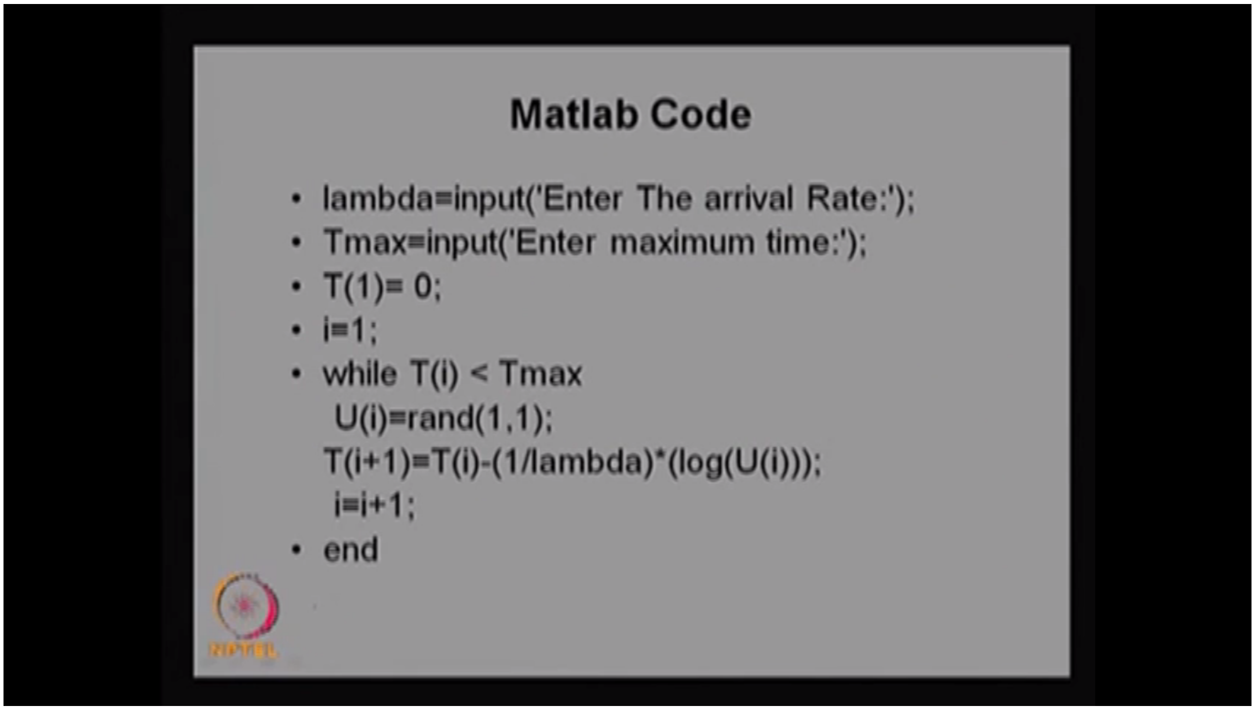
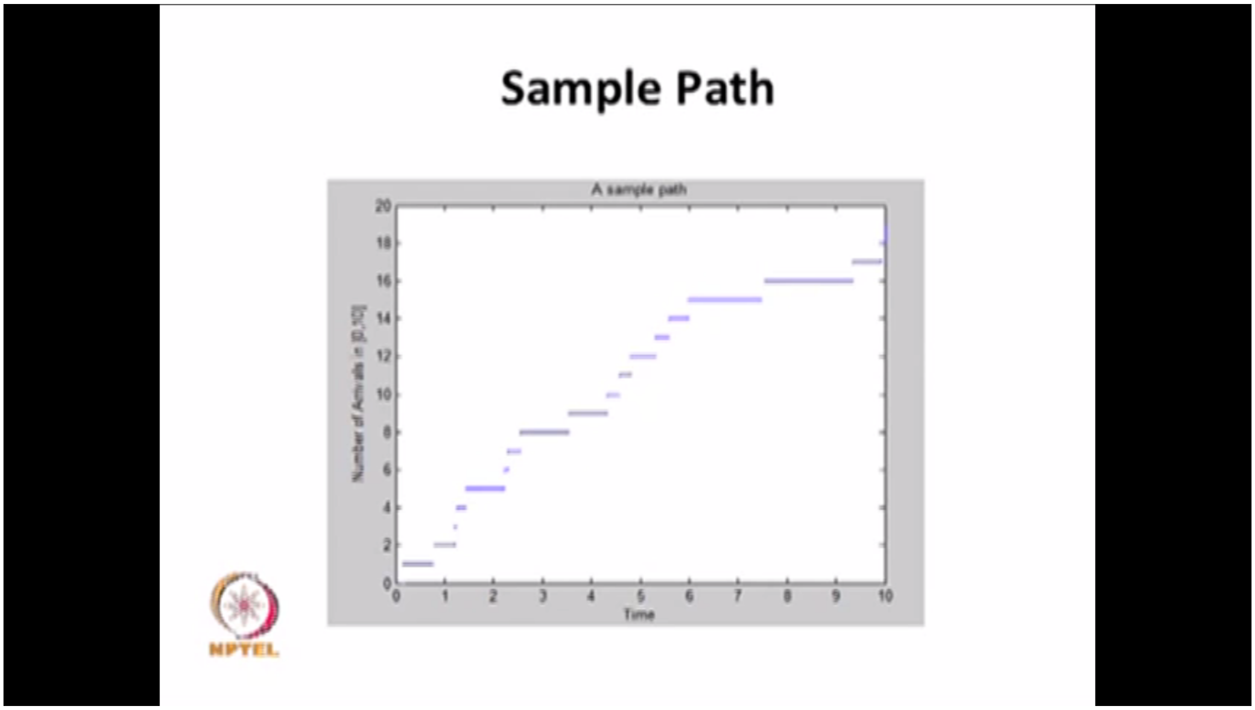


