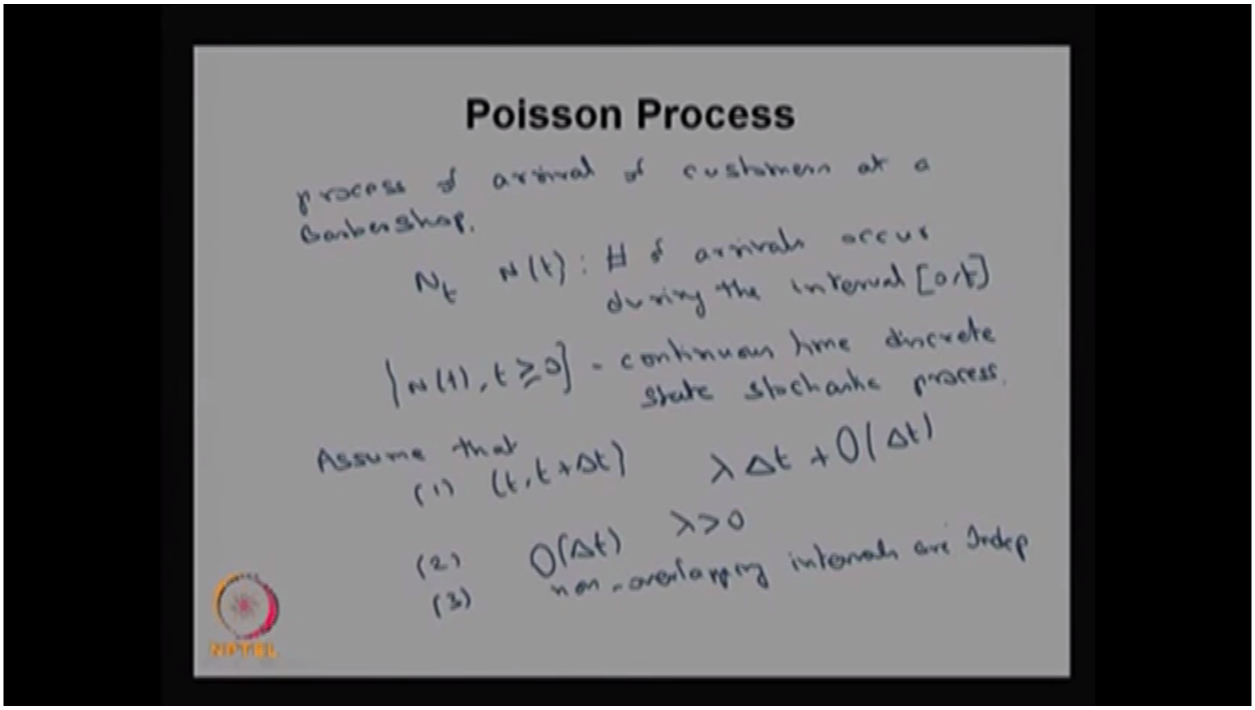


So till now we have discussed what is the discrete time arrival process. Now we are going to discuss the continuous time arrival process that is a Poisson process. So in this lecture I am going to develop what is the Poisson process and how we can get the Poisson process from the scratch.

Suppose you consider the process of arrival of customers at a barber shop. So this is the same example we have discussed in the beginning of this course also. So over that time how many arrivals is going to take place that is going to be a random variable. So let Nt, N suffix t or in some books they use N of t so the N of t denotes number of arrivals occur during the interval 0 to – the closed interval 0 to t. That means we are defining a random variable N of t that denotes a number of arrival occurs during the interval 0 to t. For fixed t, N of t is going to be a random variable therefore N of t over the time because t is great or equal to 0 this is going to be a since the possible values of a T that is the parameter space is going to 0 to infinity. Therefore, this is going to under the classification of a continuous parameter or continuous time and the possible values of N of t for different values of t that is going to be takes a value 0 or 1 or 2 therefore it is going to be a countably infinite therefore this is going to be a continuous time or continuous parameter discrete state stochastic process.

