

Slide 1: I am going to explain the discrete time Markov chain with the three simple examples, the first example is as follows. A factory has two machines and one repair crew. Assume that probability of any one machine breaking down at a given day is α , so the α is the probability. Assume that if the repair crew is working on a machine, the probability that they will complete the repairs in two more day is β . For simplicity ignore the probability of a repair completion or a breakdown taking place except at the end of a day. That means we observe the system at the end of the day, how many working machines in the system. Let the X_n be the number of machines in operation at the end of the inner n th day. Assume that the behavior of X_n can be modeled as a Markov chain. So based on the information available here the machine can be break down and we have only one repair person and that the probability of... he can do the repair in a day, that probability is β and $1 - \beta$ is the probability that he cannot be able to complete the repair of machine in a day. And the random variable X_n is, it denotes how many machines are in the operation at the end of the day. Therefore the possible values of X_n since we have two machines, the possible values of X_n will be 0, 1 or 2. So this will form a state space capital S . So the S consists of the element 0, 1 and 2. And the X_n over the N , it is going to form a discrete time Markov chain, because it is a discrete time... a discrete state stochastic process and also, based on the clue, the number of machines are working in any day depends on how many machines are working on the previous day and how many things are under repair and so on. So the dynamics of the number of machines in operation depends only on the number of machines working in the previous day not all the previous, earlier days. Therefore the memoryless property is satisfied by the stochastic process X_n , therefore this is called a discrete time Markov chain. Our interest is to find what is the one step transition probability matrix with the assumption that the X_n is the time homogeneous also. Since it is a time homogeneous DTMC, therefore we are trying to find out what is the one step transition probability matrix for the given time homogeneous DTMC.

Slide 2: So this is the one step transition probability matrix P_N and the possible states are 0, 1 and 2 and suppose the system was in the state 0, 1 or 2 in the N th step, where the system will be in the $N + 1$ step. Therefore this is the possible values of $X_n + 1$ and this is the possible values of X_n and this one-step transition probability matrix will give, suppose the system was in this state in the N th step, what is the probability that it will be in these states in the $N + 1$ step. So the first index will give what is the probability that 0,0 in one step, that means in their N th step, number of working machines are 0 and what is the probability that, in the $N + 1$ step also 0 machines are in working condition. That means all are under repair, all two machines are under repair and the probability of crew is going to be not repair, that is going to be $1 - \beta$, therefore the probabilities $1 - \beta$. In one step what is the probability that the number of working machines going from 0 to 1, that is because of the crew is finishing the repair

in a day and that probability is β . And since he can do only one repair in a day therefore the possibility of repairing more than one machine in a day, it is not possible, it's a rare event and the probability is going to be 0, therefore P_{02} is going to be 0. Similarly now we can visualize, what is the probability that number of working machines is one in the N th step and what is the probability that the 0 machines will be working in the $N + 1$ step. That is possible with the two independent things. The one machine can be failed and the other machine cannot be finishing the repair. Therefore the crew is not finishing the repair that probability is $1 - \beta$ multiplied by one machine is going to be repaired. Therefore the total number of machines working will be 0 in the $N + 1$ step, that is $\alpha \times (1 - \beta)$. And similarly you can evaluate the other element also, and for example the system is going from the state 2 to 0 that is nothing but at N th step two machines are in the working condition and the $N + 1$ th step 0 machines are in the working condition that means both the machines got repair, got failed in the same day, therefore that probability is $\alpha \times \alpha$, that is a probability both the machines got failed in the same day, therefore in the next day the number of working machine is going to be from 2 to 0. Like that you can visualize the other elements also. The same one-step transition probability matrix can be visualized with the state transition diagram. And the state transition diagram, you have to make with the state space as other teasers or the notes and the weights of the directed arcs are nothing but the one-step transition probability of system is moving from one state to other states, those are going to be the weights. If the probabilities are 0s, then no need to draw the directed arc from that particular note to the destination note. So first you start with the notes as the possible values of the state space. So you list out all the state space as a note, now by seeing the one-step transition probability matrix you should make the arc from 0 to 0, self-loop is allowed if the probability is going to be greater than 0. So you should draw the self loop from 0 to 0 with the arc value $1 - \beta$ and you should draw the arc from 0 to 1 with the arc weight β and you should not draw any arc from 0 to 2 because that probability is going to be 0. Therefore 0 to 0 that probability is $1 - \beta$ and 0 to 1 it is going to be β and there is no arc from 0 to 2 because that probability is 0. And similarly now we can go for filling the second row. So 1 to 0 is $\alpha \times (1 - \beta)$, 1 to 1 is $1 - \alpha \times (1 - \beta) + \alpha \beta$, and 1 to 2. So you have all three probabilities are greater than 0 therefore 1 to 0 that arc is $\alpha \times (1 - \beta)$ and 1 to 1 is $1 - \alpha \times (1 - \beta) + \alpha \beta$, 1 to 2 is $\beta \times (1 - \alpha)$. Similarly you can draw the arc from 2 to 0 that is α^2 and 2 to 1 and so on. Therefore 2 to 0 that is α^2 and 2 to 1... 2 to 1 is $2\alpha \times (1 - \alpha)$ and 2 to 2 that is $1 - \alpha^2$. So the state transition diagram is a pictorial view of a one-step transition probability matrix. This has nothing to do with the initial probability distribution. It gives only information about whenever the DTMC is the time homogeneous. Suppose the system starts from one particular state, what is the probability that the

system will move into the another states with the probability and it won't give more than that information, but this much information is useful when you are going to study the properties of the discrete time Markov chain as well as when you are... want to find out the limiting distribution that is the distribution of X_n has tends to infinity, the diagram will be very useful to conclude whether the limiting distribution exists or not, if it exists, whether it is going to be unique or not and so on. So those things can be visualized easily by seeing the state transition diagram.