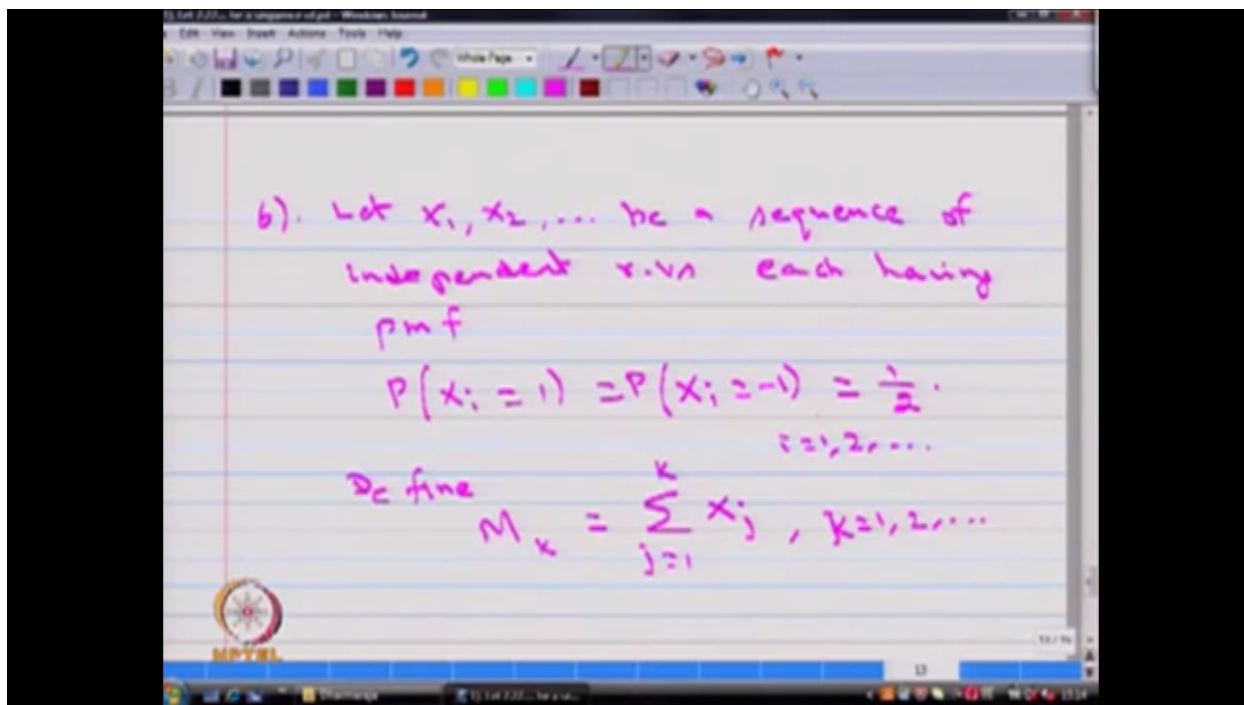


Now we move into next example, example 6. Let  $X_1, X_2$  so on be a sequence of independent random variables each having probability mass function probability of  $X_i$  is equal to 1 that is same as probability of  $X_i$  takes a value minus 1 the probability is 1 by 2. This is valid for that means it's a sequence of iid random variables and they are discrete a type. Define  $M$  suffix  $K$  as the sum of first  $K$   $X_i$  random variables. So this running index is  $K$  is equal to 1, 2, and so on. So we are defining a sequence of random variable  $M_K$  by summing first  $K$   $X_i$  random variables.

For a fixed integer  $n$  we define another sequence of random variable that is denoted by  $W$  superscript  $n$  of  $t$  that is nothing but 1 divided by square root of  $n$   $M$  suffix  $n$  times  $t$ . This is for all  $t$  greater or equal to 0 such that  $n$  times  $t$  is an integer. So we are defining another sequence of random variables  $W$  superscript  $n$  of  $t$  that is 1 divided by square root of  $n$  time  $n$  of  $t$  where  $n$  of  $t$  is integer. So this is valid for all  $t$  greater or equal to 0.



If you find out the mean and variance for the difference of the random variable of  $n$  of  $t$  minus  $W_n$  of  $s$  for  $0$  less than or equal to  $s$  less than or equal to  $t$  this quantity will be  $0$ . That means  $W_n$  of  $t$  is  $1$  divided by square root of  $n$   $M_n$  of  $t$  and the way we define the  $M_n$  of  $t$  that is a summation of  $X_i$ 's and the probability of  $X_i$  is equal to  $1$  and the probability of  $X$  is equal to minus  $1$  minus  $1$  is  $1$  by  $2$  therefore the mean of  $X_i$ 's are going to be  $0$  because of that the expectation of or mean of  $W_n$  of  $t$  minus  $W_n$  of  $S$  that is equal to  $0$ . Also if you evaluate the variance of  $W_n$  of  $t$  minus  $W_n$  of  $S$  by finding first variance of  $X_i$ 's using that you find out the variance of  $M_n$  of  $t$ . Then find out the variance of  $W_n$  of  $t$  minus  $W_n$  of  $S$  that is going to be  $t$  minus  $S$ . it needs calculation of expectation of  $X_i$ 's square then using expectation of  $X_i$ 's square and the expectation of  $X_i$ 's you can find out the variance of  $X_i$ 's using variance of  $X_i$ 's you can find out the variance of  $W_n$  of  $t$ . Then you find out the variance of  $W_n$  of  $t$  minus  $W_n$  of  $S$ . By using mean and variance for fixing fix  $t$  greater or equal to  $0$  as  $n$  tends to infinity we can conclude  $W_n$  of  $T$  tends to a random variable  $X$  and this convergence takes place in distribution using CLT one can conclude  $W_n$  of  $t$  converges to the random variable  $X$  and the convergence in distribution where  $X$  is normal distribution with the mean  $0$  and the variance  $T$ . Using central limit theorem one can prove  $W_n$  of  $T$  converges to  $X$  in distribution where  $X$  is a normal distribution with the mean  $0$  and the variance  $t$ .

This result is very useful in Brownian motion and the same problem will be discussed in detail when we are discussing the model of Brownian motion.