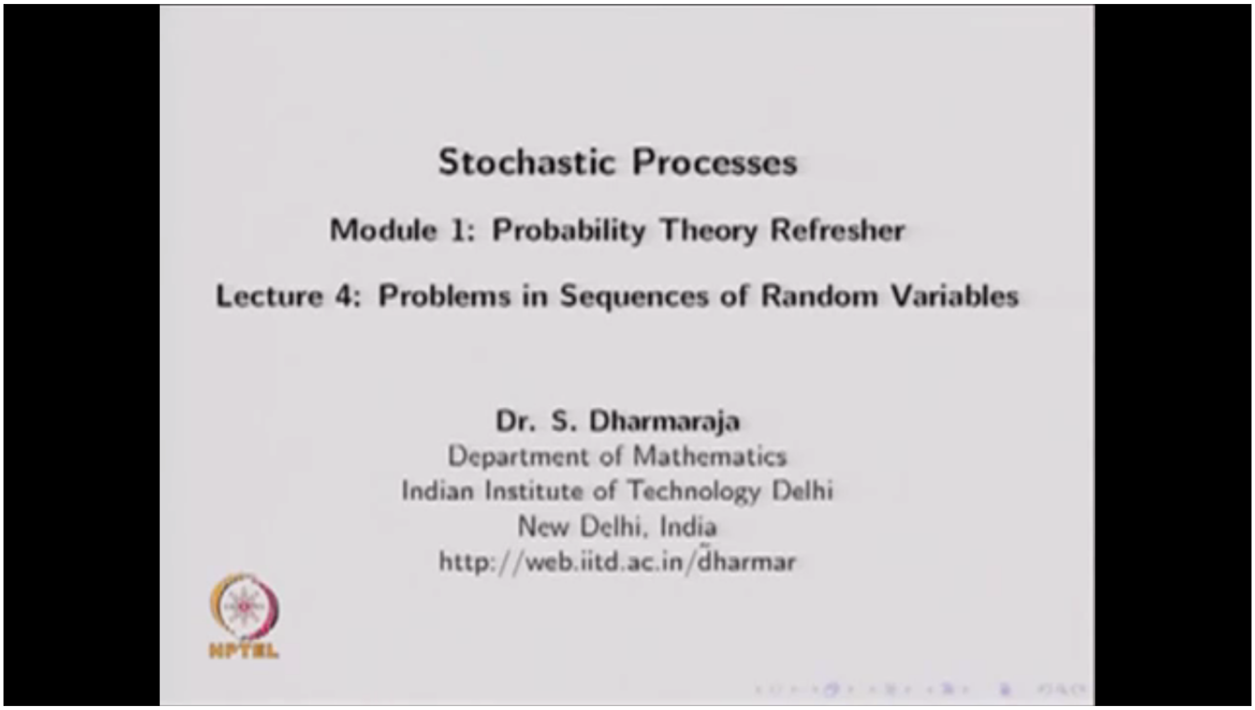

National Programme on

Technology Enhanced Learning

Video Course on

Stochastic Processes -1

by

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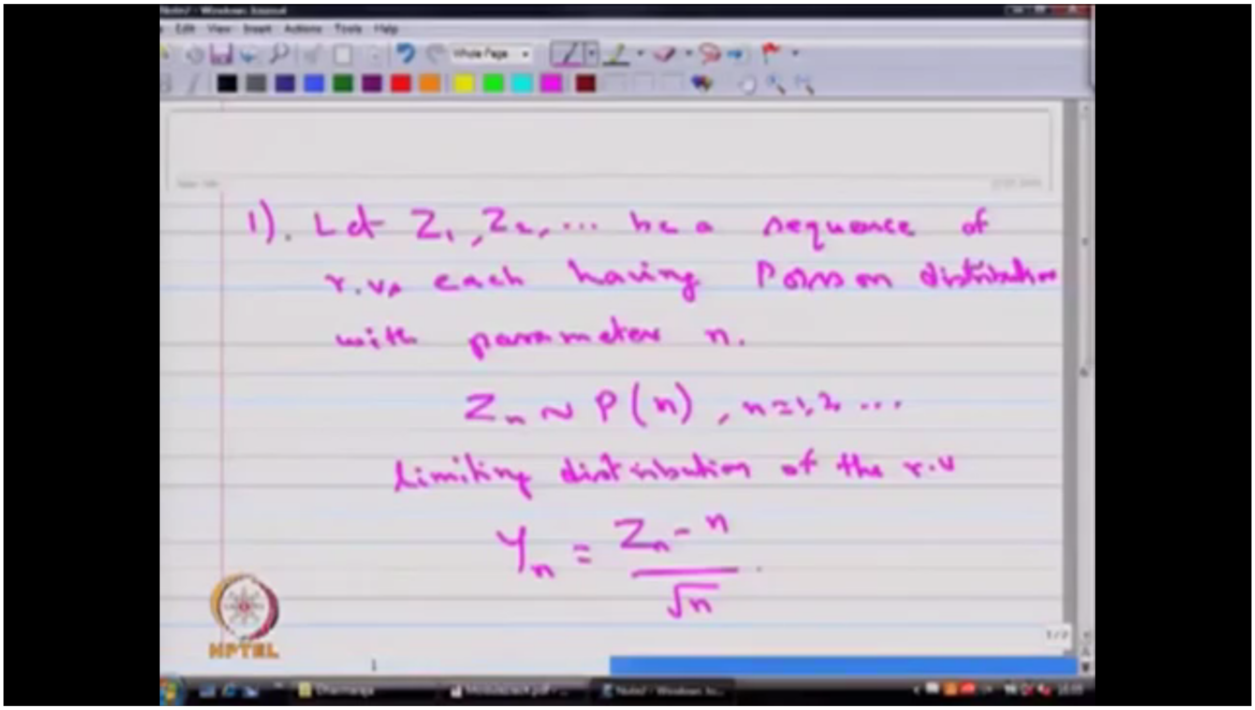
Module 1: Probability Theory Refresher

Lecture #4

Problems in Sequence of Random Variables

So this is a stochastic processes model 1 probability theory refresher. Lecture four problems in sequence of random variables. As a illustrative examples we are going to discuss four problems in this lecture. The first problem let Z1, Z2 so on be a sequence of random variables each having Poisson distribution with parameter n that is Zn is Poisson distribution with the parameter n for n is equal to 1, 2, 3, and so on. Our interest is to find the limiting distribution of the random variable that is defined as Y suffix n that is Zn minus n divided by square root of n.

