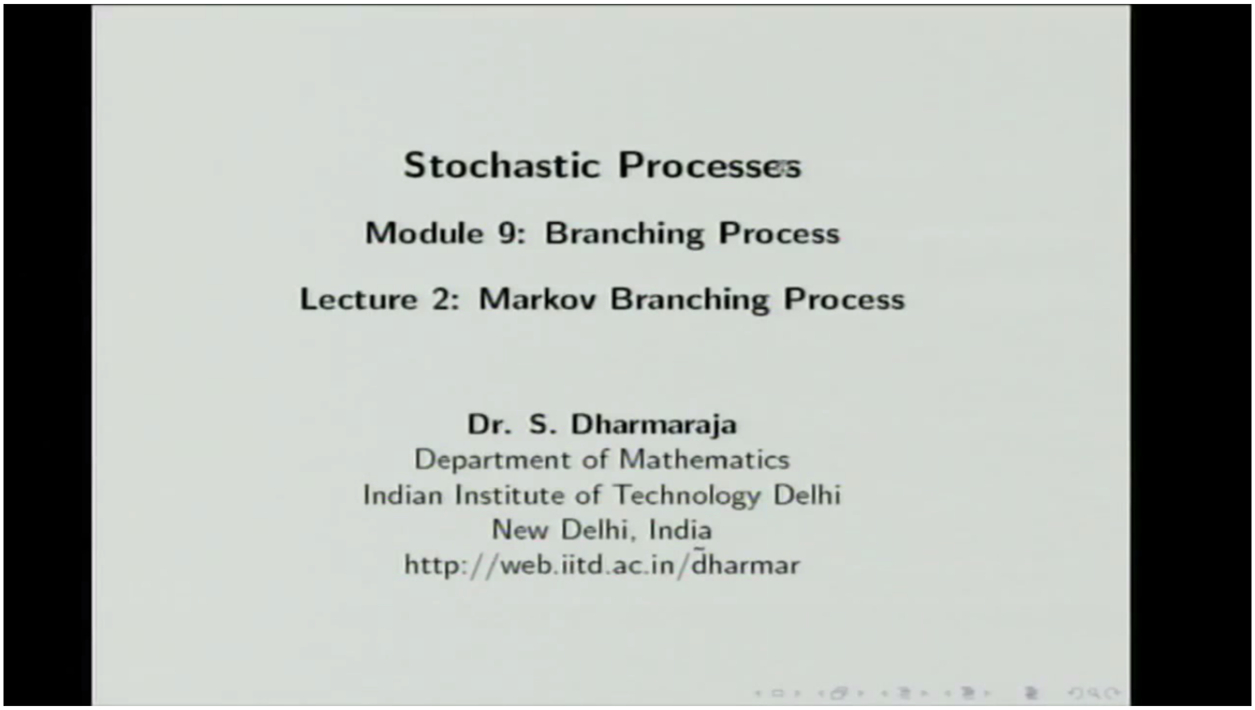


National Programme on

Technology Enhanced Learning

Video Course on

Stochastic Processes - 1

By

Dr. S Dharmaraja

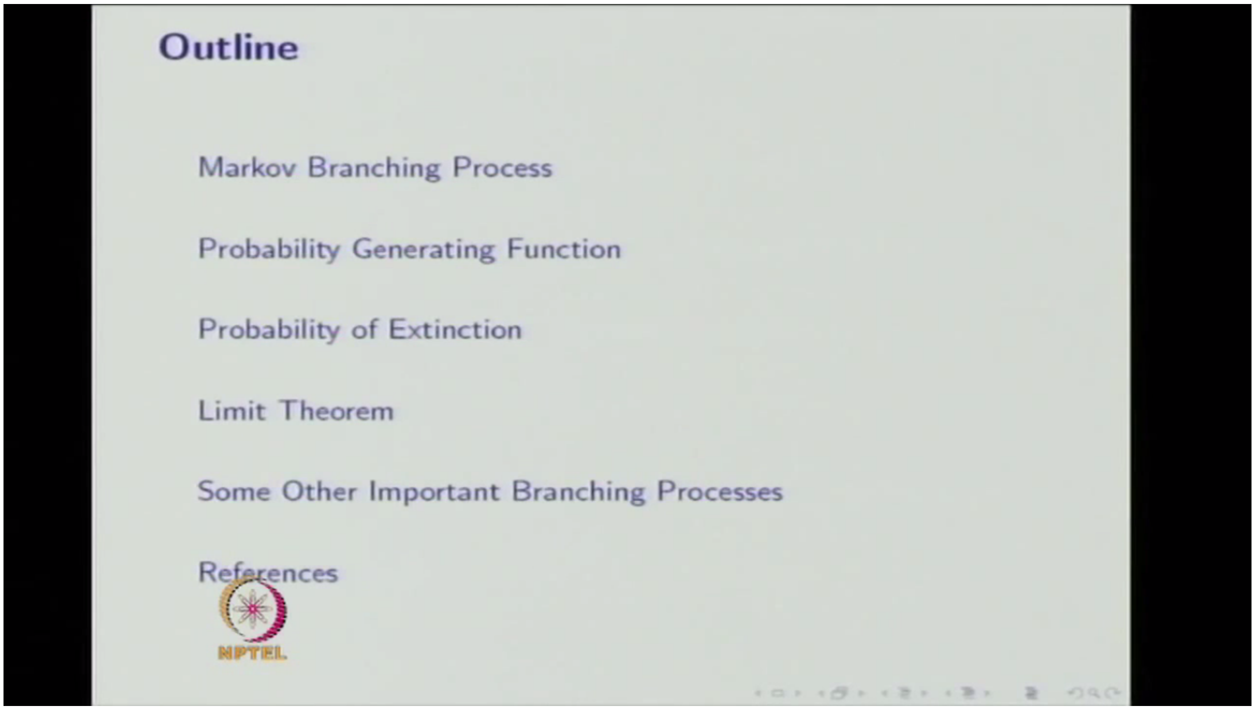
Department of Mathematics, IIT Delhi

Module# 9

