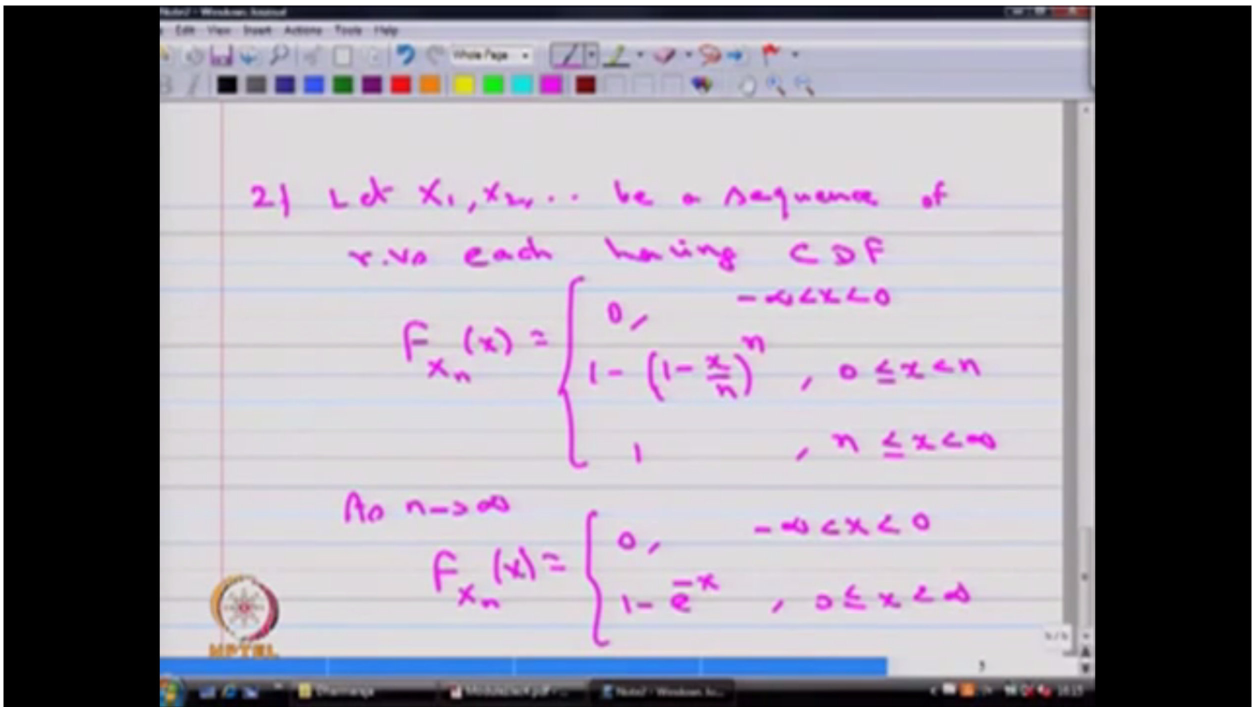
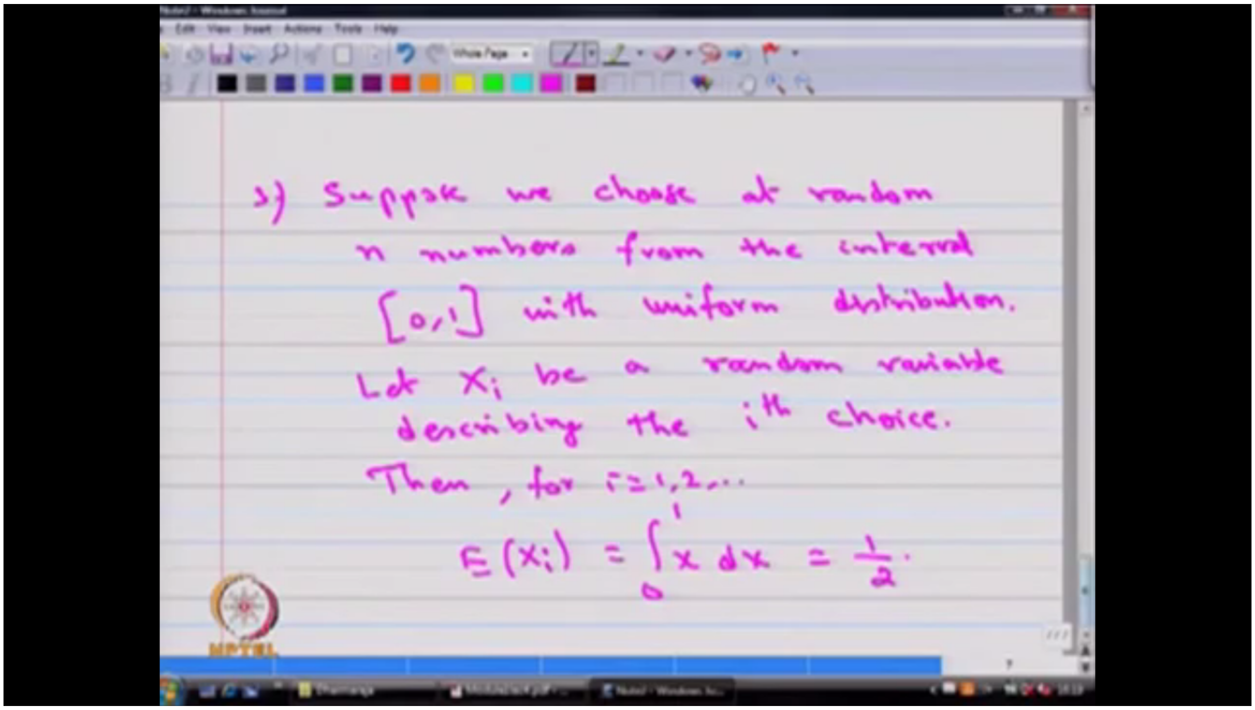
Next example. Let X1, X2 and so on be a sequence of random variables each having CDF, Cumulative Distribution Function, F suffix Xn of X 0 from minus infinity to 0 and it takes a value 1 minus 1 minus X by n power n, for X is lies between 0 to n. From n onwards till infinity the value is 1. So this is the cumulative distribution function for the random variables X i's. It's a function of n therefore I have made it F suffix X of Xn that means this is a CDF for the random variable n for every n you have this form.