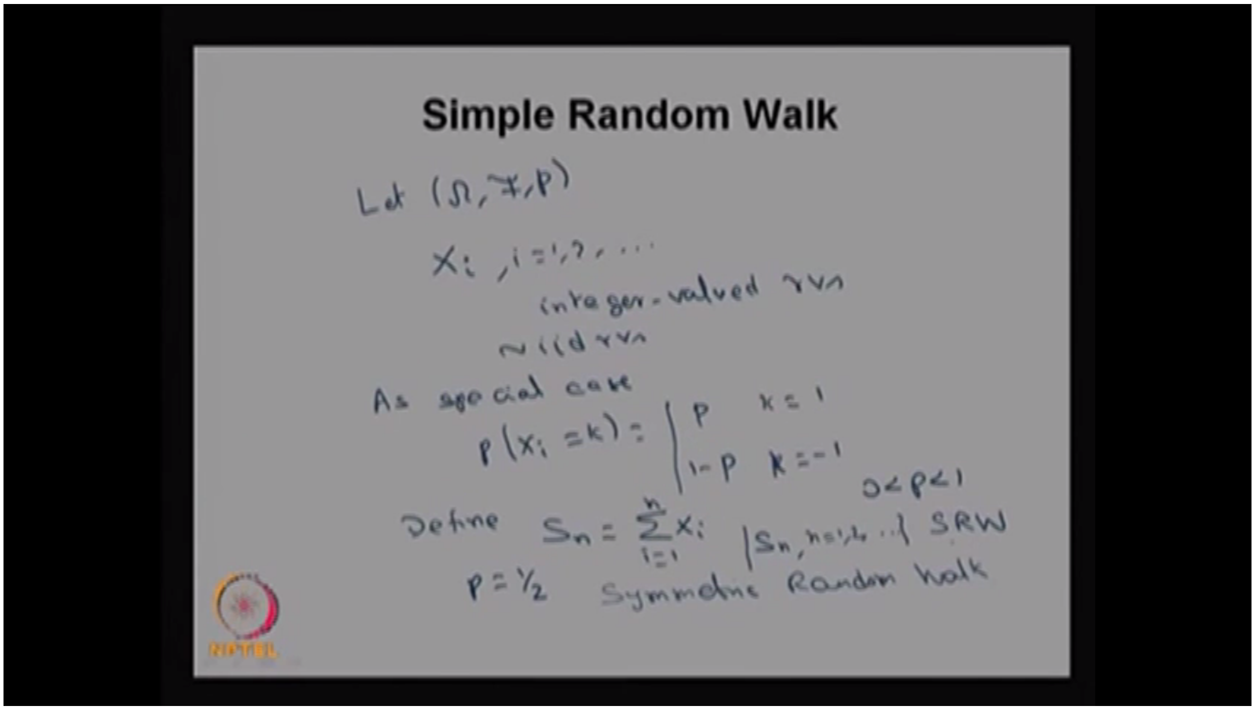
Now I am going to explain how we can create the sample path of the Poisson process using the MATLAB code. So since I said the Poisson process is related with the inter-arrival times are exponential distribution so I can start with the time 0 there is no customer in the system and I can go for what is the maximum time I need the sample path then I can keep on create the random variables. From the random variable I can generate the exponentially distributed the time event. Then I can shift the time event by T of i plus 1 by adding the next exponentially distributed time event. Then I can go for plotting the sample path.