Branching Process

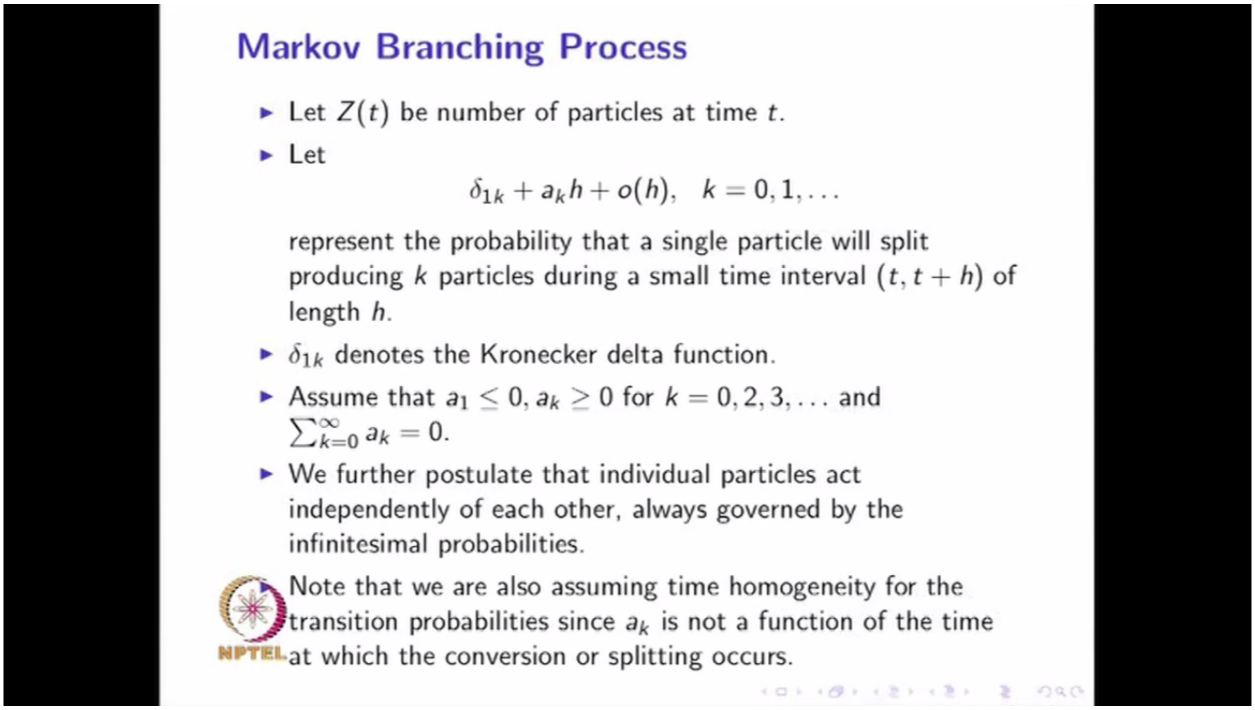
Lecture # 2

Markovian Branching Process

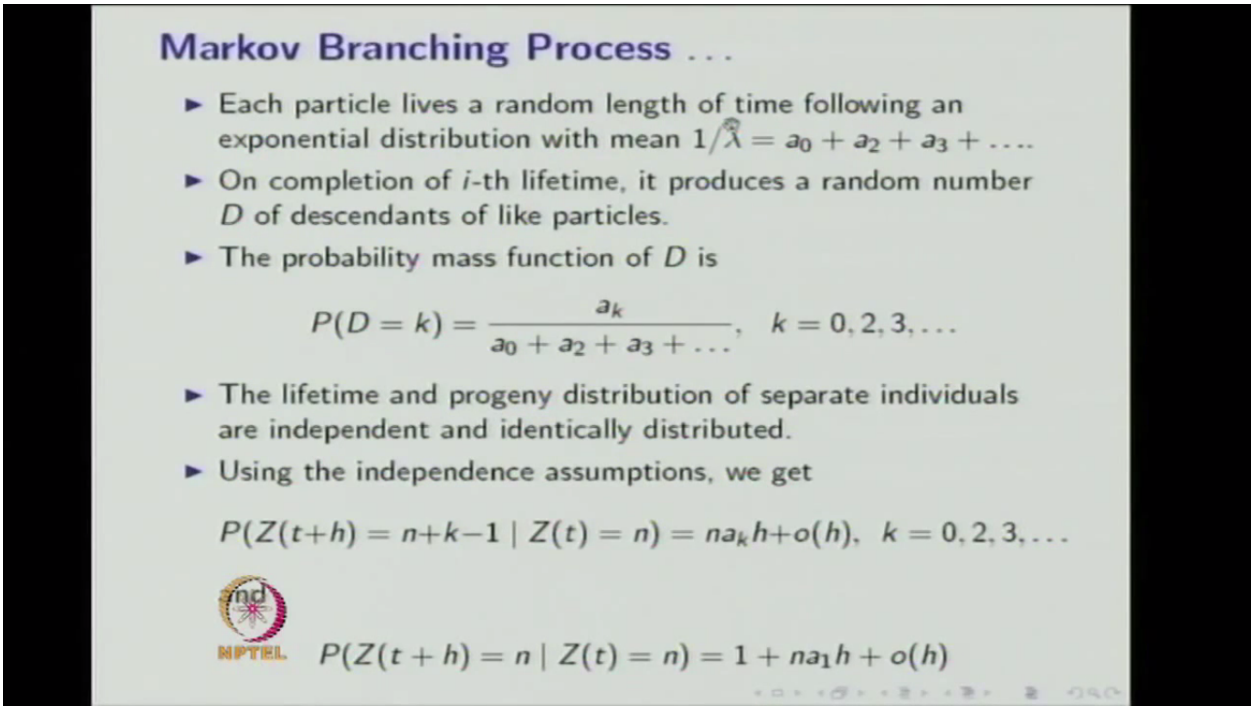


This is a stochastic processes module 9 branching processes. in the lecture one we have discussed a definition and the examples of branching processes. Important discrete type branching process. Galton-Watson process is discussed in detail. We found mean and variance of Galton-Watson process. Then we find the probability of extinction for the Galton-Watson branching process.

