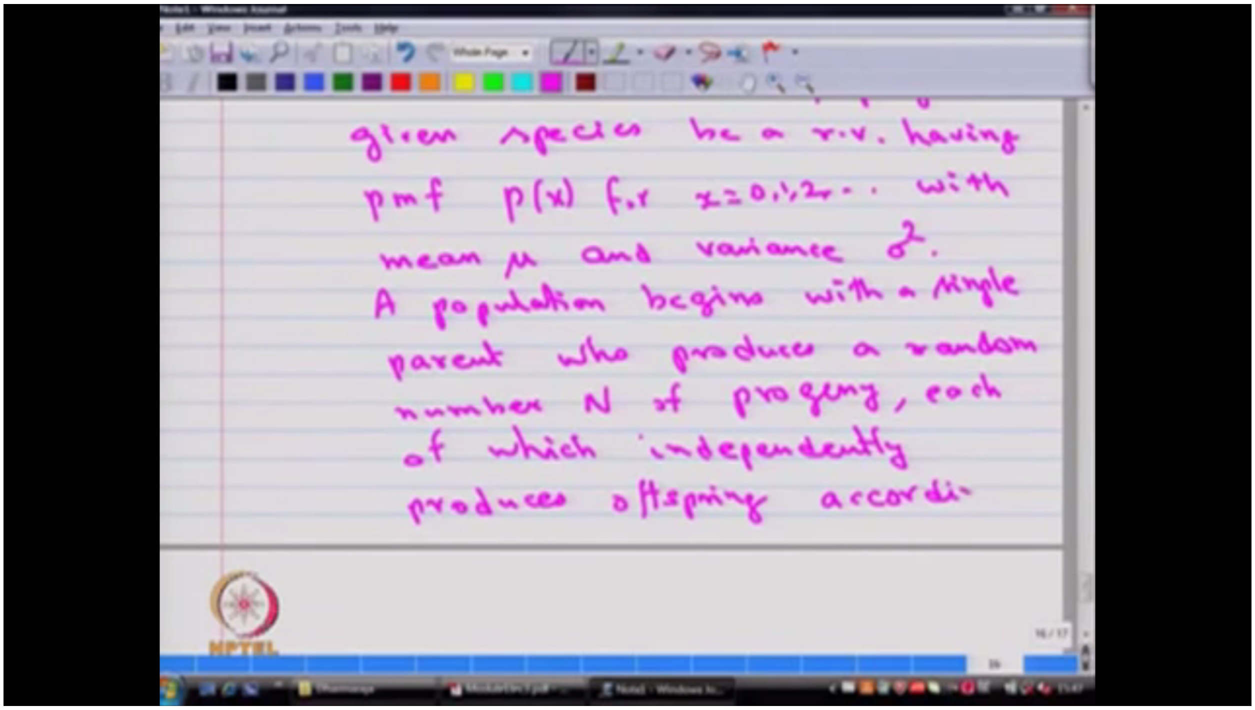
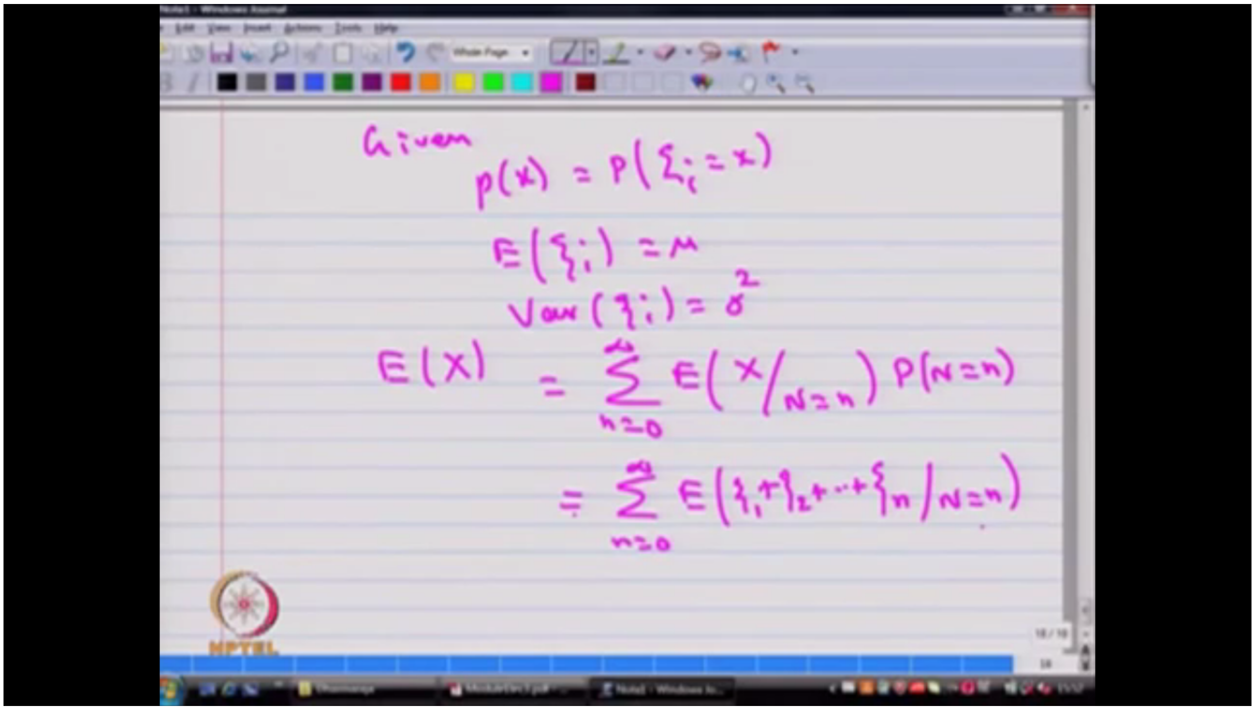
The next example. Let the number of offspring of a given species be a random variable having probability mass function P of X for X equal to 0, 1, 2 and so on. With mean mu and variance square. A population begins with single parent who produces a random number say N of progeny each of which independently produces of spring according to P of X that is a probability mass function to form a second generation. Find the mean and variance of number of descendants in the second generation.