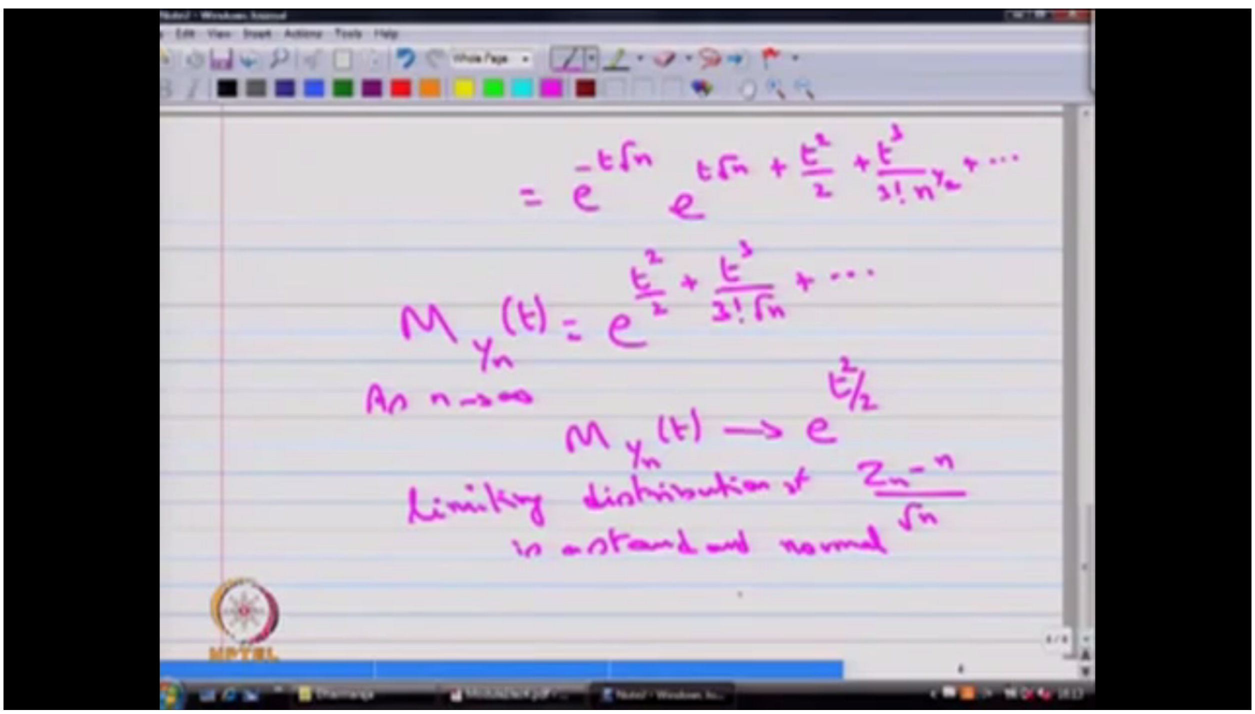


So given Zn Poisson distribution with the parameter n we can find out the MGF of Zn. MGF of Zn is nothing but expectation of e power Zn of t that is same as summation k is equal to 0 to infinity e power k times t e power minus n n power k by k factorial because it is a expectation of e power Zn of t where Zn is Poisson distribution with the parameter lambda therefore this is going to be k is equal to 0 to infinity this one. So you can take an e power minus n outside so the remaining term becomes k is equal to 0 to infinity e power t multiplied by n the whole thing power k by k factorial. That is same as e power minus n e power n times e power t that can be rewritten as e power n times e power t minus 1.

Now we will find out the MGF of the random variable Yn where Yn is Zn minus n divided by square root of n. Therefore the MGF of the random variable Yn as a function of t that becomes MGF of Zn minus n divided by square root of n function of t. That is same as expectation of e power Zn minus n divided by root n multiplied by t. You know the rules of moment generating function. The constant is out. So you can use that logic. So it becomes e power minus t times root n because nt by root n therefore it becomes t times root n. Then MGF of the random variable Zn use another rule of a moment generating function instead of t it becomes a t divided by square root of n. So that is same as e power minus t times the root n just now we found what is the moment generating function of Zn.

So use the same thing but replace t by t divided by square root of n. Therefore, this becomes e power n times wherever the t you replace t by t by square root of n. So t by square root of n minus 1. Therefore, you can further simplify by expanding e power t by n that means you keep this e power n you expand only e power t by square root of n. That is 1 plus t divided by square root of n then the next term will be t square by 2 times n and the next term will be t cube divided by 3 factorial n power 3 by 2 and so on. The last term is so this is the expansion of e power t by square root of n minus 1.

So close the bracket. That is same as e power t times square root of n multiplied by so this 1 and plus 1 and minus 1 will be canceled. So you will get e power n times t by square root of n that becomes a t of square root of n and the next term becomes t square by 2 then it becomes t cube by 3 factorial n power 1 by 2 and so on. Therefore this becomes e power t square by 2 plus t cube by 3 factorial square root of n and so on. Our interest is to find out the limiting distribution of Yn. So this is the moment generating function of Yn for n. So as n tends to infinity because of our interest is to find out the limiting distribution as n tends to infinity the moment generating function of Yn becomes e power t square by 2. If you recall the moment generating function for standard distributions one can conclude this is the MGF of a standard normal distribution. Therefore, we conclude the limiting distribution of Yn is standard normal distribution. That is the limiting distribution Zn minus n divided by square root of n is a standard normal distribution.



So this problem is very important in the renewal processes. Therefore we discuss this example as how to find the limiting distribution of some standard random variables.