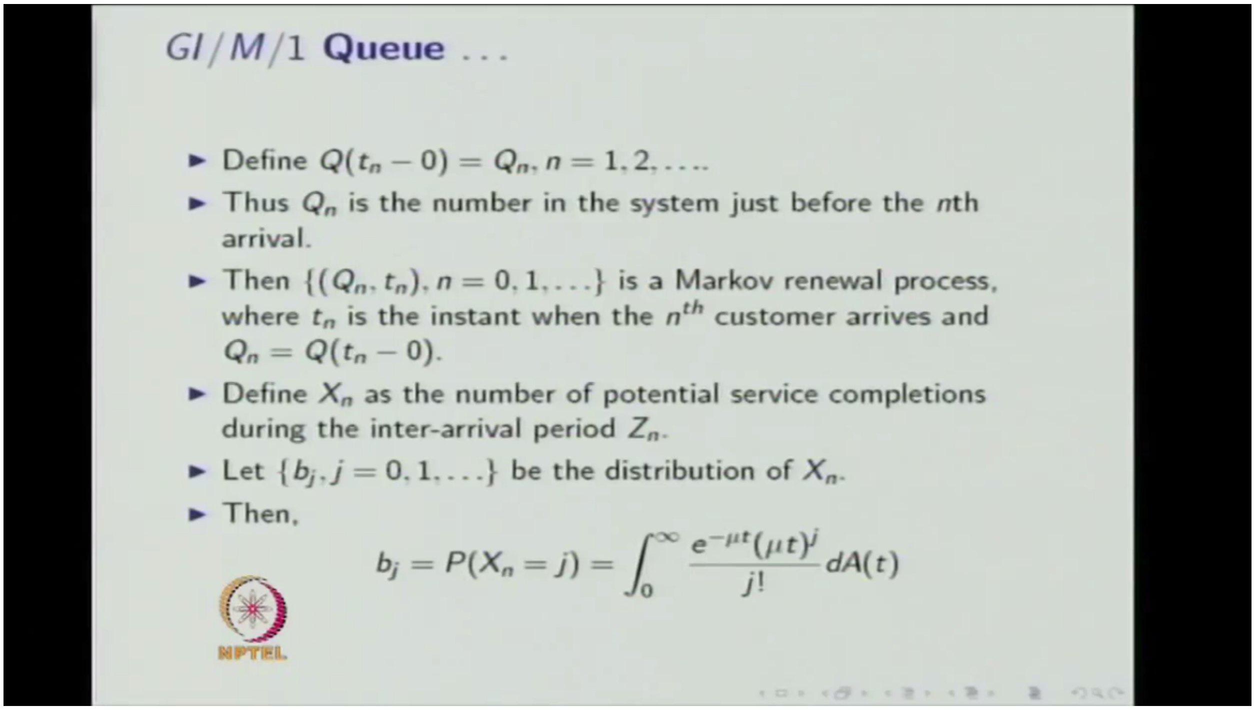
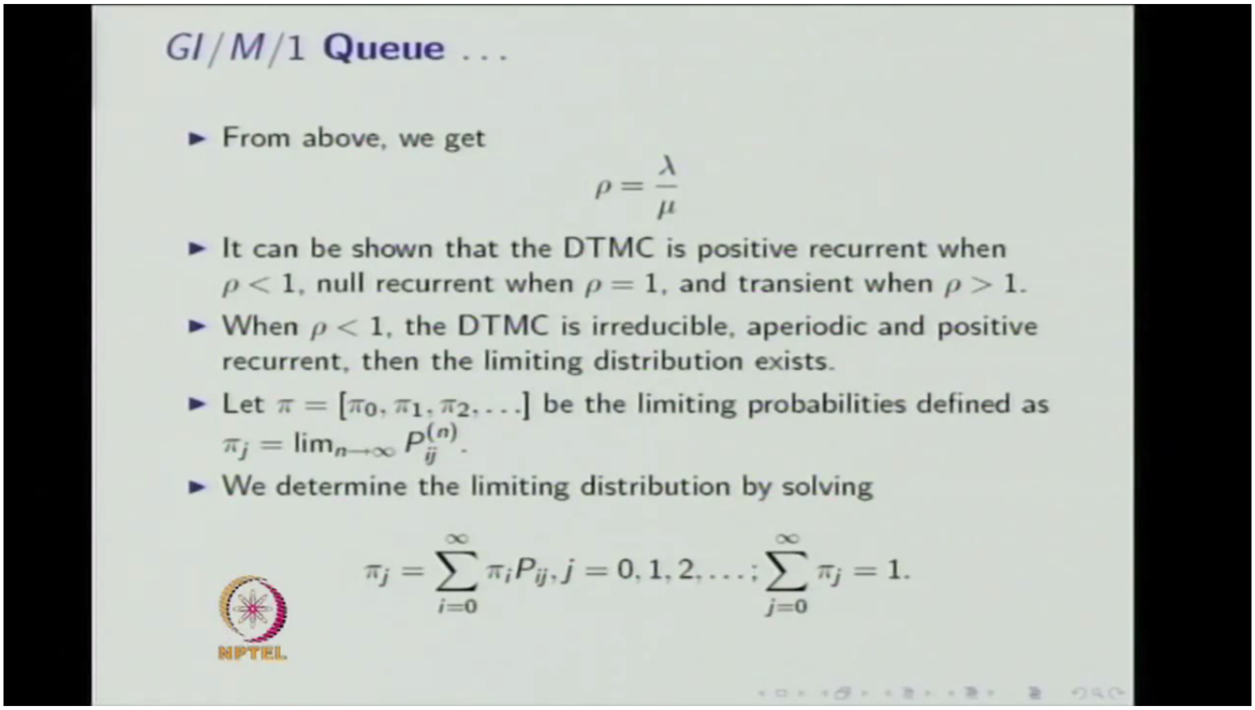
