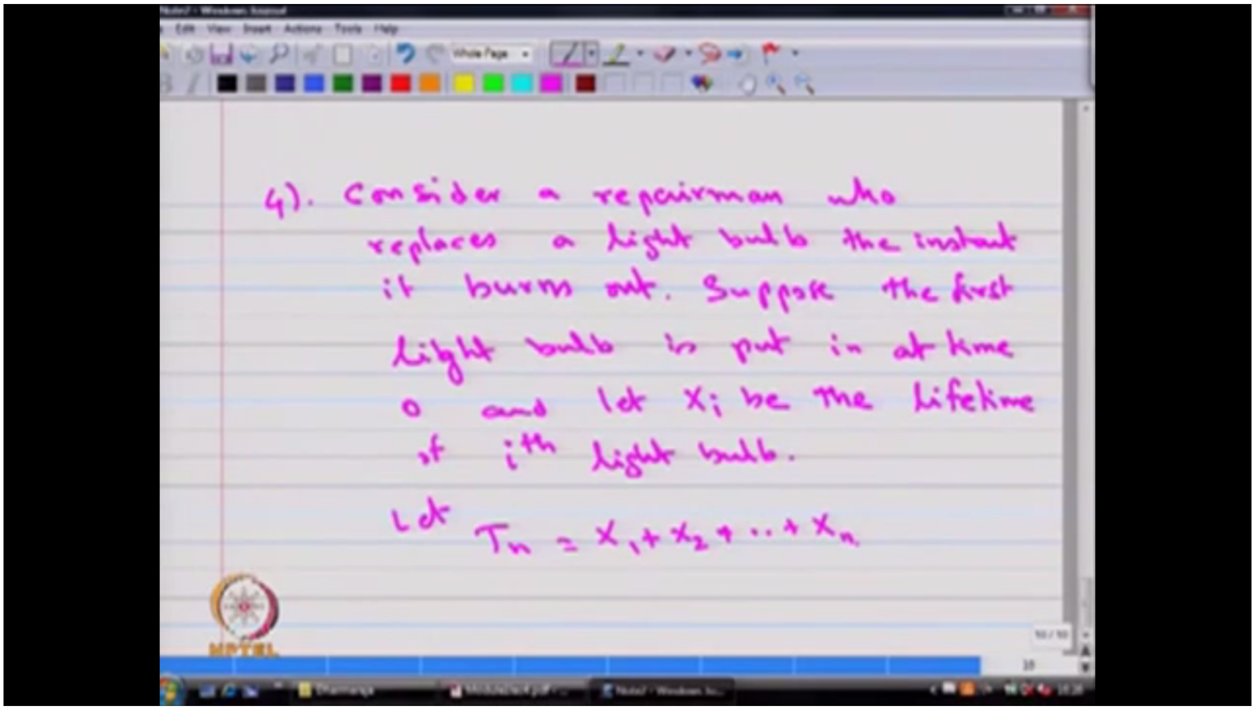