So let me read out the question again that the number of offspring of a given species be a random variable having a probability mass function P of X for X is equal to 0, 1, 2, and so on with the mean mu and the variance Sigma square. A population begins with the single parent who produces a random number say N of progeny each of which independently produces offspring according to P of X to form a second generation. Find the mean and variance of number of descendants in the second generation.

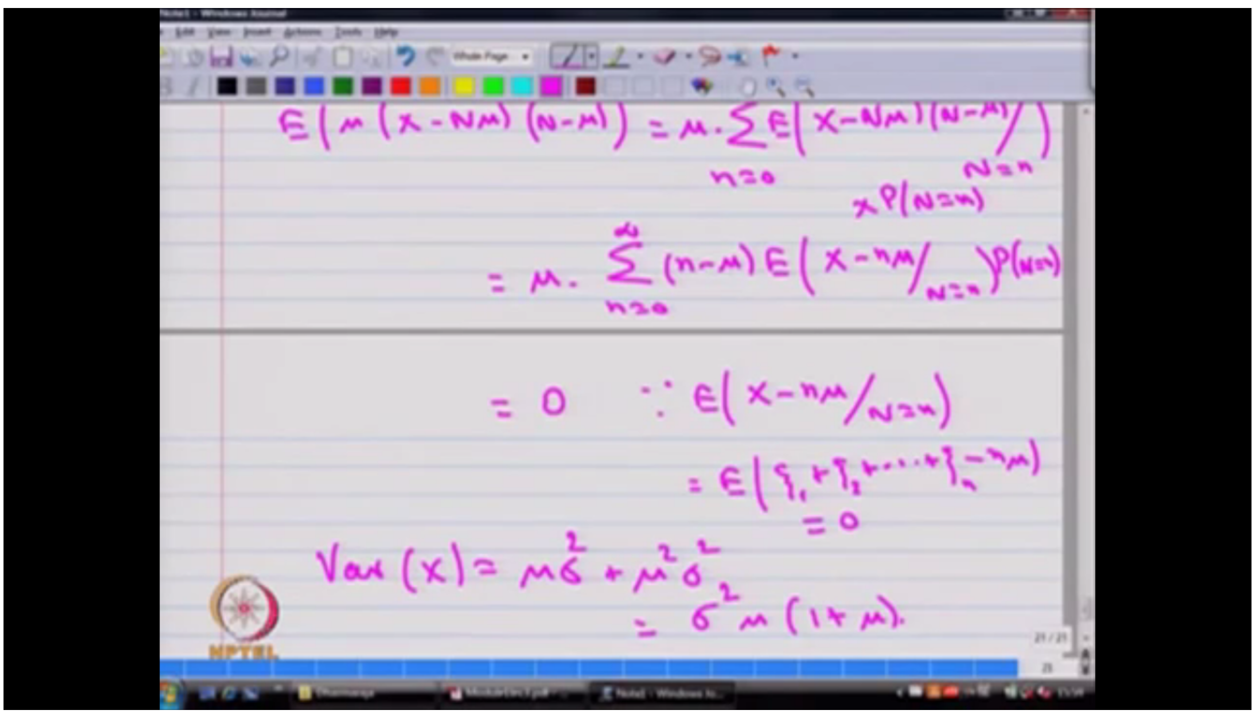
Let X is psi 1 plus psi 2 plus psi suffix N where psi i is the number of progeny of the ith offspring of the original parent. Given the P of X is nothing but the probability mass function for the random variables psi i's and also the expectation of psi i's that is equal to mu and variance of psi i's that is equal to Sigma square. Psi i's are iid random variables. Our interest is to find out what is the expectation of X where X is -- our X is a psi 1 plus psi 2 plus and so on till psi suffix N. Here N is also a random variable that is important.