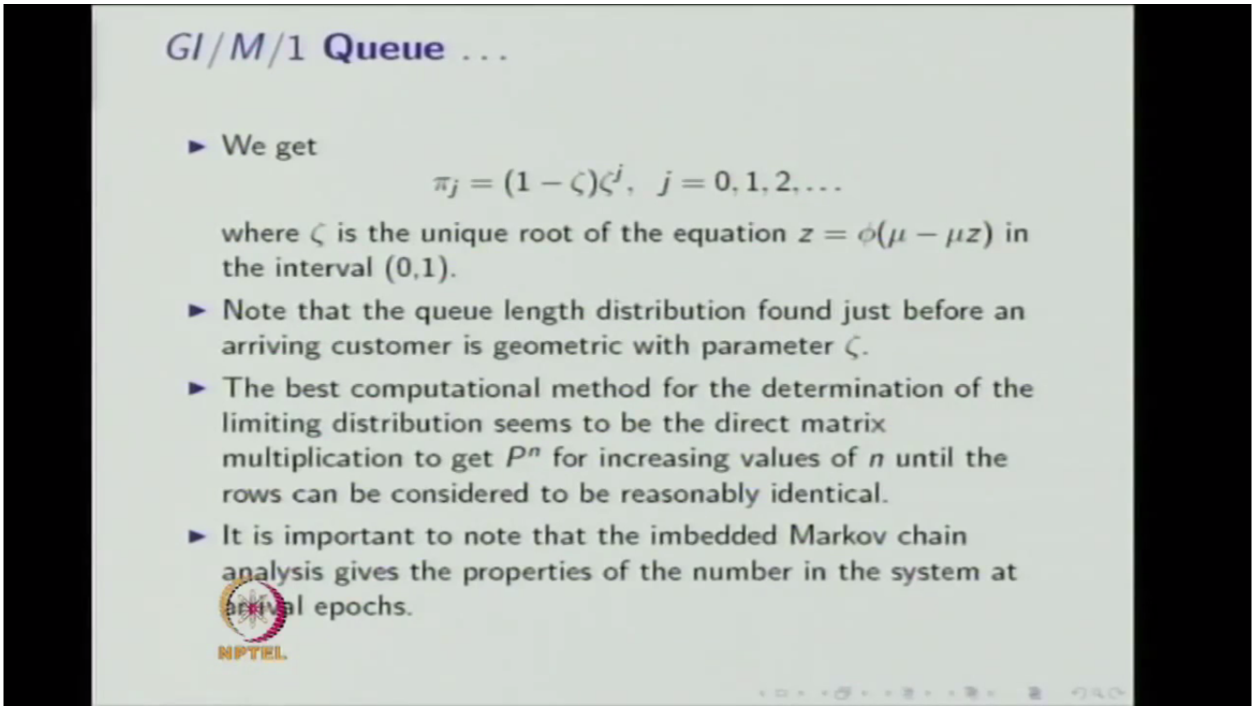


Mean number of inter-arrival time.