So this is the one sample path in which over the time from 0 to 10 the number of arrivals occurs in the interval 0 to 10 in the form of that means there is a 1 arrival occurs at this time therefore the N of t values is incremented by 1 and it is taking the same value and when at the second arrival occurs then the increment is taken by 2 and so on. And if you see carefully the sample path you can find out the increment is always by 1 over that time and there is no two arrival or more than one arrival in a very small interval of time and you can come, you can able to see the inter arrival time that is going to be exponentially distributed with the parameter lambda whatever the lambda I have chosen in this sample path. So this is the way the sample path of the Poisson process look like.

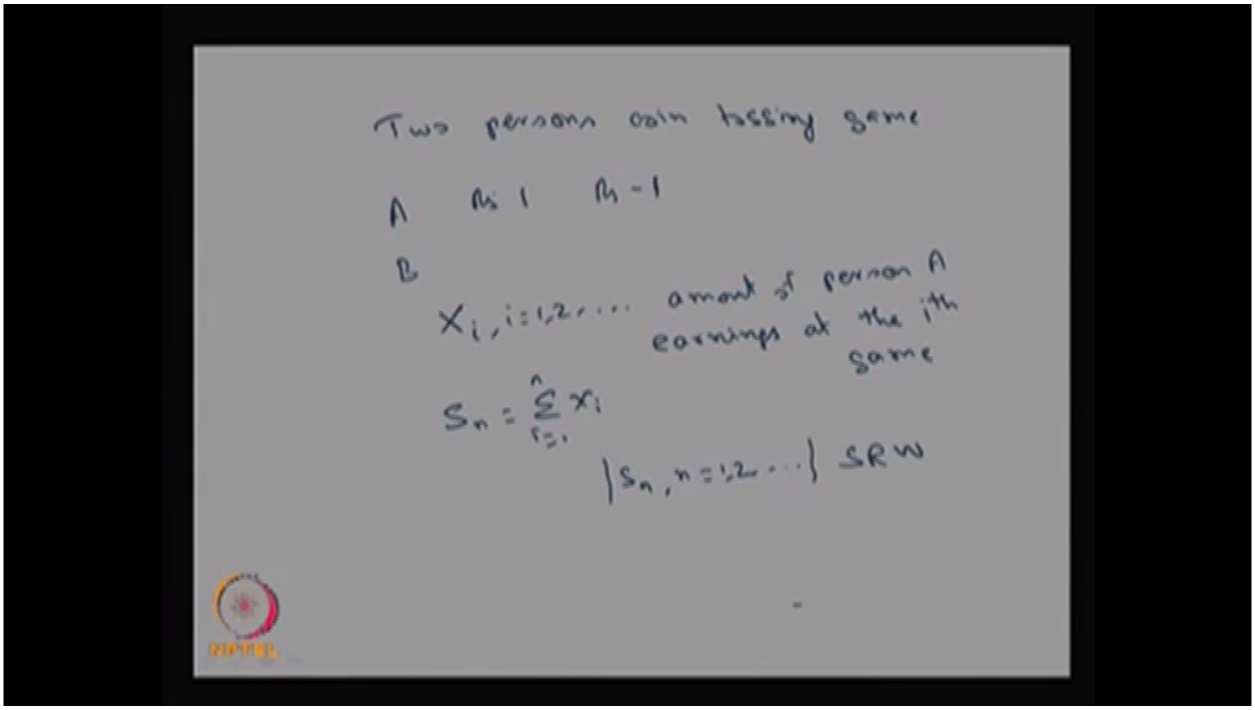
Now we are going to discuss the third type of stochastic process that is simple random walk. So how we can create the simple random walk let me explain. You have a probability space. From the given probability space you define a sequence of random variable Xi's and those random variables are integer valued random variables. Each Xi's are integer valued random variable. Not only that all the Xi's are IID random variables and each one is a integer valued discrete type random variable. As a special case I can go for the random variable Xi takes a value 1 or minus 1 with the probability P and 1 minus P. This is a special type of random walk. In general I am going to define in general random walk also. As a special case I will go for the random variable Xi takes the value 1 with the probability P and Xi takes the value minus 1 with the probability 1 minus P where the P can take the value 0 to 1.