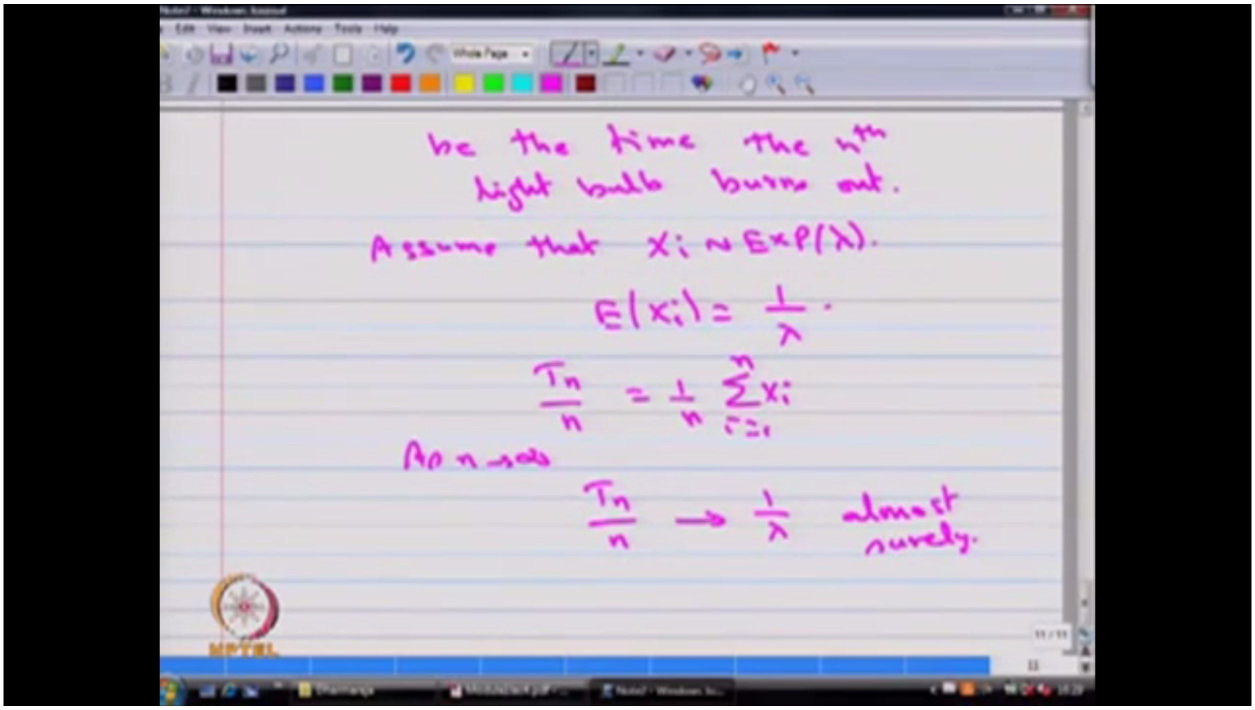