This is lecture 2. In this lecture we are going to discuss a Markov branching process. This is a very important branching process of a continuous type. This we are going to start with the probability generating function. Then we are finding the probability of extinction and we discuss the limit theorem. Finally, we are going to discuss some other important branching processes at the end.

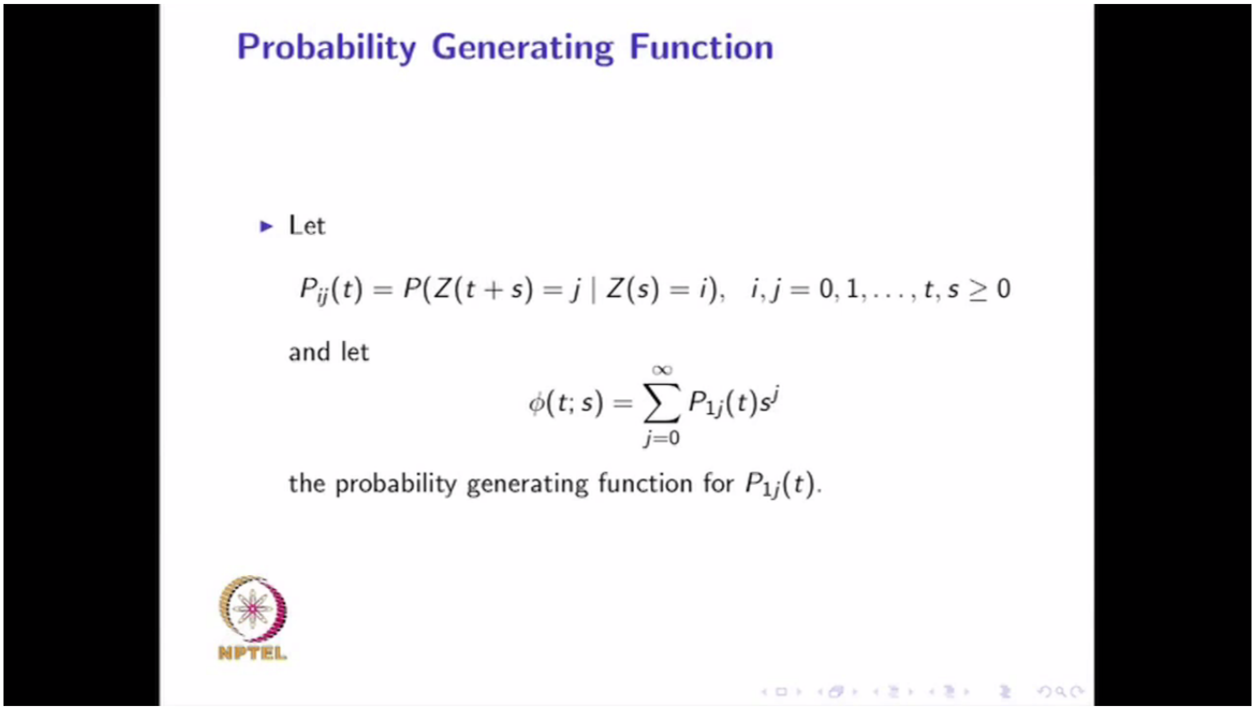


What is a Markov branching process? Let Zt be the number of particles at time t. The sequencer is Z of t the collection of random variables Z of t for t great or equal to 0 form a Markov branching process with the following assumptions. Let delta 1,k plus ak times h plus order of h for k is equal to 0, 1, 2, and so on the process the probability that a single particle will split producing k particles during a small time interval t to t plus h of length h. Delta of 1,k is the chronicle delta function. Assume that a1 is less than or equal to 0 for k 0, 2, 3, and so on ak's are great or equal to 0 and summation of ak's starting from 0, 1, 2, and so on that will be 1, that will be 0. We further postulate that individual particles act independently of each other always governed by the infinitesimal probabilities. Note that we are also assuming time homogeneity for the transition probabilities since ak is not a function of time at which the conversion of split occurs. Since ak is not a function of the time at which the split occurs.