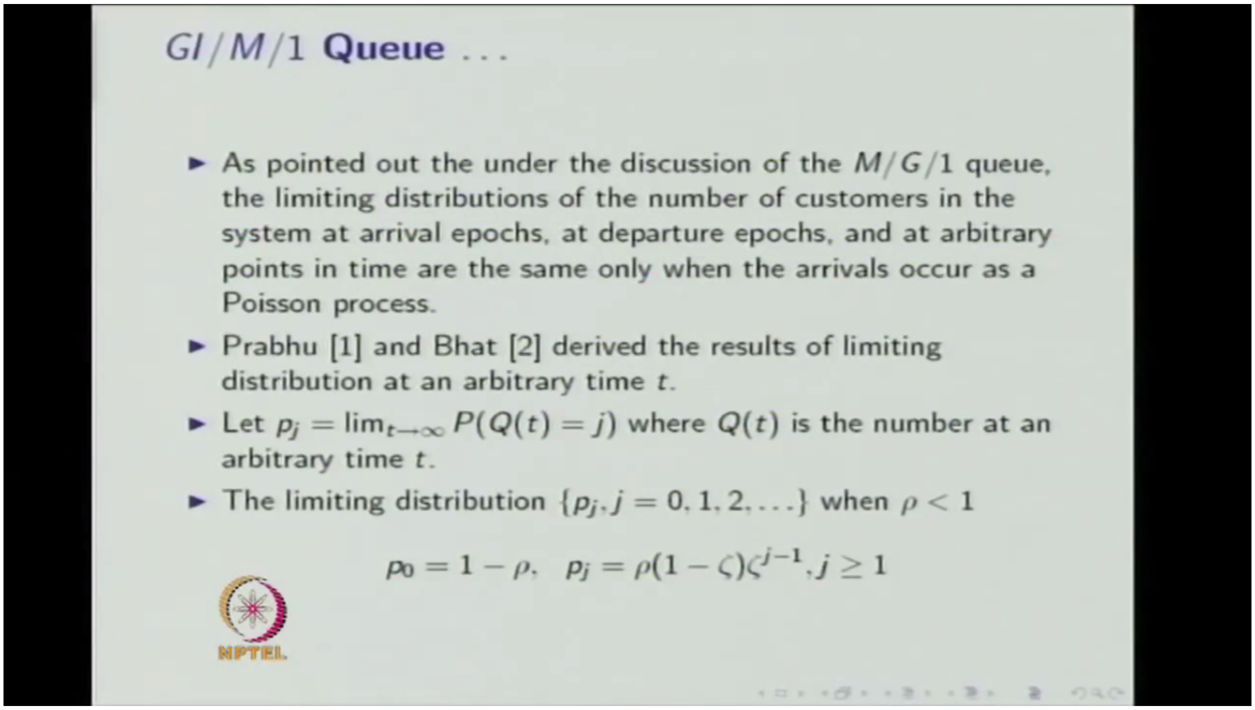
So that is nothing but the mu by lambda. Now we can define the traffic intensity that is nothing but Rho. Rho is equal to arrival rate divided by the service rate. From above you can get Rho is nothing but lambda by mu. It can be shown that the DTMC is a positive recurrent when Rho is less than 1. We have already made it it is irreducible and a periodic now we are giving the condition for a positive recurrent when Rho is less than 1 given DTMC is a positive recurrent. If Rho equal to 1 it is a null recurrent. If Rho is greater than 1 then it will be a transient. That means when Rho is less than 1 all the states are positive recurrent. Therefore the DTMC is said to be a positive recurrent. Similarly when Rho is 1 all the states are a null recurrent therefore the DTMC will be a null recurrent and similarly for Rho is greater than 1.