Now I am going to define the random variable Sn that is nothing but sum of Xi's sum of first n Xi's that is going to format the random variable Sn and the stochastic process Sn or the stochastic sequence Sn for different values of n this will form a simple random walk. The Sn is going to form a simple random walk. Why it is simple because it is going to take integer valued random variable and each values are going to take - each random variable is going to take the value 1 or minus 1 therefore this is going to be call it as a simple random walk. In general the k can take the any integers accordingly you'll end up having Sns are going to be a random walk and I am going to give another special case when P is equal to half that means each Xi random variable takes a value 1 with the probability half or minus 1 with the probability half then that random walk is going to be called as a symmetric random walk. Why it is symmetric? Because with the probability half it takes a forward one step or with the probability half it takes a backward one step therefore that type of a random walk is called a symmetric random walk. In general if it takes a value 1 or minus 1 then it is called a simple random walk if k can take any integers then it is going to be call it as a generalized random walk.

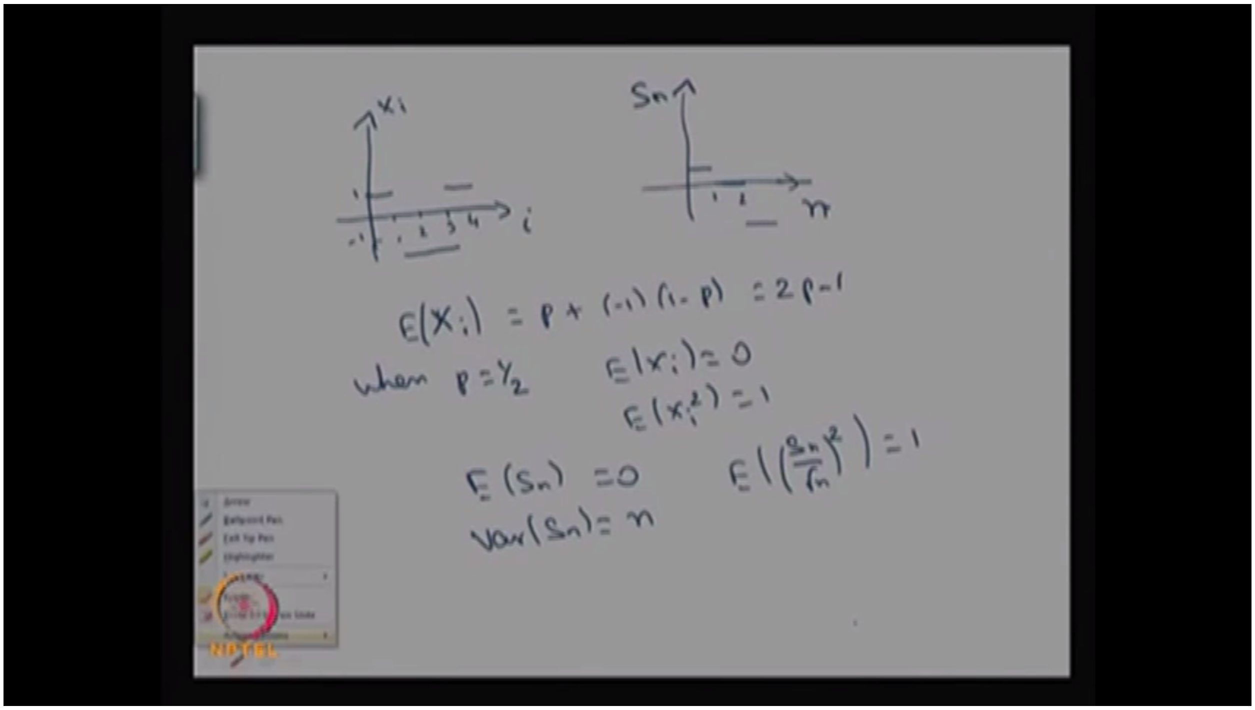