As n tends to infinity we get F suffix Xn of X that becomes zero from minus infinity to zero and it takes a value 1 minus e power minus X from zero to infinity as n tends to infinity the CDF of the random variables Xn becomes 0 between the interval minus infinity to 0 and the value becomes 1 minus e power minus lambda X where X is lies between 0 to infinity.



Suppose X is a random variable with the CDF that is FX of X that is 0 between the interval minus infinity to 0 and 1 minus e power minus X where X is lies between 0 to infinity then one can conclude Xn converges to X in distribution since the sequence of FX suffix n of X tends to F of X for X is greater or equal to 0 and the value is 1 minus e power minus X. Hence one can conclude the sequence of random variable Xn converges to the random variable X in distribution. Here the X is a exponential distribution with the parameter 1. So this is one example of how the sequence of random variable converges to a random variable in distribution.

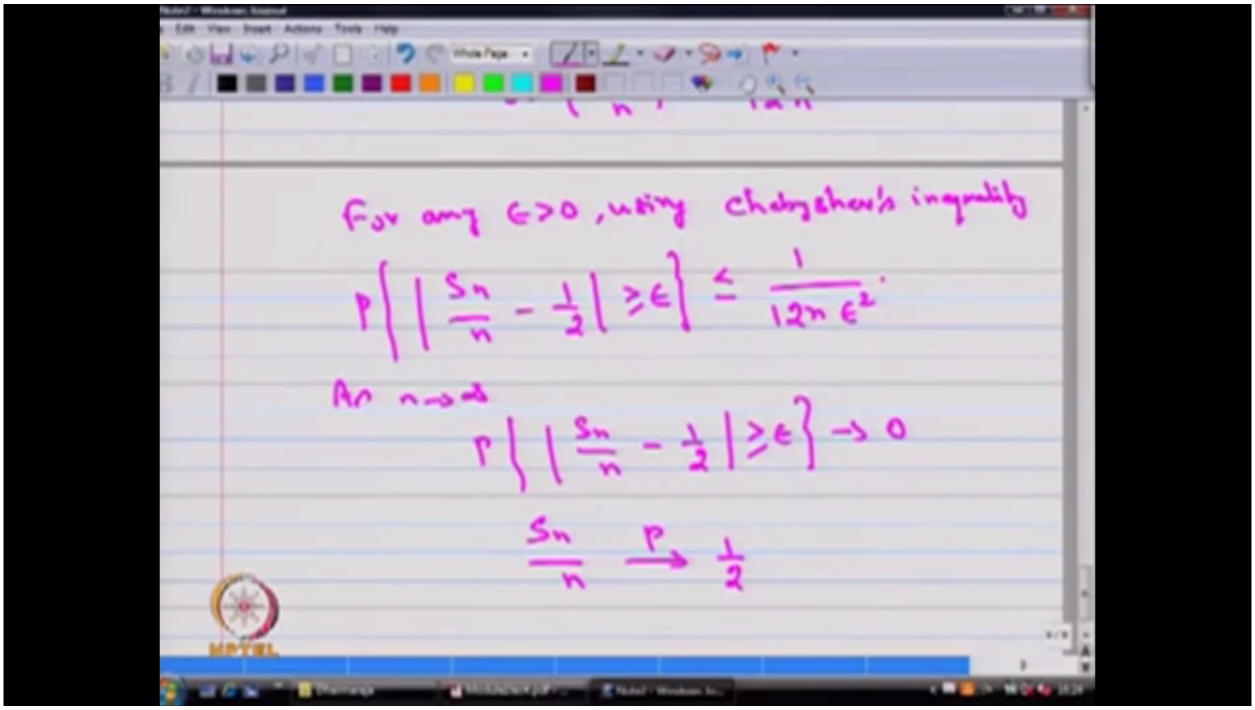
Next I will move into the third example. Suppose we choose at random n numbers from the interval 0 to 1 with uniform distribution. Let Xi be a random variable describing the ith choice. Then for i is equal to 1, 2 and so on you can find out what is the expectation of Xi's that is nothing but the integration from 0 to 1 X times the probability density function for uniform distribution with the interval 0 to 1 that is 1. Therefore X into dx if you compute the expectation of Xi's is going to be 1 by 2.



Similarly one can evaluate the variance of Xi's that is nothing but 0 to 1 x square dx minus the mean square expectation of X square minus expectation of X whole square so the expectation of X square is 0 to 1 x square dx. So if you evaluate this quantity that is 1 by 3 minus 1 by 4. So if you simplify you will get 1 by 12. If you remember the formula of variance of a uniformly distributed random variable between the interval A to B then the variance of Xi's X is nothing but you can get it and by substituting the value of A is equal to 0 and B is equal to 1 you will get 1 by 12.