Therefore, this expectation can be computed as N is equal to 0 to infinity the conditional expectation of X given N takes a value small n multiplied by the probability mass function of capital E that is same as summation n is equal to 0 to infinity that is same as expectation of psi 1 plus psi 2 and so on plus psi suffix n given N and takes a value n multiplied by the probability of N takes a value n. That is same as you know that expectation of psi i's are mu, therefore this becomes mu can be taken out then summation n is equal to 0 to infinity n times the probability of N takes a value n that is same as mu and the expectation of the random variable N that is also mu therefore it is mu into mu that is equal to mu square. Now we can compute the variance of X the same way that is expectation of X minus mu square because the random variable expectation is mu square, the whole square. Further this can be simplified by expectation of X minus N times mu plus N times mu minus mu square the whole square. One can expand therefore, you will get it is expectation of X minus N times mu the whole square plus expectation of mu square multiplied by N minus mu the whole square and the third term becomes 2 times expectation of mu multiplied by X minus Nmu into N minus mu by taking mu outside. Now you can evaluate the first quantity that is expectation of X minus N mu whole square using the conditional expectation by making summation n is equal to 0 to infinity expectation of X minus mu whole square given N takes a value n multiplied by the probability of N takes a value n. If you substitute the way you have done the expectation and so on finally you will get the answer that is Sigma square summation n is equal to 1 to infinity n times probability of N is equal to n. That is same as the summation n times Pn is same as the mean that is mu therefore this becomes mu times Sigma square. Even though I have skipped a one or two steps one can get a Sigma square summation n times Pn that is same as mu therefore it is mu Sigma square.

Similarly you can work out the second expectation that is an expectation of mu square multiplied by N minus mu whole square. That is same as mu square is constant. The expectation of N minus mu whole square that is nothing but variance of a random variable N. And the variance of a random variable N is Sigma square therefore it is mu square Sigma square. Now we have to evolve  the third expectation that is a expectation of mu times X minus Nmu multiplied by N minus mu. Here also one can use the conditional expectation that is mu times summation n is equal to 0 to infinity expectation of X minus Nmu multiplied by N minus mu condition N takes a value n multiplied by probability of N takes a value n. So if you simplify you will get a mu times summation n is equal to 0 to infinity n minus mu expectation of X minus y nmu given N takes value n multiplied by probability of N takes a value n. that is same as 0 because expectation of X minus nmu given N takes a value n that is nothing but expectation of psi 1, psi 2 and so on plus psi minus nmu that expectation quantity is going to be 0 because expectation of pis i's are going to be mu and this is expectation of psi 1 plus psi 2 and so on plus psi n minus n times mu therefore that becomes 0.

Hence the variance of X substitute all these three variance results, all these expectation results in the above this expression. Therefore, you will get a variance of X is going to be mu times Sigma square plus mu square Sigma square and third term is going to be 0 therefore, you will get a sigma square mu multiplied by 1 plus mu. This problem is useful in branching processes. Therefore I discussed in this lecture. Even though there are many more problems of a similar kind we have chosen a few problems for the illustrative  purpose and we are going to come across the similar problems in the course also, therefore I have chosen some five problems to discuss as an illustrative example.




Here is the reference for this lecture.