So this random walk can be created in a simple example of two persons coin tossing game also. This simple random walk can be explained by the example 2 persons coin tossing example in which you have a person A and B. if at the end of the coin tossing if A is going to head then he is going to win rupees 1 or if A is – at the end of the nth coin tossing if it is going to get the tail then he is going to be loose. In this game if A wins then B gives rupees 1 to A and if A loses then A gives rupees 1 to B.

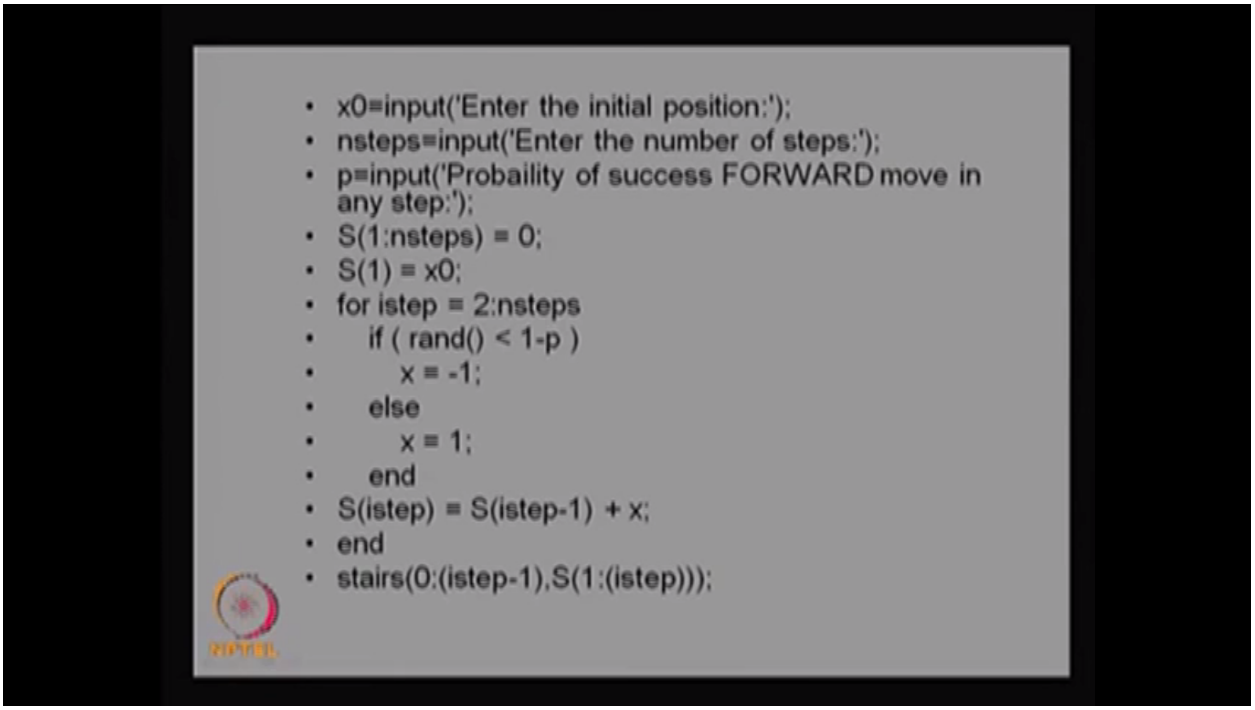
So accordingly I can go for creating a random variable Xn or Xi for i is equal to 1, 2, and so on therefore Xi denotes what is the amount of the person A earning at the ith game. Similarly we can construct a stochastic process for player B and calculate the measures of interest. I can go for creating a random variable Sn is nothing but summation of Xi's where i is equal to 1 to n therefore the Sn denotes what is the amount earned by the person A at the end of nth game. That's what's the total amount. So the Xi denotes how much he is going to earn at the end of each game whereas the Sn is going to be the total amount to earn by the person A at the end of first n games. Therefore this Sn is going to form a simple random walk where the Xi's are going to take an integer value with the value 1 and minus 1 with the probability P it is going to take the value 1 or it is going to take the value minus 1 with the probability 1 minus P. So I am just relating the simple random walk with a simple scenario of two persons a coin tossing game.



