

Slide 1: This is the model 2 of stochastic processor. In this model what we are going to discuss is the definition then followed by the simple stochastic process. And this model consists of two lectures and here this is the first lecture in which we are going to describe the stochastic process then we are going to discuss the classification of a stochastic process followed by few simple examples, which arises in the real-world problem.

Slide 2: So the content of this lecture is going to be, as I said, let me first give the definition of a stochastic process, then I will explain how to create or how to develop the stochastic process and how to, what is the meaning of a parameter and the state space, then I am going to give what are all the approaches in which the stochastic process can be described and the classification of a stochastic process based on the parameter and the state space, and then at the end of this lecture we are going to discuss some of the few simple stochastic processes and the summary of the lecture 1., and there are few reference books also listed for this course to be... preparation.

Slide 3: What is stochastic process. Let me give their definition. Let  $\Omega$  be a given probability space. That means you know what is a random experiment, from the random experiment you know what is the  $\Omega$  and from the collection of possible outcomes you got the Sigma algebra that is  $\mathcal{F}$  and you have probability measure also, therefore this triplet is going to be the probability space and you have a given probability space. From the given probability space you have the collection of random variables that is  $X$  of  $\mathcal{T}$ , where  $T$  is belonging to capital  $\mathcal{T}$ , defined on the probability space that is  $\Omega \in \mathcal{F} \in \mathcal{P}$  that is called stochastic process. That means you have a probability space, from the probability space you have collected random variables with the  $t$  belonging to capital  $\mathcal{T}$  and this collection is going to be called as a stochastic process. Now the question is whether we can create only 1 stochastic process or how to create a stochastic process from the sigma-algebra, that means suppose you have a  $\Omega$  from the  $\Omega$  you can always create here sigma-algebra that is a capital  $\mathcal{F}$ , that is a collection of subsets of  $\Omega$  satisfying the condition. If you make a union of a few elements then they... if you make the elements... if you take a few elements then the union of elements is also belonging to one of the element and if you take any one of the elements in the  $\mathcal{F}$  then the complement is also belonging to  $\mathcal{F}$ . So if these conditions are going to be satisfied then that collection of subsets of  $\Omega$  is going to be called as sigma-algebra. So from the  $\Omega$  we have created here random variable that is  $X$  of  $\mathcal{T}$ , that is nothing but a random variable, that is nothing but a real valued function, which is defined from  $\Omega$  to  $\mathbb{R}$  such that it satisfies the condition  $X$  of  $\mathcal{T}$  of inverse of minus infinity to the closed interval  $X$  that is belonging to  $\mathcal{F}$  for all  $X$  belonging to  $\mathbb{R}$ . That means whatever we the  $X$  belonging to  $\mathbb{R}$ , if the inverse images from minus infinity to some point  $X$  ray, if that is belonging to capital  $\mathcal{F}$  then that real valued function is going to be called as a random variable. Like that if you may get different random variable for

different  $\mathcal{T}$ 's, where all the  $\mathcal{T}$ 's are belonging to, so I can go for  $\mathcal{T}_1$  or  $\mathcal{E}_s$ , so all the  $\mathcal{T}$ 's are belonging to capital  $T$ . So that means if I have a collection of random variables for the different values of  $\mathcal{T}$ , then that collection is going to be called as a stochastic process. Now the question is whether we can create only one stochastic process from a given probability space or more than one stochastic process can be created from the same probability space? The answer is yes, you can always create more than one random variable from the same probability space, that means for a different collection of capital  $\mathcal{T}$ , you can have a different stochastic process. More than one stochastic process can be created from one probability space. Now the next question, if I change the Sigma algebra what happens? if I change the Sigma algebra capital Findings, then I may land up collecting some other stochastic process in which those real valued function is going to be a random variable for that... that particular  $\Omega$  and  $F$  and  $P$  and that for a given probability space, the stochastic process is going to be changed for a different collection of  $\mathcal{T}$  belonging to capital  $T$ , that means once you know the  $F$  then you will have some collection of random variable that will form a stochastic process. If you change the another  $F$ , then you may get the different stochastic process and also for a given probability space you can have more than one stochastic process by the way you define collection of random variable, the way you have a capital  $\mathcal{T}$ , accordingly you will have a different stochastic process. Now the way I have given the collection of random variable, I can say it in a different way, that is a stochastic process also... is also defined as a function of two arguments that is  $X$  of  $W, T$  where  $W$  is belonging to  $\Omega$  and  $T$  is belonging to capital  $T$ , that means the same way I can define the collection of random variable as a collection of  $W, T$ , where  $W$  is belonging to  $\Omega$  and  $T$  belonging to capital  $T$  and this is also going to be form it as a stochastic process. That means always the  $W$  is belonging to  $\Omega$  that means the  $W$  is belonging to the possible outcomes and the  $t$  is belonging to capital  $t$  and this is going to set the given probability, this is going to set the stochastic process. The other names for the stochastic process are going to be Chance process, there are some others who use the word Chance process. There are some authors they use a notation that is called Random process. So either the stochastic process can be called it as a Chance process or the Random process also. Now what we are going to see, once you have a collection of random variable. So based on the  $X$  of, the values of  $X$  of  $T$  and the values of the different values of  $\mathcal{T}$ , we are going to define what is a parameter space and what is state space.

Slide 4: What is the meaning of a parameter space? The set... the set... we use the notation capital  $T$ , that is called the parameter space, the set capital  $T$  is called the parameter space and it is usually represented as the time, most of the time or it can be represented as the length or it can be represented as a distance and so on. So usually we go for  $T$  as the time so the set  $T$  is called the parameter space. Similarly I can define the state space as the the set, capital  $S$ , that is nothing but all possible values of  $X$

of  $T$ , for all  $T$ . So this set is called the state space.  $X_T$  is a random variable from  $\Omega$  into  $A$  suffix  $T$  where  $A$  suffix  $T$  is a subset of capital  $R$ . Then the  $A_T$ s are going to be the elements of... it is going to be contained in the real line, then the  $S$  is nothing, but union of  $T$  belonging to capital  $T$ , all the  $A_T$ s that is going to form a state space. That means for a fixed  $T$  you will have a collection of a possible values that is going to be the  $A_T$  and for variable  $T$  you collect all the Union and that possible values of  $X_T$  is going to form a set and that set is called the state space. Similarly the all possible values of a small  $t$  belonging to capital  $T$  and that set is going to be called as a parameter space. So based on the parameter space and the state space, we can go for classification. Now I can explain, what are all the possible values of  $S$  can take. So this  $T$  is going to be the collection of capital  $T$  therefore this can be a finite that means accountably finite or it could be countably infinite also. or it could be uncountably many elements of a small  $t$  so that set can be a finite set or it could be countably infinite or it could be uncountably many elements also.  $T$  can also be multi-dimensional set. Similarly the state space capital  $S$ , that can be a same way it could be finite or it could be countably infinite or it could be uncountably many elements. So since the state-space are going to be the collection of all possible values of  $X_T$  and  $X_T$  is a real-valued function and then it is going to be a random variable therefore these elements are going to be always the real numbers. So either it could be a finite elements or it could be accountably infinite elements and it is going to be uncountably many elements that means it could be a set of intervals on a real line or it could be the whole real line itself.