So this is the N of t over the T greater or equal to 0 that is going to be a continuous time discrete state stochastic process.

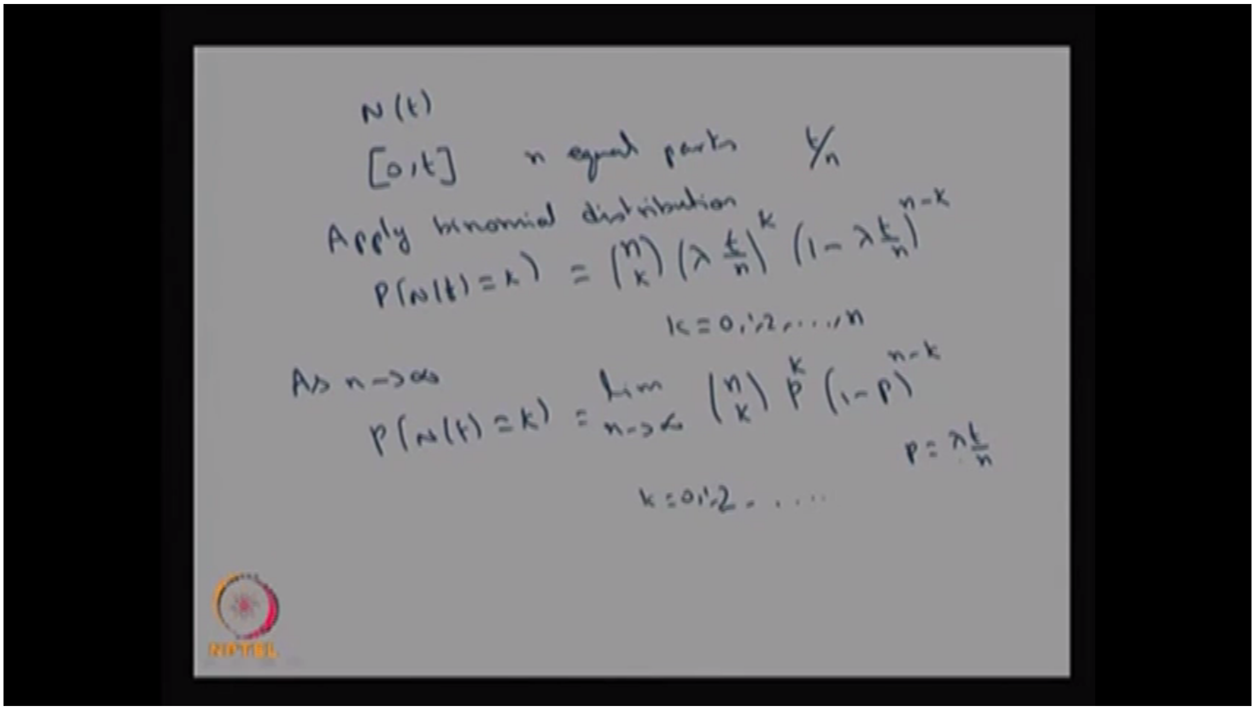
Now we are going to develop the theory behind the Poisson process. To create the Poisson process you need a few assumptions so that you can able to develop the Poisson process. The first assumption in a small negligible interval if the interval is a t to t plus delta t if the small negligible interval is t to t plus delta t then the probability of one arrival is going to be lambda times delta t plus O of delta t. The probability of one arrival occurs during the interval t to t plus delta t is going to be lambda times that smaller interval delta t plus order of O delta t. Here the lambda is going to be strictly greater than zero and we are going to discuss what is lambda and so on in the later after this explaining the Poisson process. So here the lambda is going to be a constant and which takes the value greater than zero and O delta t means has a delta t tends to zero the order of a delta t that is going to be tends to 0 as delta t tends to 0. So this is the first assumption.

The second assumption the probability of more than one arrival is going to be a order of delta t in a same interval t to t plus delta t more than one arrival in this small negligible integral that probability is order of delta t that means as a delta t tends to zero this values is going to tends to zero.