So our interest is to find out the limiting distribution. So when Rho is less than 1 the DTMC is irreducible a periodic and positive recurrent along with the condition b naught is greater than 0 and b naught plus b1 is less than 1. with this condition it is reducible. With Rho is less than 1 it is a positive recurrent we can easily verify it is a periodic. Hence , the limiting distribution exists and it is unique and that will be independent of initial stat i. Therefore, Pij will be a limit n tends to infinity the probability of i to j in n steps. So define the limiting distribution probability vector as a Pi it's consists of Pi naught, Pi1, Pi2 and so on where Pij's are defined in this form limit n tends to infinity P of i to j in n steps. We determine the limiting distribution by solving Pi is equal to Pip and the summation of Pi's is equal to 1. So the first one is a homogeneous equation including this normalizing condition you have a system of non-homogeneous equation. So all this non-homogenous – so all these system of non-homogeneous equation to get the limiting probabilities. We get the limiting probabilities are one minus Psi, Psi power j where Psi is the unique root of the equation Z is equal to the Laplace stieltjes transform of the inter-arrival distribution with the variable mu minus mu Z in the interval 0 to 1.

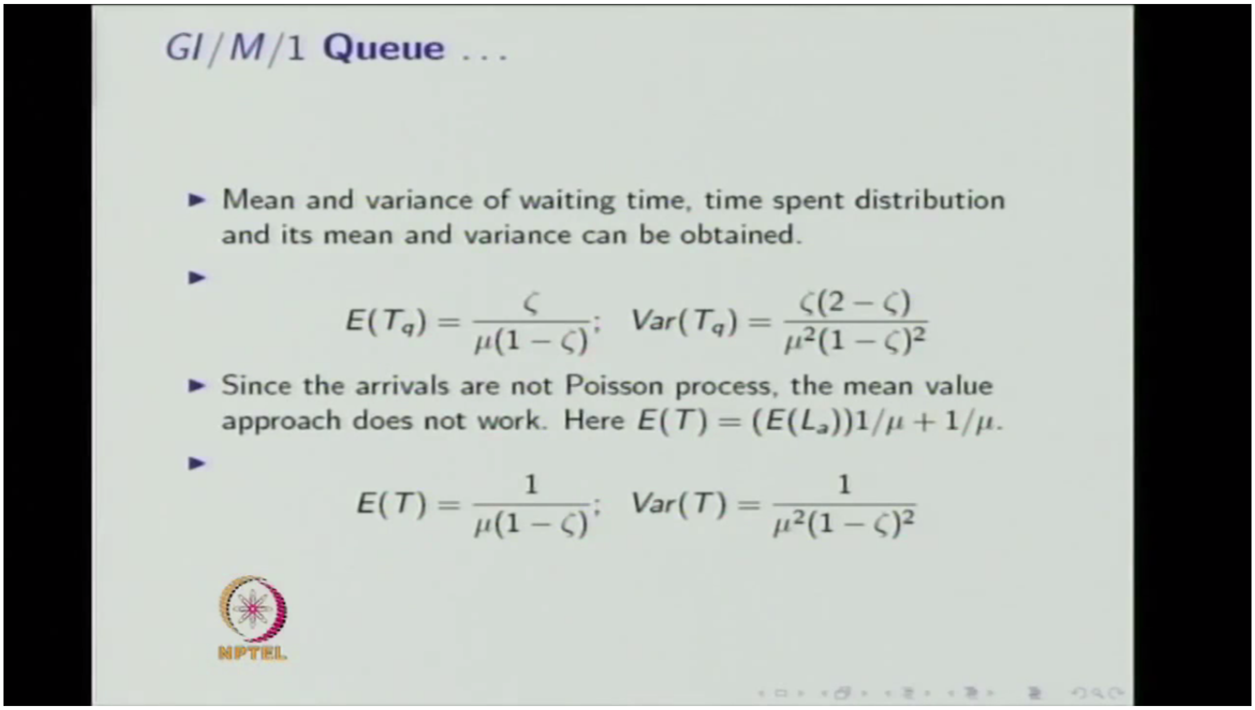


That means first you have to solve the equation Z is equal to Psi of mu minus mu Z and you know what is a Psi of mu minus mu Z from the beta Z. Solve the equation in the interval 0 to 1. So the unique root you have to substitute as a Psi then substitute Psi in the Pij and that will be the – since the unique root is in the interval 0 to 1 therefore the Pij 1 minus Psi Psi power j that will form a probability mass function for the limiting Rho, limiting distribution. Note that the queue length distribution found just before arriving customer is geometric distribution with the parameter Psi. The limiting distribution which we got it that is just before arriving customer the queue side the queue length distribution found just before an arriving customer which is a geometric distribution with the parameter Psi. The best computational method for determination of the limiting distribution seems to be their direct matrix multiplication to get P power n for increasing values of n until the Rhos can be considered to be reasonable identical. Whenever for a larger n P power n has the identical rows it means the limiting distribution exists. Therefore, the best computation method is find the P power n for larger n until the rows can be considered to be a reasonably identical. It is important to note that the embedded Markov chain analysis gives the properties of the number of -- number in the system at arrival epochs. The Q of t for t greater than or equal to 0 that is a discrete state continuous time stochastic process whereas Qn for n greater – n is equal to 0, 1, 2, and so on that is a embedded time homogeneous discrete time Markov chain and that is Qn is nothing but Q of tn minus 0 that is nothing but the number of customers in the system just before the nth arrival. Hence, it is important to note that the embedded Markov chain analysis gives the properties of the number in the system at arrival epochs not the departure epochs or not at the arbitrary time instants. It gives the embedded Markov chain analysis gives the properties of the number in the system at only at the arrival epochs. As pointed out the under – the discussion of MI/M/1 queue the limiting distribution of the number of customers in the system at arrival epochs at departure epochs and at arbitrary points in time are the same only when the arrivals occurs as the Poisson process. So in the MI/G/1 queue the limiting distribution of the number of customers in the system at arrival epochs, departure epochs and arbitrary time points are same because the arrival occurs in the Poisson process for inter arrival follows independent exponential distribution with the same parameter whereas in the GM GIM/1/ queue model the embedded Markov chain results gives the limiting distribution of the number of customers in the system at the arrival epochs only. That is not same as the limiting distribution at the departure epochs, that is also not same as the arbitrary time points.