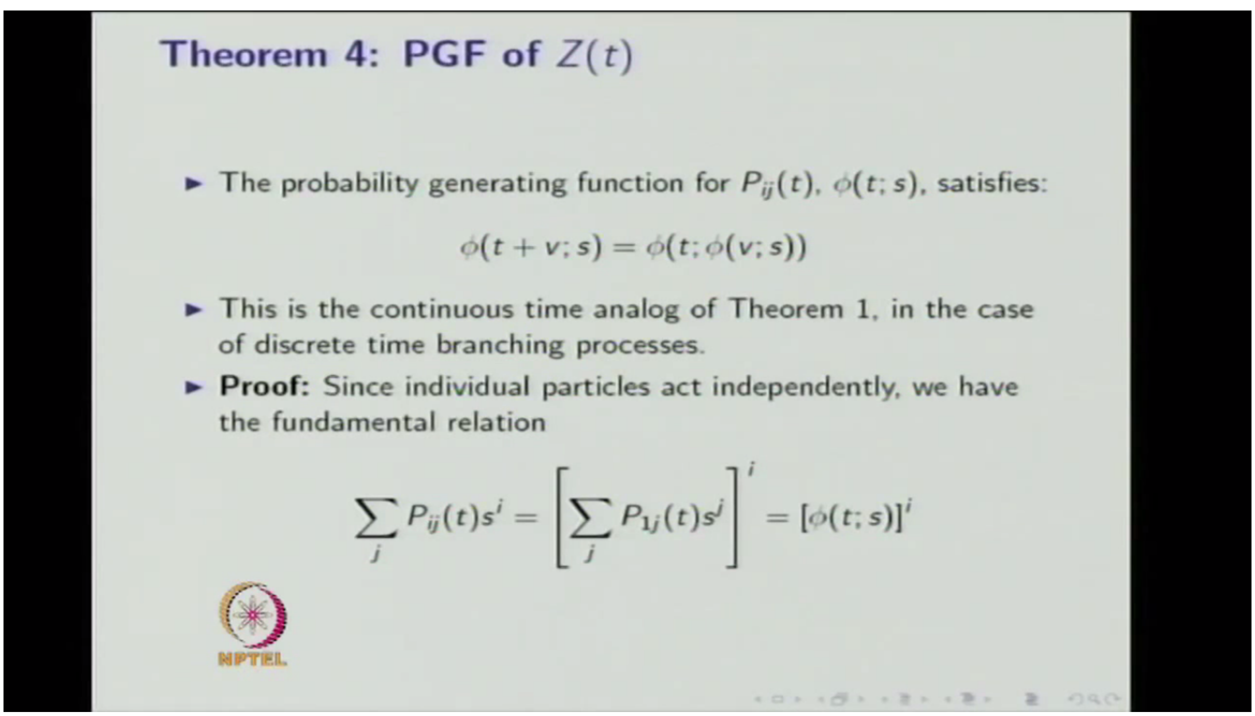


The similar assumptions we have taken care in the discrete type of branching process also. Each particle leaves a random length of time following an exponential distribution with the mean 1 by lambda that is same as a naught plus a2 plus a3 and so on. On the completion of ith lifetime it produces a random number D of descendant of life particles. The probability mass function of D is probability that D is equal to k will be a suffix k divided by a naught plus a2 plus a3 and so on. The lifetime and the progeny distribution of separate individuals are independent and identically distributed. Using the independent assumptions we get the conditional probability of Z t plus h is equal to n plus k minus 1 given the Z of t was n that is same as n times a suffix k of h plus order of h. We know that a small order of h means small order of h divided by h tends to 0 as h tends to infinity as h tends to 0. Order of h divided by h tends to 0 as h tends to 0 and for k equal to 1 the probability of Z of t plus h is equal to k given Z of t is equal to n probability of Z of t plus h is equal to n given Z of t is equal to n that will be 1 plus n times a1h plus order of h. So for k 0, 2, and 3 we have a separate expression. For k equal to 1 we have a different expression.