If you see the sample path of the Sn first I can go for what is the sample path of each Xi's. Each Xi's can take the value 1 or minus 1 therefore it is going to take the value 1 or minus 1 therefore if X1 takes a value 1 it is 1 if X2 takes the value minus 1 it is like this. If X2 takes the value minus X3 takes the value minus 1 then it is here. If X4 takes the value 1 then it is like this. So this is a sample path of XI over the i. The way I have given the Xi's now I can go for, sorry now we can go for writing what is the possible values of n and what is the possible values of Sn. So since X1 is equal to 1 therefore S1 is going to be 1 and X2 is going to be minus 1 therefore it takes a value 1 plus minus 1 therefore it is going to be 0 and X2 is going to be minus 1 therefore S2 is X3, S3 is going to be minus 1 and the X4 is going to be 1 therefore it is going to be again 0. So this is the way the sample path goes over the n. So this is the one sample path for the possible values of Xi's takes a value 1 and minus 1 accordingly I have drawn the sample path of Sn over the n.

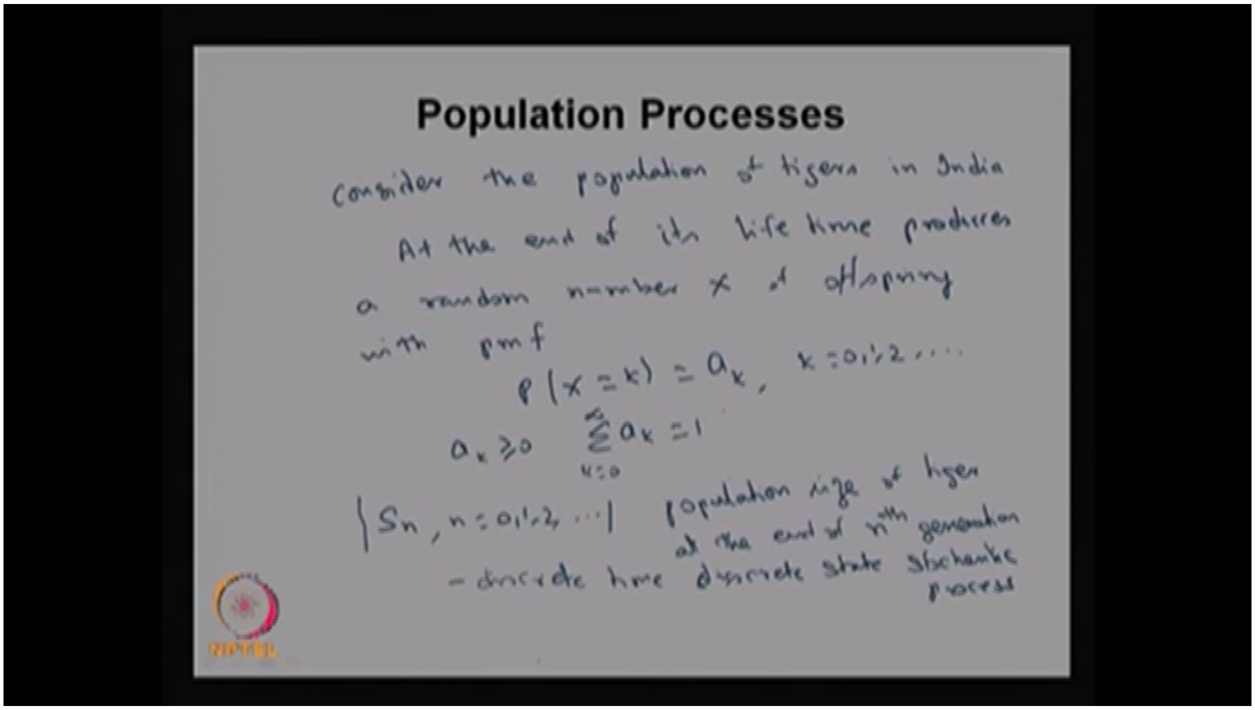


Since the Xi's are going to take the value 1 and minus 1 and with the probability P and with the probability 1 minus P takes a value minus 1, I can go for finding out what is the expectation of Xi that is nothing but Xi is equal to P plus minus 1 times 1 minus P. Therefore this is nothing but a 2P minus 1. So when I go for discussing the symmetric random walk when the P is equal to half then the expectation of each Xi is going to be 0 and also I can able to find out what is the E of Xi square that is going to be 1. Not only that when P is equal to half I can able to find out what is the expectation of Sn that is going to be 0 and the variance of Sn is going to be n and I can go for writing what is the expectation of Sn by root n power n power 2 that is going to be 1. So the way I have got the result for expectation of Xi's and the expectation of Sn I can go for what is the limiting distribution of Sn. So using central limit theorem I know what is the mean for each Sn and I know what is the variance of each Sn and also therefore using a CLT I can able to conclude Sn divided by square root of n minus the mean of this random variable is zero divided by the standard deviation is going to be 1 and this as n tends to infinity this will be a standard normal distribution where Z is going to be a standard normal distribution as n tends to infinity and this convergence is via distribution. That means I can able to conclude the distribution of Sn by square root of n as n tends to infinity in distribution this sequence of random variable will converges to the standard normal in distribution.