Then the third assumption occurrence of arrivals in a non-overlapping intervals are mutually independent. Non-overlapping intervals are independent. So this is a very important assumption that means what is the probability that the arrival occurs in a non-overlapping intervals that probability is the same as the product of a probability of arrival occurs in the each interval. Therefore it is going to satisfies the independent property. Occurrence of events in non overlapping intervals are mutually independent. Therefore the probability is going to be probability of intersection of all those things is same as the probability of individual probability and their product.

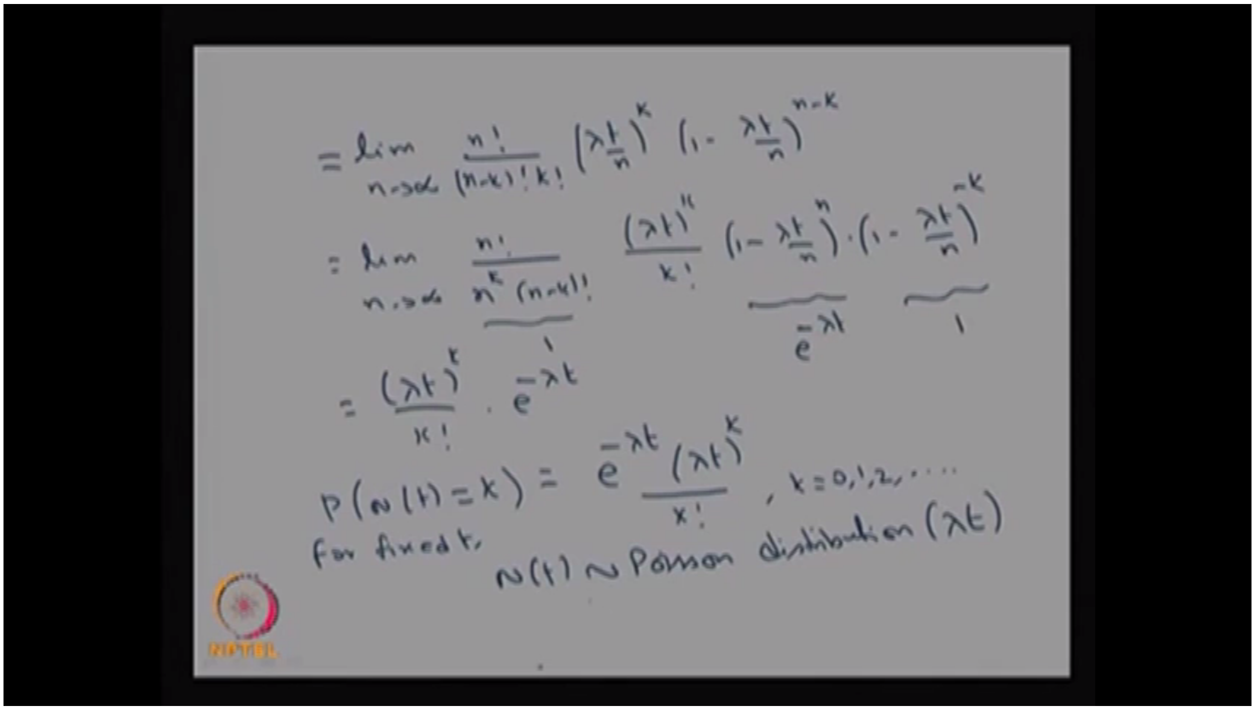
So with these three assumptions we are going to develop the Poisson process. So what I am going to do since I started with the random variable N of t is the number of arrivals in the interval 0 to t I am going to partition the interval 0 to t into n equal parts. I'm going to partition the interval 0 to t into n equal parts. Since I made it the interval 0 to t into n equal parts then each will be of the length t by n and since I made the assumption the non-overlapping intervals are independent and the probability of one arrival is lambda times delta t and the probability of more than one arrival is order of delta t and so on therefore I can apply binomial distribution. The way I have partition the interval 0 to t into n pieces therefore this is going to be of n intervals of interval length t by n therefore I can say what is the probability that I can able to find out what is the probability that k arrivals takes place in the interval n intervals of each length t by n what is the probability that k arrivals takes place therefore the possible values of k is going to be 0 to n and I can able to find out by using the binomial distribution what is the probability that n of t takes a value k. Since non-overlapping intervals are independent and each probability of what, sorry probability of one arrival is lambda times delta t where delta t is a t by n so each interval behave as a Bernoulli trial whether the arrival occurs or there is no arrival and like that you have n such independent trials. Therefore, the sum of n independent Bernoulli trials land up binomial trials therefore by using the binomial distribution I can able to get what is the probability that n of t takes a value k that is what is the possible nck ways and what is the probability of arrival takes place in one interval that is lambda times this interval length is t by n lambda times t by n power k and what is the probability of no arrival takes place in each interval that is 1 minus lambda times t by n power n minus k. So this is the way I can able to get what is the probability that a k arrival takes place in the interval 0 to t by partitioning n intervals so this is a probability. But the way I made a partition n equal parts so now I had to go for what is the result has n tends to infinity. That means my interest is what could be the result if the n tends to infinity of k - of what is the probability that nt takes a value k as n tends to infinity therefore the running index for k is going to be 0, 1, 2, and so on what is the probability of nt takes a value k. That means in the right hand side I had to go for finding out as n tends to infinity what is the result for the right hand side what is the probability of nt takes a value k.