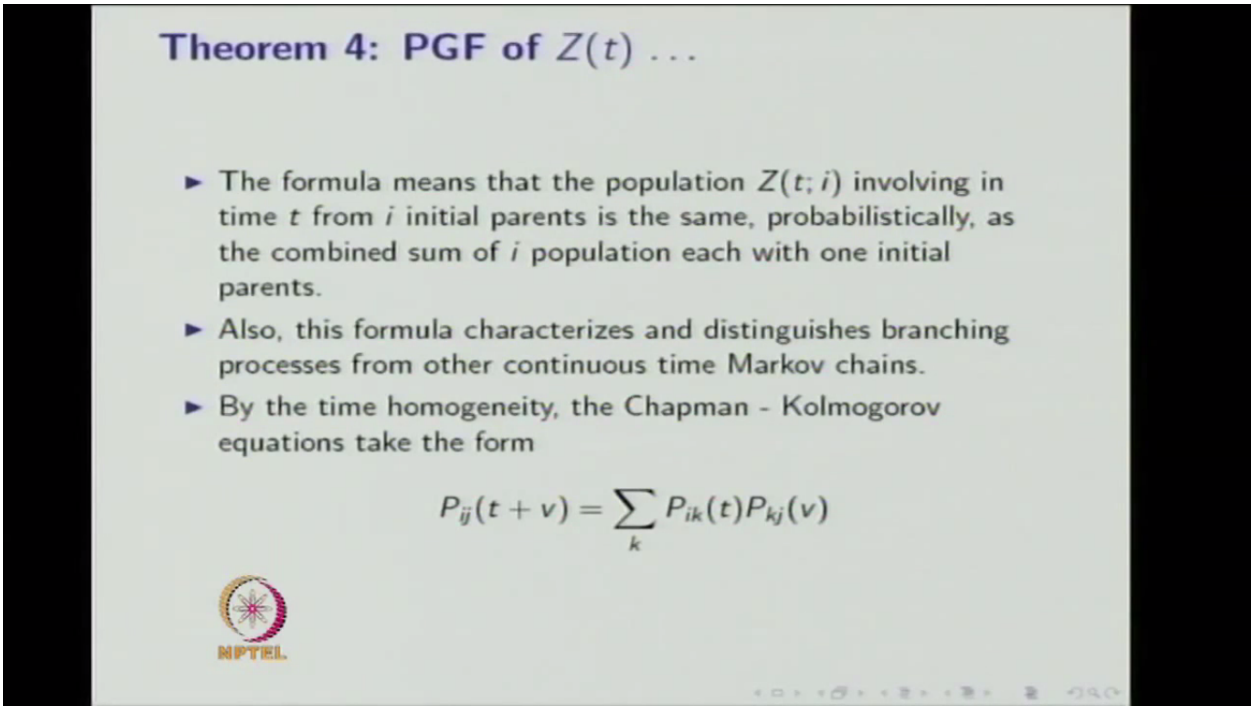
Now using the conditional probability we are going to define the probability generating function.