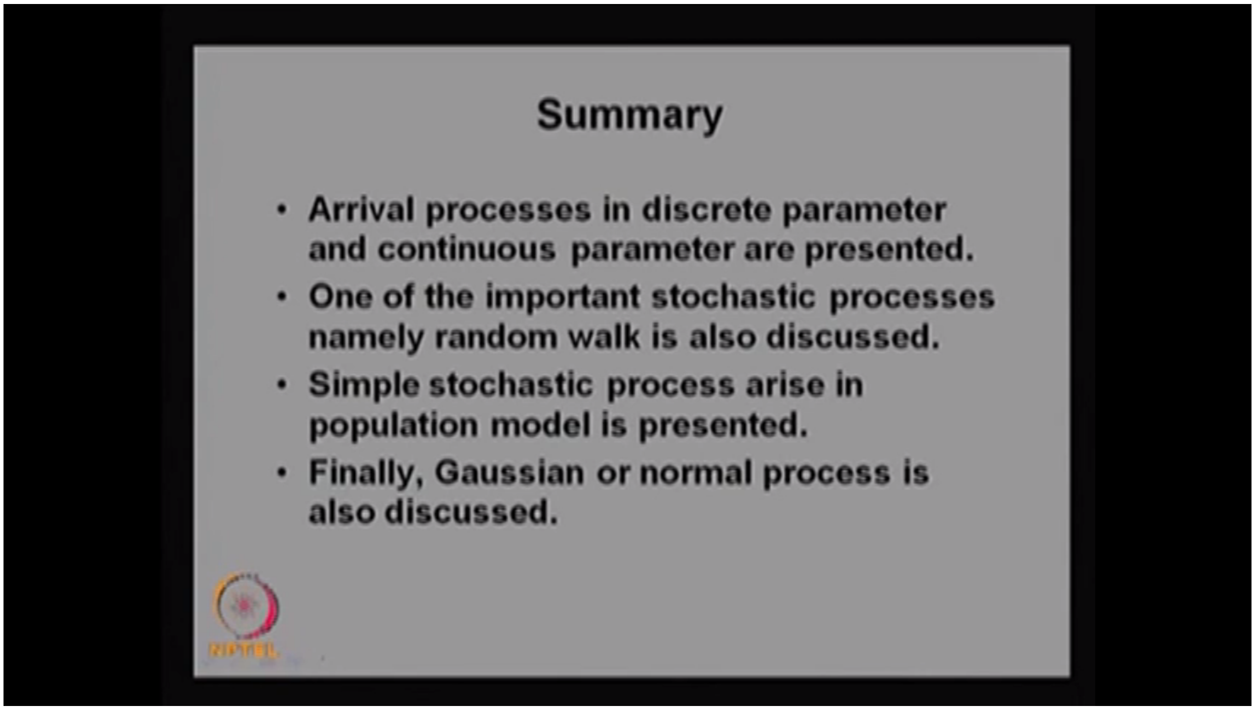


I can go for creating what is a sample path of the simple random walk by using the MATLAB code. So for that I have to fix what is the initial position and what is the maximum number of the steps I would like to go for finding the sample path and what is the probability of success in each – what is a forward move probability. Accordingly it is going to take the value 1 with the probability P and it is going to take the value minus 1 with the probability 1 minus P. So I am giving the value of a P only and then I am just going for the possible values of Sn by adding the 1 or minus 1. Accordingly I am just writing the sample path of Si's. So if you see the sample path over the time 0 to 20 and each Xi's are going to take the value 1 or minus 1 accordingly the Sn is going to take the same value or incremented by 1 or decremented by minus 1 according to the values of Xi's therefore this is going to be the one path which is depicted using the MATLAB code.

So this is the earlier I have shown the same graph. This is the Sn as n tends to infinity here you can see the different sample path for as n tends to infinity you can find out what is the distribution of Sn divided by square root of n as n tends to infinity also and this figures it has a three different sample path and one can observe what is the amount of a person A has as n tends to infinity that depends on whether he is going to take the positive value or he is going to have the negative value depends on the first few games. That it can be observed from this diagram. The first few results whether he is going to gain by one rupee or he is going to lose by one rupee accordingly the possible values of Sn will go as n tends to infinity.



Now we are going to discuss the fourth simple stochastic process that comes in the population model. Now we will see the fourth simple stochastic process arises in the population model. You consider a population of tigers in India. So that is going to be for over the time this is going to be form a stochastic process so I am going to make the assumption at the end of its life time it produces a random amount, random number X of offspring with the probability mass function that is the