Slide 1: So based on the values of the way I have explained the random variable or the stochastic process is going to be X of W,T, where W is belonging to Omega and T is belonging to capital T. There are two approaches, can define the stochastic process. The first one that is we name it as a case one, I can say it as the collection of random variable as a family… family of random variables as X.T where T is belonging to… T is belonging to capital T. So this is the way I can create the random variable and this is the easier… easier approach in the sense, once I know the different T for fixed T, it is going to be a one random variable and I have collected a family of random variable for different values of small t, therefore this is the way we can create the stochastic process and this is the easier approach also.

Slide 2: The next one that is a case two, that is nothing but has a set of functions on capital T, that is nothing but collection of X W, for W is belonging to Omega that means I have made a function on capital T and once I fix one W I will have a one function and if I fix another W, where W is nothing but a possible outcomes, therefore if I have a different possible outcome, that is going to be create a different stochastic process. There are.. therefore I can create a stochastic process of X of W,T either fixing T or fixing the W accordingly I can have a two different ways of creating a stochastic process and the case two, the way I have made a collection of random variable by fixing the W, then I made a function of… functions on T, therefore this is going to be the method of realization of the process or it is going to be called as a trajectory or it can be called as a sample path or it can be called as a sample function also. So these are all the different ways the case two can be called that means once you know the one possible outcomes, therefore you are tracing the stochastic process along the one possible outcomes, therefore that is going to be called as a realization of the process or the trajectory or the sample path. So the conclusion is you can always define a stochastic process as a collection of random variable for different value of T or you can go for a collection of functions on T for different values of the possible values of possible… possible outcomes that is a W belonging to Omega. So these are all the two approaches, can create the stochastic process.

Slide 3: Not only we can go for making a one dimensional random variable or one dimensional stochastic process so we can create a stochastic process it could be one dimensional or it could be a two dimensional or it could be a N-dimensional also. So first we have discussed what a stochastic process and how to create the stochastic process, whether it exists and so on, then we have given the parameter space and the state space, then we have given… then we have given what are all the ways we can create the two different approaches, you can create the stochastic process and how we are discussing what is the dimension of the stochastic process. So either the default, it could be a one-dimensional or it could be two-dimensional or it could be N dimension. Let me give a one simple example in which it is going to be a two dimensional. That means I have a random variable X of T that is going to be X1 of T, X2 of The, in which X1 of T is nothing but the maximum temperature and the X2 of T could be minimum temperature. The maximum and minimum temperature, possible of a place at any time T and this together is going to be one random variable that means this is a random vector which consists of two random variables X1 of T and X2 of T that means for fixed T, you have a one random vector X of T and therefore you have a random vector for over the T and this random vector will form a stochastic process therefore this is going to be a two dimensional stochastic process. Therefore in general you can define N dimensional stochastic process with… for fixed, for every T you have a random vector X of T that is going to be a X1 of T, X2 of T and so on, it is going to be the… and the element is XN of T that is going to be a N triple, in which each one is going to be a random variable for fixed T and this is going to be a random vector for fixed T and this is going to be a N dimensional stochastic process in which each one is going to be a one-dimensional random variable for fixed T. That means you can go for making one dimensional random variable, then you have a collection of random variable from one dimensional stochastic process or you can have a two dimensional, like that you can have a N dimensional stochastic process. In the course what we are going to discuss always, it's a one dimensional stochastic process. We can always create a complex valued stochastic process also in the form of X of T. Let me define it here, the X of T is going to be X1 of T plus I times X2 of T, where I is nothing but the complex quantity square root of minus 1. That means the X1 of T is a real valued random variable for fixed T and the X2 of T is also a real valued random variable for fixed T. The way I have made it the X of T, this is going to be a complex valued random variable for fixed T, therefore the X of T over the T that is going to be form a complex valued stochastic process, because for fixed T, X of T is going to be a complex valued random variable. The corresponding stochastic process is called a complex valued stochastic process in the one dimensional form. Like that you can go for the multi-dimensional complex valued stochastic process also, but in this course we are… what we are discussing only the real valued one dimensional random variable, most of the times, sometimes we are discussing real valued two-dimensional or N dimensional stochastic process, that too with the real valued random variable, not the complex valued.

Slide 5: So now we are going for classification of stochastic process the way I have explained the parameter space capital T, the capital T is parameter space and capital S is going to be the state space, that is nothing but the collection of possible values of X of T and the possible values of small t belonging to capital T that form a parameter space, some books they use the notation parameter set also and the capital S is going to be the state space. Now based on this we are going to classify… we are going to classify the stochastic process, suppose let us start with the capital S. Suppose the possible values of S and what is the name of the stochastic process, if… if S is going to take the only countably infinite or countably finite values, then it is going to be called as the corresponding stochastic process, is going to be called as integer valued stochastic process or we can call it as a discrete state stochastic process. So whenever the possible values of S is going to be a countably finite or countably infinite, then we say it is a integer valued stochastic process or a discrete state stochastic process. Suppose the possible values of S is going to be the real values, then we call it as a real valued stochastic process. Suppose if it takes Euclidean space with the K dimensional Euclidean… Euclidean K dimensional space, then we call it as a K dimension… k vector space, K vector stochastic process. That means the each random variable going to have a one-dimensional random variable and like that you have a K random variables for fixed T, therefore you have a K vector stochastic process, therefore it is going to be called as a K vector stochastic process in which each element is going to be a one dimensional random variable for fixed T. So the collection, the K triple values stochastic process is going to be called as a K vector stochastic process. Similarly you can go for, based on the capital T, what is the name of the stochastic process for different values of T. That means if it is going to take the value countably finite or countably infinite or it is going to take only the integers values, then we say it is a discrete parameter stochastic process or again there is another name, it is called the stochastic sequence also. Whenever the possible values of a capital T is going to be countably finite or countably infinite, then we call the corresponding stochastic process as the stochastic sequence or it is a discrete parameter stochastic process. Otherwise, if it takes uncountably many values in the capital T, then it is going to be called as a continuous parameter and it is… or it is going to be called as a stochastic process itself. Therefore based on the discretize, it uses the word sequence or if it is going to be uncountably many values of capital T, then it is going to be called as a stochastic process. So based on the classification, I can go for making a 1 table in which the possible values of S will take a column and the possible values of capital T will take a row, so either it could be a… either it could be a countably finite or countably infinite, that I uses the word discreet. If the possible values of T is going to be uncountably many, either it is set of all intervals or it will be a whole real line itself or it is going to be a union of many intervals, in that case it is going to be called as a continuous… continuous parameter. Similarly if the possible values of S is going to be a countably finite or countably infinite, then the state space is going to be called as a discrete. Similarly if it is going to be uncountably many values, then it is going to be called as a continuous. So accordingly you can classify the stochastic process into the four type, in which if the T is going to be a discrete as well as S is going to be a discrete, then it is going to be a discrete time… discrete time or discrete parameter both are one and the same. So discrete time… discrete state stochastic process. Similarly if the T is a discrete and the state space is continuous then we can call it as a discrete time continuous state stochastic process. Similarly this is going to be a continuous time discrete state stochastic process and this is going to be a continuous time… continuous time… continuous state stochastic process. That means based on the possible values of capital T and the possible values of capital S. Any stochastic process can be classified into the four types, in which it is going to be a discrete discrete or continuous continuous or discrete continuous continuous continuous based on the time and the state space.