Slide 1: This is model two, definition and a simple stochastic process and today is a lecture two of simple stochastic process. In the lecture one we have seen the definition of stochastic process and the classification of stochastic process based on time space and the parameter space. And we have given few simple stochastic processes via the classification.

Slide 2: In this lecture we are going to discuss some simple stochastic process starting with the discrete time arrival process that is a Bernoulli process and a continuous time arrival process that is a Poisson process, followed by that we are going to discuss the simple random walk then we are going to discuss a one simple population process, which arises in the branching process Then we are going to discuss the Gaussian process, so with that the lecture two will be over.

Slide 3: What is Bernoulli process? Bernoulli process can be created by the sequence of random variable. Suppose you think of the random variable XIs where I is belong… I takes the value 1, 2 and so on therefore this is going to be a collection of random variable and each random variable are XIs and you can think of XIs are going to be a IID random variables. And each is coming from the Bernoulli trials. That means each random variable is a Bernoulli distributed… each random variable is a Bernoulli distribution and with the parameter P. So the same thing can be written in the notation form XI takes the… the XIs are in the notation it is a capital B1, small p, that means it is binomial distribution with the parameters 1NP that is same as each XIs or Bernoulli distributed with the parameter 1 and P. So now I can… so this is going to be a stochastic process or we can say it is a stochastic sequence. Now I can define another random variable. For every N Sn is nothing but sum of first N random variables. Suppose you think XI is going to be the outcome of the Ith trial so the XI can take the value 0 or 1 that means with the probability the X… each XI can take the value K if K is equal to 0 with the probability 1 minus P and K taken, K can cake the value 1 with the probability P. Therefore each, since each XIs are IID random variable, you can come to the conclusion Sn is nothing but binomial distribution with the parameters N,P. Suppose you assume that XI is going to be number of whether the arrival occurs in the Ith trial or not, if XI takes the value 0, that means that no arrival takes place in that Ith trial. If XI takes the value want, that corresponding to the Ith trial there is an arrival. So the Sn represents, Sn denotes the number of arrivals in N trials. So now you can create a stochastic process with Sn, where N takes a value 1, 2 and so on. Therefore this is going to be a binomial process. So the XIs takes a value 0 or 1 with the probability 1 minus P and Patient, each one is going to be Bernoulli distributed therefore this is going to be a Bernoulli process. This XIs are going to form Bernoulli process. The way you have created Sn is equal to sum of first N random variable and each Sn is going to be a binomial distribution with the parameters N and P. Therefore this Sn, that sequence of Sn for N is equal to 1, 2, 3 binomial process. Therefore since you have collected arrivals over the… over the possible values of 1 2 and so on therefore this is going to be a… one of the discrete time arrival process. So similarly we are going to explain what is the continuous time arrival process, whereas here binomial process, this is going to be a discrete time arrival process.

Slide 4: Suppose you would like to see the trace of Sn, so before you go to the trace of Sn, we can go for what is a trace or sample path of XI. For different values of N is equal to 1, N is equal to 2, N is equal to 3 and so on, if you see each XIs takes a value 0 or 1, therefore it can take the value 0 or X1 can take the value 1 or X2 can take the value 0 or this can take the value 1, again it can take the value 1 and 0, so the possible values of XIs are going to be 0 and 1 therefore each XIs can take the value 0 in the horizontal line or it can take the 1 till you get the next trial. Similarly if you make the sample path or the trace of Sn… the sample path or the trace of Sn, since Sn is going to be a sum of first N random variable, therefore the… based on the XI takes the value suppose the X1 takes a value 0 and suppose X2 take the value 1 and suppose X3 takes a value 1 and so on, so since the X1 is equal to 0 therefore S1 is 0 then at S2 is same as X1 plus X2, therefore he takes a value 1 and S3 is equal to X1 plus X2 plus X3, therefore that is going to be again, you are adding the values therefore it is going to be a 2, therefore this is 1 and this is 2. So based on the X4 it is going to be 0 or 1, either it can take the value 2 itself or it can go to the 3. Therefore if you see the sample path of Sn, it is going to be incremented, either incremented by 1 or it takes the same value till the next year. Therefore not only you can find out the Sn you can… not only you can find out the sample path of Sn, you can get the mean and variance, because each Sn is going to be a binomial distribution with the parameters N and P, therefore the expectation of Sn is going to be N times P and the variance of Sn is going to be N times P into 1 minus P. So you can be able to see the sample path of XIs as well as Sn over the different values of N. In discrete-time, sample paths are sequences. I can also define the new random variable capital T is nothing but number of… number of trials up to and including the first that means suppose it takes a value N that means for subsequent N minus 1 trials I got the failures or no arrival takes place in the subsequent N minus 1 trial and the Nth trial I get the first arrival, that means the T is a random variable to denote how many trials to get the first success or the first arrival… or the first arrival. So if it is going to take the first arrival in the Nth trial, then the probability of T takes the value N, that is same as 1 minus P to And minus 1 x P, because all the trials are independent and subsequent N minus 1 trial gives no arrival and the Nth trial you get the first arrival. Therefore this is going to follow a geometrical… geometric distribution with the parameter P. So since you know the distribution of T you can find out the mean and variance, because the mean of a geometric distribution is going to be 1 divided by P and the variance of T is going to be 1 minus P divided by the P square.

Slide 5: Similarly I can go for finding out what is the probability that till Nth trial, I didn't get the first or I didn't get the first arrival, so if N plus Mth trial, if I am getting the first arrival, what is the probability that it is going to take after M trials to get the first arrival, that probability you can able to get, that is same as the probability of the T takes the value M. So this property is called… this property is called memoryless property. Since it is geometrically distributed and the geometric distribution satisfies the memoryless property, that can be visualized in this example, the probability of T-minus N is equal to M given that the T takes the value greater than N, that is same as what is the probability that the T takes the value small MRI, that means the right-hand side result is independent of N and it is the same as the distribution of, that means the residual arrival… number of arrivals that is same as the original arrival distribution therefore this satisfies the memoryless property. So this is the geometric distribution, satisfies the memoryless property in the discrete time and there is another distribution satisfies the memory less property in the continuous time, that is exponential distribution. So the way I have related the binomial distribution from the Bernoulli process then I get the binomial process, also I was able to create, the geometric distribution, you can create or you can develop the Pascal distribution or negative exponential distribution, the way I have defined the capital T is going to be the number of trials to get the first success or first arrival, instead of that if I make another random variable to go for, how many trials are needed to get the Rth success, where R take… can take the value greater than or equal to 1. If it is the Rth, first success is going to happen in the Nth trial. If R is greater than 1, then I can go for defining what is a negative binomial distribution for that particular random variable. If R is equal to 1 then that is land up to be the same, the random variable capital T.