probability of X takes the value k that is Ak where it satisfies Ak's are going to be great or equal to 0 and the summation is going to be 1 and also I am making the assumption all the offsprings act independently of each other and at the end of their lifetime individually can have a pregnancy accordance with the probability mass function, the same probability of Xi's takes a value k with this Sn we'll form a discreet the time and the discrete state stochastic process where Sn is the population size of a tiger at the end of nth generation and if you see the sample path of a Sn over the different generation suppose you make it a S0 is equal to 0 and suppose you make it S1 is equal to X1 and suppose X1 takes the value 3 and then the second generation S2 is going to be X1 plus X2 plus X3 and suppose you make it X1 takes the value 3 and X2 takes the value 0 and the X3 takes the value 1 then we have a S2 is going to take the value 4. So if you see the sample path of a Sn over the n it is going to take the value 1, then it is going to take the value 3, then it is going to take the value 4 and so on and this is a sample path of the population size of a tiger over the nth generation and this is going to form a discrete time discrete state stochastic process.



And there is another stochastic process Gaussian process that I'll discuss in the later lectures and in this lecture we have covered the arrival process of the two type. One is the discrete time and the another is a continuous time arrival process and we have also discussed the random walk and we have discussed a simple stochastic process arises in the population model and the Gaussian process that I will discuss later. And the references books are which.

So with this I will complete the model two of definition and the simple stochastic processes. Thank you.

Reference Books

- J Medhi, "Stochastic Processes", 3rd edition, New Age International Publishers, 2009.
- U Narayan Bhat, "Elements of Applied Stochastic Processes", John Wiley & Sons, 2nd edition, 1984.
- S K Srinivasan and K M Mehata, "Stochastic Processes", Tata McGraw-Hill, 2nd edition, 1988.
- S Karlin and H M Taylor, "A First Course in Stochastic Processes", Academic Press, 2nd edition, 1975.