Now we are finding - now we are going to discuss the limiting distribution at the arrival – at the arbitrary time points. Prabhu and Bhat derived the results of limiting distributions at a arbitrary time t. Let Pj is the probability that limit t tends to infinity the probability Qt is equal to j. So this is nothing to do with the embedded Markov chain Qn. We are finding limit at t tends to infinity probability that Qt is equal to j where Qt is a number of customers in the system at arbitrary time t. The limiting distribution exists whenever the Rho is less than 1 and probability that no customer in the system in a long-run are the limiting in a long-run that P naught is equal to 1 minus Rho and the Pj is equal to Rho times 1 minus Psi Psi power j minus 1 for j is equal to 1, 2 and so on. So this is the limiting distribution at arbitrary time.

To determine the distribution of waiting time of a customer we need a distribution of a number of customers in the system at the time of that customers arrive if you want to find out the distribution of waiting time. Replace Rho by Psi in the MM1 results since the queue length the distribution is same. Our interest is to find out the waiting time distribution since the limiting time distribution at arrival epochs is the same as the limiting distribution of MM1 queue model. Therefore, you can replace Rho in the MM1 results by Psi to get queue length distribute, to get the waiting time distribution. Therefore, by replacing Rho by Psi in the moment results of waiting time distribution you get the waiting time distribution for GI/M/1 queue as the CDF of waiting time distribution is 1 minus Psi times e power minus mu times 1 minus Psi times t for t greater than or equal to 0. For t is less than 0 it will be 0. So this is the waiting time distribution for GI/M/1 queueing system.



First we found the limiting distribution at the arrival epochs. Next we find the limiting distribution. Next we discuss the limiting distribution at the arbitrary time points from the Prabhu and Bhat results. Then we have discussed the waiting time distribution. Now we are discussing the moments of queue Psi. The mean and variance of waiting time, time spent distribution and its mean and variance can be obtained. Once we know the limiting distribution at the arrival epochs as well as you know the waiting time distribution you can find the mean and variance of waiting time by adding mean, you can find the mean of a time spent also. You cannot use the mean value approach because the arrivals are not Poisson process. So you can find out the average time spent in the system that is E of t will be average number of customers seen by at the arrival epochs that is ELa multiplied by 1 by mu plus the average service time that is 1 by mu will give the average time spent in the system. Since the arrivals are not Poisson processes, not Poisson process we cannot use the mean value approach. By simplification you can get already we know what is the average – what is the distribution of number of customers seen at the arbitrary, sorry, already we know the limiting distribution at the arrival epochs. So we can find the mean from those results. Then multiply by 1 by mu plus 1 by mu will give the average time spent in the system. Then you can find the variance of time spent in the system also. The first this one the mean and variance of waiting times since you know the waiting time distribution you can find the mean and the variance.