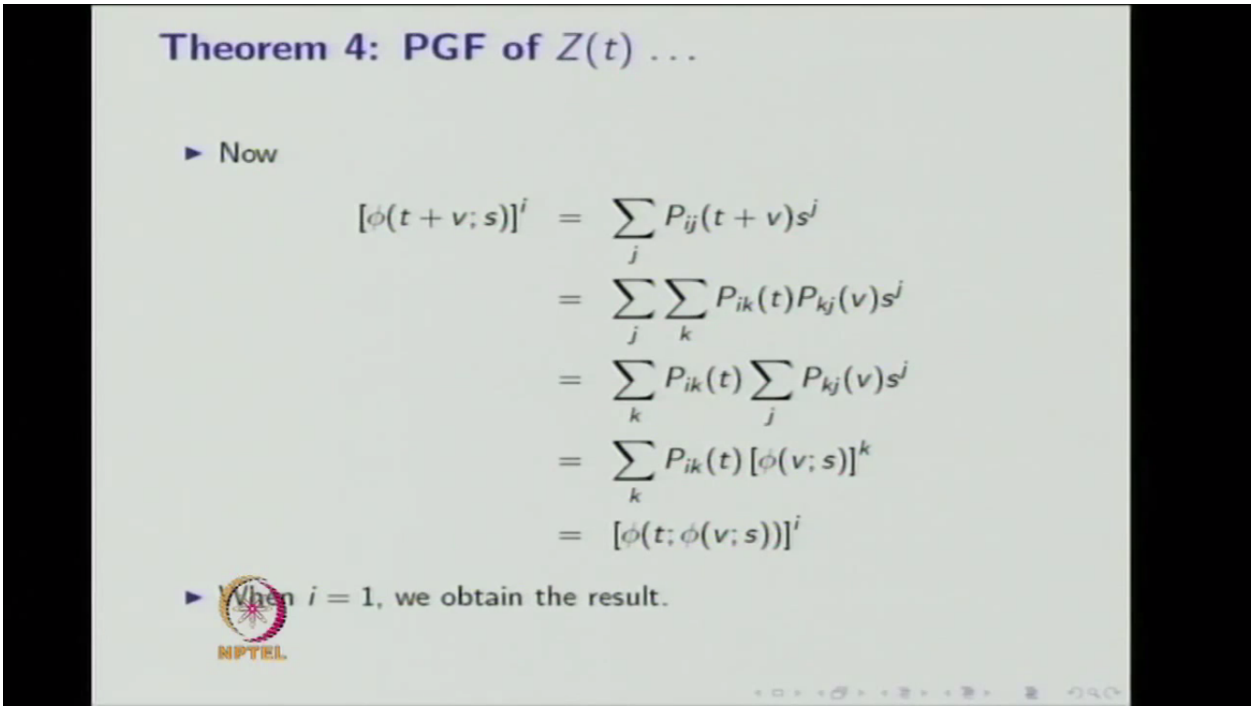
Now let Pi,j of t is nothing but the conditional probability of a probability of Zt plus S is equal to j given Z of s was i. Using these we define the probability generating function that is nothing but Psi of t,s the two variables summation over j Pij of t s power j. This will be the probability generating function for Pij of t where Pij of t is a transition probabilities.