We take n tends to infinity because we need to study the limiting behavior of the stochastic process. So that is same as limit n tends to infinity of nck I can make it as a p power k where p is going to be lambda times at t by n and 1 minus p power n minus k.



Now I have to find out what is the result of our limit n tends to infinity of this expression nck p power k 1 minus p power n minus k where p is going to be lambda times t by n. If I do the simple calculation let me explain the limit n tends to infinity that is same as limit n tends to infinity of nck I can make it as a n factorial, n minus k factorial and k factorial and that is a lambda t by n power k and that is 1 minus a lambda t by n power n minus k and that is same as the limit n tends to infinity of n factorial and here this n power k I can take it outside and the n minus k factorial and lambda t power k and divided by k factorial. So this k factorial I take it inside and the power 1 minus lambda t by n power n minus k I split it into 1 minus lambda t by n power n into 1 minus lambda t by n power minus k.

So now I can look as n tends to infinity this is nothing to do with the n therefore lambda t power k by k factorial will come out so this result is going to be lambda t power k by k factorial and this will land up as n tends to infinity this is going to be e power minus lambda t and this will land up 1 and this is also land up 1 as n tends to infinity. Therefore, I may land up e power minus lambda t. Hence the final answer of what is the probability that k arrival takes place in the interval 0 to t that is going to be e power minus lambda t and the lambda t power k by k factorial and the possible values of k can be 0, 1, 2, and so on.



For fixed t if you see this is same as for fixed t it is going to be a random variable. For all possible values of t it is going to a stochastic process. So for fixed t the n of t is a random variable and that probability mass function is e power minus lambda times t lambda t power k by k factorial. So lambda is a constant for fixed t lambda into t that is going to be a constant. Therefore, the right-hand side looks like the probability mass function of the Poisson distribution. Therefore for fixed t the n of t is Poisson distribution. The random variable n of t for fixed t it is going to be a Poisson distribution with the parameter lambda times t. Lambda is a constant and for fixed t, t is a constant. So lambda multiplied by the t again this is going to be a constant therefore for fixed t it is going to be a Poisson distribution with the parameter lambda multiplied t therefore for possible values of t the n of t is going to from a stochastic process and since for fixed t it is going to be a Poisson distribution the collection of a random variable and each random variable is a Poisson distribution. Therefore this is going to be call it as the Poisson process.

The way I have – we have explained earlier each random variable is a Bernoulli distributed random variable. The collection of random variable is a Bernoulli process. Similarly each Sn is going to be a binomial distribution therefore the collection is going to be a binomial process. The same way for fixed t, it is going to be a Poisson distribution therefore that collection is going to be call it as Poisson process.

So now we have developed n of t is going to be a Poisson process.