Now I move into the fourth example. Consider a repairman who replaces a ligh tbulb the instant it burns out. Suppose the first light bulb is put in at time 0 and let X suffix i be the lifetime of ith light bulb you define the random variable Tn is a sum of n Xi's where Xi's are iid random variables. Xi be the lifetime of the ith light bulb and when Xi's are iid random variable you are defining Tn is X1 plus X2 plus Xn and so. So the Tn be the time of time the nth light bulb burns out because the Tn is a X1 plus X2 and so on till Xn therefore T be the time the nth light bulb burns out.

Assume that Xi's is exponential distribution with the parameter lambda. We know that already Xi's are iid random variable. Now I am making the further assumption Xi's follows exponential distribution with the parameter lambda. That means you know what is the mean of this random variable. Since it is exponential distribution with the parameter lambda, this becomes 1 divided by lambda.

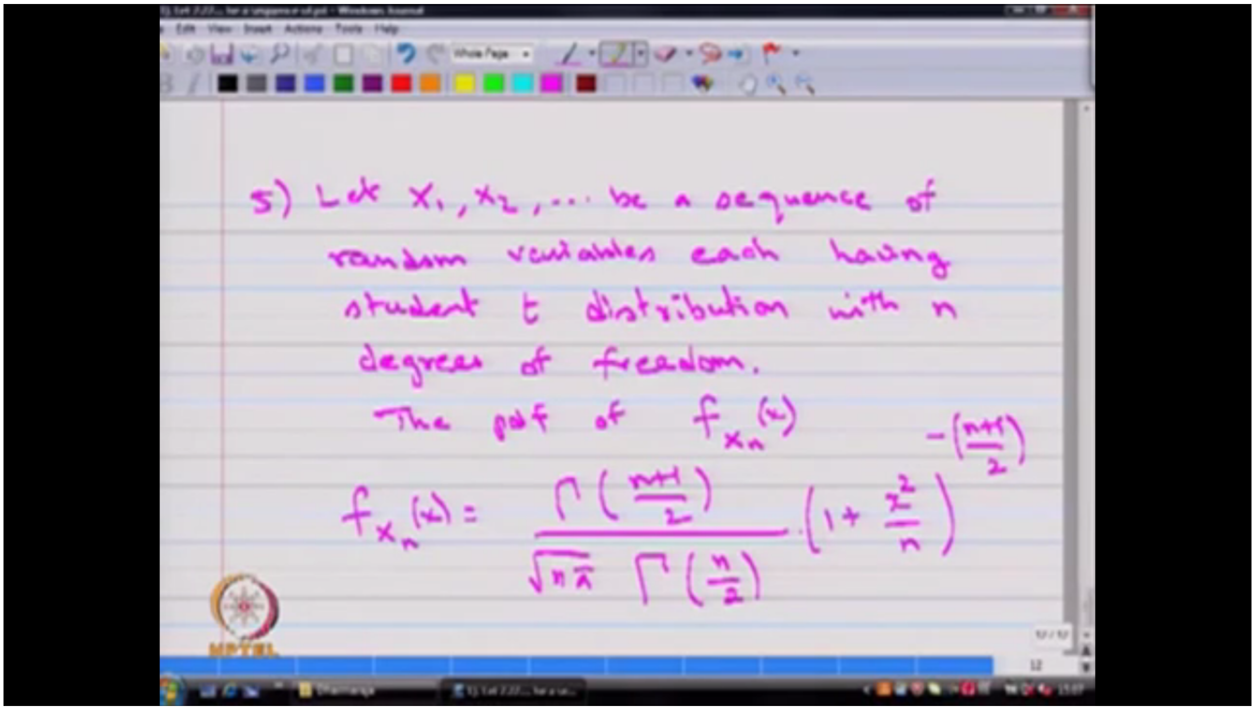
Also one can use the result Tn by n that is nothing but 1 divided by n summation of Xi's where i is running from 1 to n. As n tends to infinity one can prove Tn by n tends to 1 divided by lambda that is a mean of the random variable Xi almost surely. I'm not proving here the way you do the sequence of random variable converges to another random variable convergence takes place in probability or in distribution or in [Indiscernible] [00:04:43] mean or almost surely one can prove this that Tn by n converges to 1 by lambda almost surely.



That means we can conclude the random variable X1, X2 and so on obeys strong law of large numbers because the Tn by n that is nothing but 1 by n summation of Xi's that converges to the value 1 by lambda almost surely we can conclude the sequence of random variable Xi's obeys a strong law of large numbers. Even though in this problem I made the assumption Xi's follows the exponential distribution with the parameter lambda in general the lifetime can be any distribution.