Let S suffix n be X1 plus X2 and so on till Xn. One can find mean and variance of S because you know the mean and variance of Xi's using that you can find out what is the mean of Sn but our interest is not finding the mean of Sn. Our interest is to find out the mean of Sn by n that is basically suppose Xi's are the samples then Sn divided by n is nothing but the sample mean. So expectation of Sn divided by n that becomes 1 by 2. Similarly if you calculate variance of Sn by n that becomes 1/12 times n because the variance of Xi's is 1 by 12 so the variance of Sn is a summation of Xi's from 1 to n therefore variance of Sn by n becomes 1/12 times n.

For any epsilon greater than 0 using Chebyshev's inequality one can conclude the probability of absolute of Sn by n minus 1 by 2 greater than or equal to epsilon that is less than or equal to 1 divided by 12 times n epsilon square. I am using the Chebyshev's inequality by knowing mean of Sn by n is 1 by 2 and variance of Sn by n is [Indiscernible] [00:09:03] I get this inequality. Now as n tends to infinity the probability of absolute of Sn by n minus 1 by 2 which is greater than or equal to epsilon will tends to 0 because epsilon is in the – n is in the denominator because n is the denominator as n tends to infinity this probability tends to 0. That is nothing but Sn by n tends to the value 1 by 2 and this convergence takes place in probability. The sequence of random variable Sn by n converges to 1 by 2 in probability. Therefore, we say the sequence of random variable Xn for n is equal to 1, 2, and so on obeys the weak law of large numbers because the Sn by n converges to 1 by 2 in probability therefore we say the sequence of random variables say Sn obeys the weak law of large numbers.



So that is the intention of giving this example.