be a discrete type. Therefore the given example the Sn is going to be the discrete time Markov chain. So in this model we are going to study the discrete time Markov chain. The next model, model 5 we are going to discuss the continuous time Markov chain.

So in general whenever the stochastic process satisfying the Markov property it will be called Markov process. So based on the state space the Markov process is called as a Markov chain and based on the parameter space it is called a discrete time Markov chain or continuous time Markov chain accordingly discrete type or continuous type.