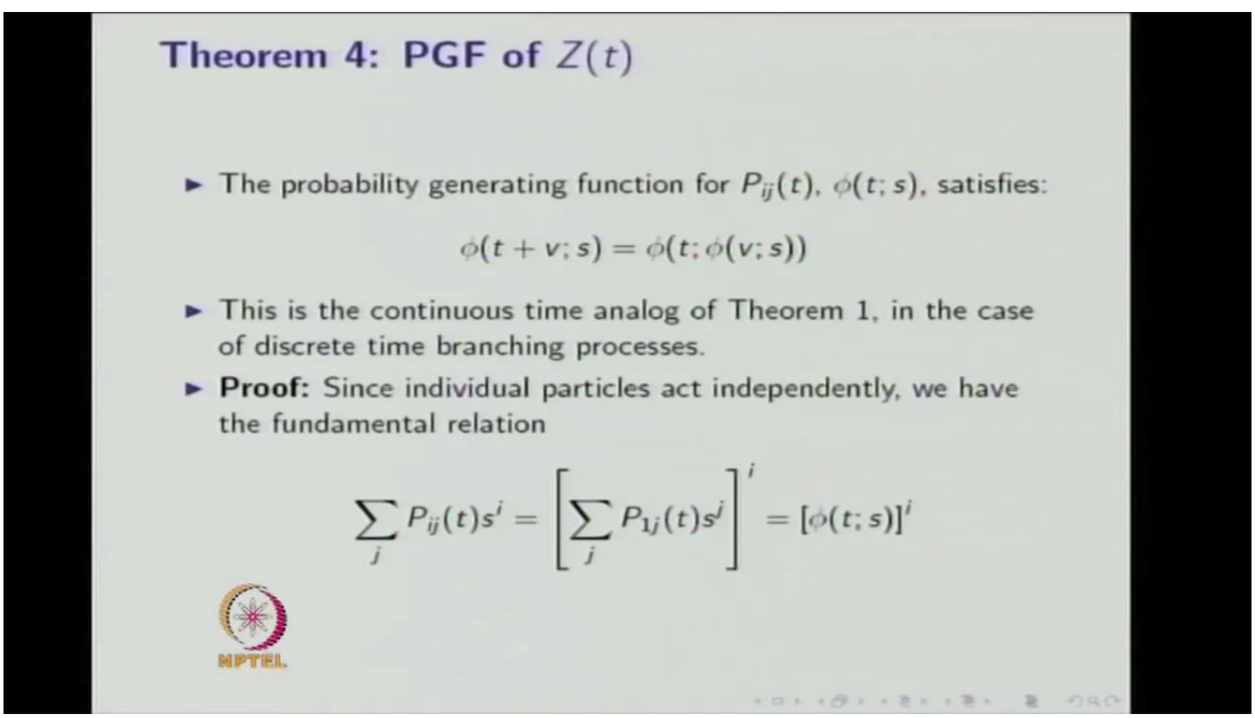
Now we are going to discuss the probability generating function of Z of p in the theorem four. Already first three theorems are discussed for the discrete type of branching process. So here we are going to discuss the fourth theorem. the probability generating function for Pij of P there is [Indiscernible] [00:07:57] t,s satisfies Psi of t plus v comma s that is same as Psi of t, Psi of v,s. This is a continuous time analog of theorem 1 in the case of discrete time branching processes. We will discuss the proof. Since individual particles act independently we have the fundamental relation the probability generating function for Pij of t that is nothing but summation over j instead of the transition probability i to j it is a transition probability of 1 to j of t s power j the whole power i that is same as the probability generating function power i.



The reason is the formula means that the population Z of t, i involving in time t from i initial parents is the same probabilistically as the combined summer of i population each with one initial parents. Therefore, the left hand side is the probability generating function for Pij of t that is same as making summation over j the probability transition probability of P 1,j of t s power the power i. That is nothing but the probability generating function for Pij power i. Also this formula characterizes and distinguishes branching processes from other continuous-time branching continuous-time Markov chains. By the time homogeneity the Chapman Kolmogorov equations take the form the one step transition probability of Pi,j t plus v can be returned in the form of summation k Pi to k of t then P k to j of v because it satisfies the time homogeneity one can write the Chapman Kolmogorov equations. Because this is a Markov branching process.



Now the probability generating function of the time t plus v the whole power i that is same as the summation j Pij of t plus v power j. So using Chapman Kolmogorov equation you can write the Pij of t plus v is a summation over k Pik of t Pkj of v. That is same as summation over k Pik of t summation over j Pkj of v s power j. We know that this is nothing but the probability generating function for Pkj of v that is same as the probability generating function of v,s power k and this will be written as the probability generating function of Psi of t, Psi of v,s the whole power i.



When you substitute i is equal to 1 we get the result because Psi of t plus v, s is same as Psi of P Psi of v,s . when you substitute i is equal to 1 in this equation you will get Psi of t plus v,s is same as Psi of t, Psi of v,s whole power i. When i is equal to 1 you will get Psi of t plus v,s is same as Psi of t, Psi of v,s.