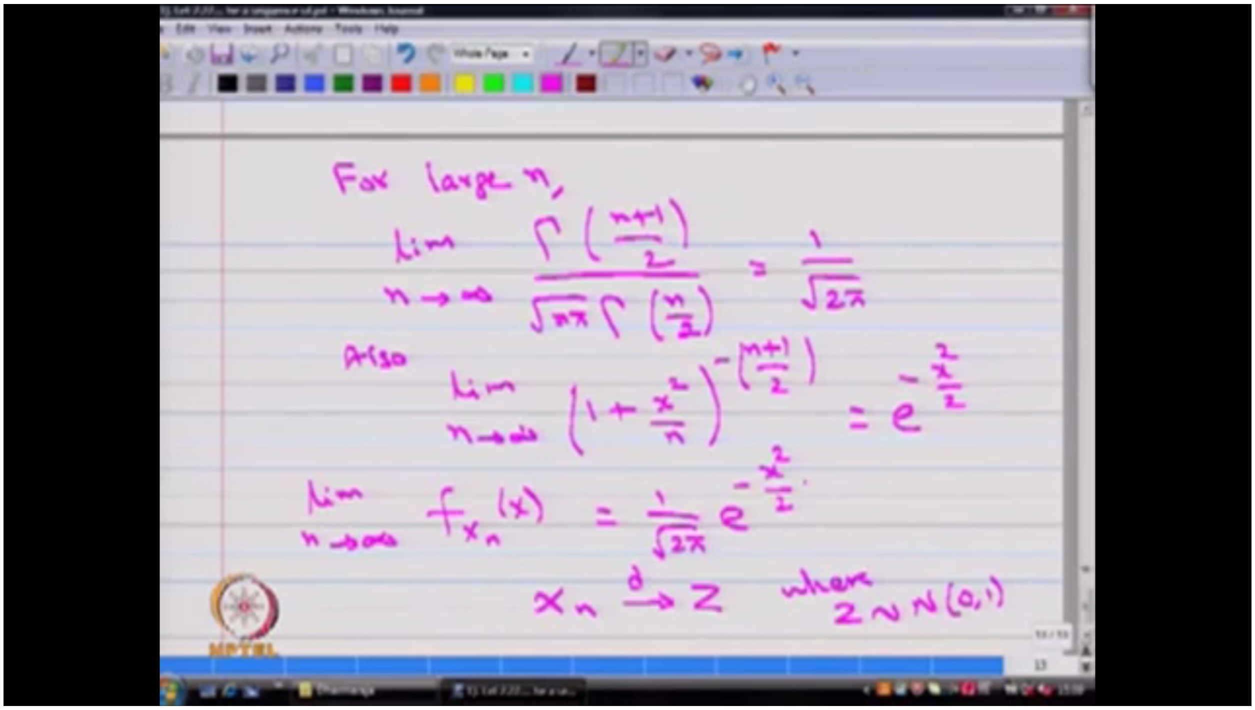
So this problem will be discussed in detail in renewal processes. So as such here we are making the assumption of distribution of Xi's exponential distribution therefore I made it a convergence takes place almost surely to the value 1 by lambda this can be generalized.

There are many more problems of the similar kind but we are discussing only few problems. Therefore, we can use a similar logic of finding the moment generating function then concluding the distribution and finding the limiting distribution or you verify whether the sequence of random variable convergence takes place in mean, convergence takes place in probability or convergence takes place in distribution or converges in **arithm** or convergence almost surely this can be used in any problem of the same way what I have done it here. And I have not discussed any problem in the central limit theorem but that will be used many times. Therefore I have not given any problems for the central limit theorem.



Let X1, X2 so one be a sequence of random variables each having student t distribution with the n degrees of freedom. Our interest is to find out the limiting distribution of the student t distribution. We know that the probability density function of f of X for the random variable Xn is given by Gamma of n plus 1 by 2 divided by square root of n times pi multiplied by Gamma of n by 2 multiplied by 1 plus X square by n power minus n plus 1 by 2. So this is the probability density function of a random variable Xn. Our interest is to find out the limiting distribution of the random variable Xn. For larger n we have the results limit n tends to infinity of Gamma of n plus 1 by 2 divided by square root of npi of Gamma of n by 2 is 1 divided by square root of 2pi using Sterling's approximation and also limit n tends to infinity of 1 plus X square by 2 the whole power minus n plus 1 by 2 that we know that is e power minus X square by 2. Hence the limit n tends to infinity of the probability density function of the random variable Xn becomes 1 divided by square root of 2pi e power minus X square by 2.

Since the right hand side is a probability density function of a standard normal distribution, we conclude for a large n the sequence of random variables X1, X2, Xn and so on that tends to the random variable Z; this convergence takes place in distribution where Z is standard normal distribution.



So this is a simple example of the sequence of random variables each having a student t distribution. The limiting distribution converges to standard normal and that convergence in distribution.